

Research on field optimization model of heliostat based on ray tracing

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Abstract. In this paper, the average optical efficiency and output power of the tower solar thermal power station are studied. The ray tracing method is used especially to the calculation process and details of the shadow blocking efficiency and collector truncation efficiency are carefully derived. Combined with intelligent optimization algorithm, the key parameters [1][2] of the mirror field layout are optimized. Then, the main method used in this paper is ray tracing. For the calculation of cosine efficiency and shadow blocking efficiency, the coordinate transformation matrix is established in this paper, which simplifies the coordinate calculation. For the calculation of collector truncation efficiency, based on Monte Carlo simulation, and an accurate collector truncation efficiency model is established by random sampling of large samples. Finally, MATLAB software is used to calculate the efficiency of the heliostat in different coordinate positions. The average optical efficiency (62.71%), cosine efficiency (75.65%), shadow blocking efficiency (94.54%), collector truncation efficiency (98.85%), thermal output power (34.77MW), and average annual thermal output power per unit area mirror (0.5535kW/m²) are obtained. This model not only helps to understand the propagation mode of light in the heliostat field, but also can quantitatively evaluate the loss of light.

Keywords: Optical Efficiency, Coordinate Conversion, Ray Tracing, Monte Carlo Simulation.

1. Introduction

With the increasingly serious energy depletion, climate change and environmental pollution caused by the use of fossil energy, clean energy has been widely valued by the world, and gradually has the trend of replacing traditional fossil energy. For example, in 2019, the power generation of clean energy in the UK exceeded that of fossil fuel for the first time, becoming the largest power source in the UK. At present, China is the country with the largest renewable energy generation, but the proportion of renewable energy is not high. Due to the existence of the thermal storage system, the solar thermal power generation has many advantages such as good power quality, stable, continuous and adjustable output. The development of solar thermal power generation has important practical significance for promoting the development of the western region, stabilizing economic growth, cultivating the solar thermal industry chain, peaking and low power abandonment rate, and improving the power grid's ability to absorb the scenery. At the same time, it is also an important measure for China to achieve the goal of "carbon peak" and "carbon neutrality". This paper gives a kind of tower solar thermal power generation device, which is composed of heliostat, absorber and collector[3], through sunlight irradiation heliostat, heliostat will reflect sunlight and converge to the collector at the top of the absorption tower, by heating the heat conduction medium, the solar energy to electric energy conversion.

Based on the existing research results in China, this paper uses ray tracing method to model and analyze the light in the heliostat field. In this method, the rays are decomposed into multiple rays and their propagation paths are traced, so that a collector truncation efficiency model is established. Next, based on the results of the ray tracing method, the loss of light can be quantitatively assessed, especially the collector truncation efficiency. This model can help to understand how light propagates in the heliostat field, as well as to evaluate the collector's performance. In addition, since in heliostat systems, the tower structure will have a shadow effect on the mirror field. In order to analyze the source of energy loss, the shadow effect of the tower structure on the mirror field is fully considered in this paper, and a detailed shading efficiency model is established. At the same time, the energy loss

of the rear heliostat may be caused by the block of the front heliostat when receiving and reflecting sunlight. In order to analyze the energy loss more comprehensively, a model is also established to consider the mutual blocking effect between heliostat.

2. The model is built and solved

2.1. Research on coordinate transformation system

In this paper, the conversion system is mainly used to calculate cosine efficiency, shadow blocking efficiency and so on. The concrete implementation is as follows: The mirror space rectangular coordinate system is established on a mirror $O_m - X_m Y_m Z_m$, which is the center of the mirror O_m , the X_m axis is parallel to the ground, the Y_m axis is perpendicular to the X_m axis and parallel to the mirror, and the Z_m axis is perpendicular to the mirror, as shown in Fig 1:

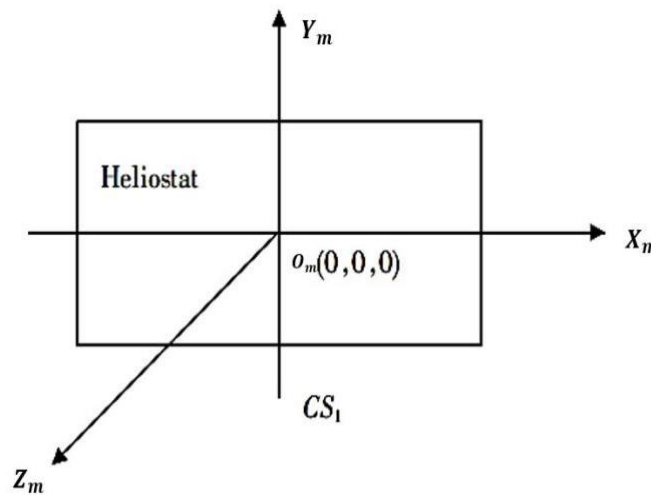


Fig. 1 Mirror coordinate system schematic

If (l_x, m_x, n_x) , (l_y, m_y, n_y) , (l_z, m_z, n_z) is the vector representation of X_m axis, Y_m axis and Z_m axis in the mirror coordinate system in the ground coordinate system, then there is a relational unit matrix T :

$$T = \begin{pmatrix} l_x & l_y & l_z \\ m_x & m_y & m_z \\ n_x & n_y & n_z \end{pmatrix} \quad (1)$$

A matrix T can be used to convert expressions of points or vectors in two coordinate systems. For example, for a point, this expression is the point's representation in the mirror coordinate system is H_m and its representation in the ground coordinate system is H'_m .

Then there is:

$$H'_m = T \cdot H_m + O_m = \begin{pmatrix} x'_m \\ y'_m \\ z'_m \end{pmatrix} \quad (2)$$

For vectors, denote the vector of a beam of light in the mirror coordinate system as \vec{V}_m , and the vector in the ground coordinate system as \vec{V}'_m . Then there is the following transformation relationship:

$$\vec{V}'_m = T \cdot \vec{V}_m \quad (3)$$

Take the rotation mechanism of the traditional heliostat as an example, its coordinate transformation relationship is as follows:

$$T = \begin{pmatrix} l_x & l_y & l_z \\ m_x & m_y & m_z \\ n_x & n_y & n_z \end{pmatrix} = \begin{pmatrix} -\sin A_H & -\sin E_H \cos A_H & \cos E_H \cos A_H \\ \cos A_H & -\sin E_H \sin A_H & \cos E_H \sin A_H \\ 0 & \cos E_H & \sin E_H \end{pmatrix} \quad (4)$$

Where is the azimuth Angle of the heliostat A_H (starting from the due east direction, measured clockwise), E_H is the elevation Angle of the heliostat (the Angle between the heliostat plane and the horizontal plane), where $E_H \geq 0$:

Remember n as the normal vector of the heliostat, $n = (x_n, y_n, z_n)$, and the expression of the heliostat azimuth Angle is:

$$A_H = \arctan \left(\frac{-y_n}{x_n} \right) + \frac{\pi}{2} \left(1 + \frac{-x_n}{|x_n|} \right) \quad (5)$$

The expression of the heliostat azimuth Angle E_H is:

$$E_H = \arcsin(-y_n) \quad (6)$$

2.2. The calculation of the direction vector of the incident light

To obtain the direction vector of incident light (sunlight),

This article is recorded D as the number of days counting from the spring equinox as, the 0 day, for example, if the spring equinox is March 21, then April 1 corresponds $D = 11$. δ is the declination Angle of the sun, expressed as:

$$\delta = \sin^{-1} \left(\sin \frac{2\pi D}{365} \sin \left(\frac{2\pi}{360} 23.45 \right) \right) \quad (7)$$

Note ST as the local time in "hours", for example, 18:45 as $ST_{(18:45)} = 18.75$. ω is the solar hour Angle with the expression:

$$\omega = \frac{\pi}{12} (ST - 12) \quad (8)$$

Marked φ as the local latitude, then:

The sun's altitude Angle is α_s , expressed as:

$$\sin \alpha_s = \cos \delta \cos \varphi \cos \omega + \sin \delta \sin \varphi \quad (9)$$

γ_s is the sun azimuth Angle (starting from true north, measured clockwise). The expression is:

$$\cos \gamma_s = \frac{\sin \delta - \sin \alpha_s \sin \varphi}{\cos \alpha_s \cos \varphi} \quad (10)$$

\vec{V}_b is the incident light direction vector, the expression is:

$$\vec{V}_b = (\sin(\gamma_s) \cos(\alpha_s), \cos(\gamma_s) \cos(\alpha_s), \sin(\alpha_s)) \quad (11)$$

2.3. Cosine efficiency model

In this paper, the cosine efficiency is described as η_{cos} , the cosine efficiency measures the influence of the Angle between the surface normal of the subject absorbing or reflecting energy and the incident light on the energy conversion, and the cosine loss represents the degree of its performance reduction.

In this paper, the main body of absorbed or reflected energy is a heliostat, and there is n , a surface normal of the heliostat, then the expression of cosine loss is:

$$\text{Cosine loss} = n \cdot \vec{V}_b \quad (12)$$

The expression of cosine efficiency η_{cos} is:

$$\eta_{cos} = 1 - \text{Cosine loss} \tag{13}$$

2.4. Shadow occlusion efficiency model

$$\eta_{sb} = 1 - \text{Shadow occlusion loss} \tag{14}$$

Where η_{sb} is the shading efficiency[4].

Shadow occlusion loss[5][6] includes two parts: one is the occlusion between heliostat and the other is the occlusion of the heliostat by the absorption tower.

The occlusion between heliostat includes two parts.

In this paper, it is recorded b as a sunbeam directed at the mirror B , which should fall on the mirror B , but because the mirror A blocks the light b , a certain shadow area is generated on the mirror, as shown in Fig 2:

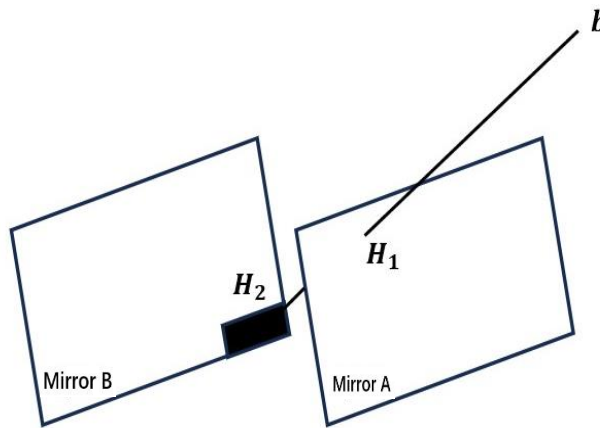


Fig. 2 Mirror A blocking the beam of light to mirror B

In order to study whether the above relationship exists, this paper makes the line b where the light is intersected with the plane where the mirror A is located at a point H_1 , and the plane where the mirror B is intersected at a point H_2 to study whether H_2 is within the scope of the mirror B . The research content is as follows:

① H_1 is the representation of the point in the mirror A coordinate system, which is converted to represent H'_1 in the ground coordinate system.

$$H'_1 = T \cdot H_1 + O_A = \begin{pmatrix} x'_1 \\ y'_1 \\ z'_1 \end{pmatrix} \tag{15}$$

② Convert H'_1 to the representation H''_1 in mirror B coordinate system.

$$H''_1 = T^T \cdot (H'_1 - O_B) = \begin{pmatrix} x''_1 \\ y''_1 \\ z''_1 \end{pmatrix} \tag{16}$$

③ Convert the direction vector of incident light b in the ground coordinate system to the mirror B coordinate system.

$$\vec{V}'_b = T^T \cdot \vec{V}_b = \begin{pmatrix} m_b \\ n_b \\ l_b \end{pmatrix} \tag{17}$$

④ In mirror B coordinate system, according to the principle of two points and one line, calculate the intersection point of light and B mirror.

The linear equation of the light:

$$\frac{x_2 - x''_1}{m_b} = \frac{y_2 - y''_1}{n_b} = \frac{-z''_1}{l_b} \quad (18)$$

Solved:

$$\begin{cases} x_2 = \frac{l_b x''_1 - m_b z''_1}{l_b} \\ y_2 = \frac{l_b y''_1 - n_b z''_1}{l_b} \end{cases} \quad (19)$$

⑤ Determine if H_2 is in mirror B .

$$\begin{cases} -3 \leq x_2 \leq 3 \\ -3 \leq y_2 \leq 3 \end{cases} \quad (20)$$

This paper is recorded b as a ray of sunlight directed at the mirror B , and the light reflected by it falling behind the mirror B is recorded as c , but because it is blocked by the mirror A , the light c ultimately cannot reach the collector at the top of the tower, as shown in Fig 3:

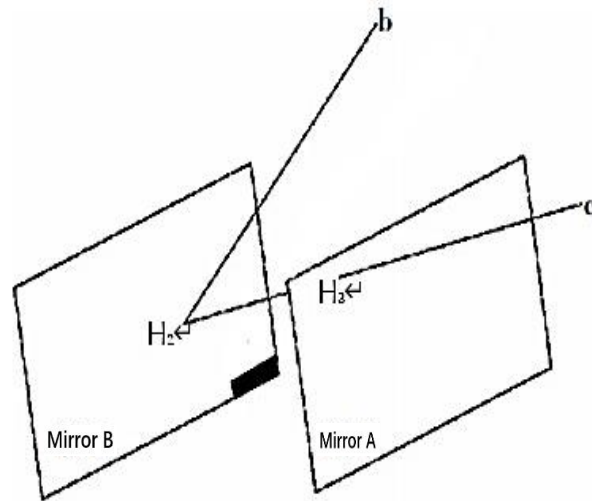


Fig. 3 Mirror A blocking mirror B reflected light schematic

In order to explore whether it is like the above situation, it is noted that the intersection point of the line where the light b is and the plane where the mirror A is H_3 , and the intersection point of the line where the light c is and the plane where the mirror B is H_2 , and whether the study H_2 is within the scope of the mirror B , the research content is as follows:

① H_3 is the representation of the point in the mirror A coordinate system, which is converted to represent H'_3 in the ground coordinate system.

$$H'_3 = \begin{pmatrix} l_x & l_y & l_z \\ m_x & m_y & m_z \\ n_x & n_y & n_z \end{pmatrix} \cdot H_3 + O_A = \begin{pmatrix} x'_3 \\ y'_3 \\ z'_3 \end{pmatrix} \quad (21)$$

② Convert H'_3 to the representation H''_3 in mirror B coordinate system.

$$H''_3 = \begin{pmatrix} l_x & l_y & l_z \\ m_x & m_y & m_z \\ n_x & n_y & n_z \end{pmatrix}^T \cdot (H'_3 - O_B) = \begin{pmatrix} x''_3 \\ y''_3 \\ z''_3 \end{pmatrix} \quad (22)$$

③ Convert the direction vector of incident light b in the ground coordinate system to the mirror B coordinate system.

$$\vec{V}'_b = \begin{pmatrix} l_x & l_y & l_z \\ m_x & m_y & m_z \\ n_x & n_y & n_z \end{pmatrix}^T \cdot \vec{V}_b = \begin{pmatrix} m_b \\ n_b \\ l_b \end{pmatrix} \quad (23)$$

④ The expression of the normal vector of the plane in which mirror B is located in mirror B coordinate system is as $N = (0,0,1)$, and the expression of the reflected light c about b in mirror B coordinate system is calculated.

$$\vec{V}'_c = \vec{V}'_b - 2 * (\vec{V}'_b \cdot N) * N \quad (24)$$

⑤ In the mirror B coordinate system, according to the principle of two points and one line, calculate the intersection of light and B mirror.

The linear equation of the light is as follows:

$$\frac{x_2 - x''_1}{m_c} = \frac{y_2 - y''_1}{n_c} = \frac{-z''_1}{l_c} \quad (25)$$

Solved:

$$\begin{cases} x_2 = \frac{l_b x''_1 - m_b z''_1}{l_c} \\ y_2 = \frac{l_b y''_1 - n_b z''_1}{l_c} \end{cases} \quad (26)$$

⑥ Determine if H_2 is in mirror B .

$$\begin{cases} -3 \leq x_2 \leq 3 \\ -3 \leq y_2 \leq 3 \end{cases} \quad (27)$$

2.5. Truncation efficiency model

The truncation efficiency of a tower type photothermal power station is defined as:

$$\eta_{trunc} = \frac{\text{Collector receives energy}}{\text{Mirror completely reflects energy} - \text{Shading loses energy}} \quad (28)$$

Because the truncation efficiency and cosine efficiency, atmospheric transmittance and so on reflect the effective degree of light and heat conduction in different stages, so "mirror total reflection energy" does not include the energy loss caused by oblique emission and atmospheric attenuation; "Mirror total reflection energy - shadow occlusion loss energy" should only include "can reach the energy near the collector".

In addition, because the sun is not a parallel beam, but a certain Angle of conical light, so after the mirror reflection, the beam will diverge, resulting in the reflected light divergence range with its travel distance increase and expand, if the collector from the heliostat too far, the reflected light will not be fully irradiated on the collector, and the light irradiated on the collector is the "collector received energy".

The divergence of the conical light and the reflected light caused by it is shown in Fig 4:

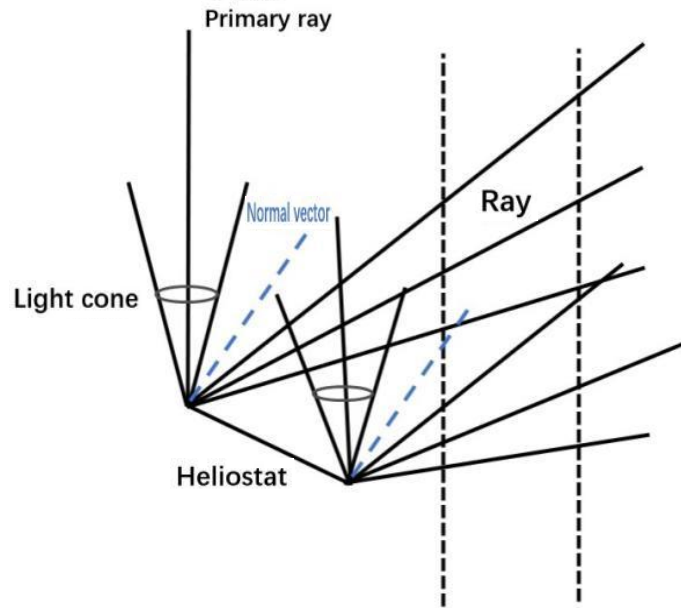


Fig. 4 Schematic diagram of the divergence of reflected light

Therefore, it is necessary to study the tapered light system before studying the phase efficiency model.

2.5.1 Solar Angle and coordinate conversion system

For the convenient representation of light in the light cone, a conical optical coordinate system is $O_l - X_l Y_l Z_l$, established in which the Z_l axis faces the center of the sun. X_l axis parallel to the ground, with the Y_l axis perpendicular to the Z_l and X_l axis.

The matrix transformation relationship between the conical optical coordinate system and the ground coordinate system is as follows:

$$T_l = \begin{pmatrix} l_x & l_y & l_z \\ m_x & m_y & m_z \\ n_x & n_y & n_z \end{pmatrix} = \begin{pmatrix} \sin \gamma_s & -\sin \alpha_s \cos \gamma_s & \cos \alpha_s \cos \gamma_s \\ -\cos \gamma_s & -\sin \alpha_s \sin \gamma_s & \cos \alpha_s \sin \gamma_s \\ 0 & \cos \alpha_s & \sin \alpha_s \end{pmatrix} \quad (29)$$

2.5.2 Monte Carlo simulation based on ray tracing

In this paper, Monte Carlo simulation[7] is used to randomly sample the reflection points, and ray tracing method is used to establish the reflected light expression corresponding to a certain light in the cone light, and explore whether the reflected light can reach the collector. Among them, Monte Carlo simulation needs to randomly sample four variables in the paper: the horizontal coordinate of the reflection point about the heliostat coordinate system, the longitudinal coordinate of the reflection point about the heliostat coordinate system, the incident Angle θ (the Angle between the incident light and the Z_l axis) and the incident light about the cone light coordinate system.

The size of the solar Angle is $32'$, that is $9.3mrad$, the half Angle of a beam of light cone is widened $4.65 mrad$, and the range of the incidence Angle is $0 \leq \theta \leq 4.65 mrad$, as shown in Fig. 5,when Monte Carlo simulation is carried out:

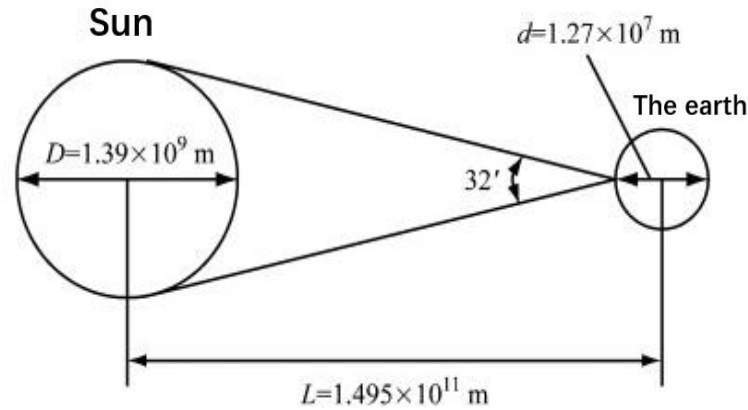


Fig. 5 Schematic diagram of the sun Angle

The coordinates of a reflection point obtained in the sampling in this paper is H_4 : After the sampling is completed, we start to track whether the reflected light of a certain ray d in the cone light reaches the collector. The research process is as follows:

- ① The direction vector of d is calculated.

$$\vec{V}_d = (\sin(\beta)\cos(\theta), \cos(\beta)\cos(\theta), \sin(\theta)) \quad (30)$$

- ② Convert d to a representation in the ground coordinate system.

$$\vec{V}'_d = T_l \cdot \vec{V}_d \quad (31)$$

- ③ Convert H_4 to the expression in the ground coordinate system.

$$H'_4 = T \cdot H_4 + O_l = \begin{pmatrix} x'_4 \\ y'_4 \\ z'_4 \end{pmatrix} \quad (32)$$

- ④ The expression of the normal vector of the plane on which the mirror is located in the ground coordinate system is n . Calculate the expression of the reflected light e about d in the ground coordinate system.

$$\vec{V}'_e = \vec{V}'_d - 2 * (\vec{V}'_d \cdot n) * n \quad (33)$$

- ⑤ According to the principle of two points and one line, calculate whether there is an intersection between the light and the collector.

The linear equation of the light:

$$\frac{x-x'_4}{m_d} = \frac{y-y'_4}{n_d} = \frac{z-z''_1}{l_d} \quad (34)$$

Collector radius is 3.5m, center height is 80m, height is 8m, so its side equation is:

$$\begin{cases} x^2 + y^2 = 3.5^2 \\ 76 \leq z \leq 84 \end{cases} \quad (35)$$

Combine the above two equations: (1) If there is no real solution, the reflected light can not be irradiated to the collector; (2) If there is only a real solution, it is necessary to analyze the coordinate point where the solution is located on the collector's surface or its back surface, if it is the surface of the light, the reflected light can be irradiated to the collector, if it is the surface of the light is vice versa; (3) If there are two real solutions, the reflected light can be irradiated to the collector.

2.6. Calculation of optical efficiency and output thermal power of mirror field

Calculate the atmospheric transmittance[8], which d_{HR} represents the distance from the center of the mirror to the center of the collector, in meters:

$$\eta_{at} = 0.99321 - 0.0001176 \cdot d_{HR} + 1.97 \times 10^{-8} \cdot d_{HR}^2 \tag{36}$$

The optical efficiency[9][10] of the heliostat η is:

$$\eta = \eta_{cos} \cdot \eta_{sb} \cdot \eta_{at} \cdot \eta_{trunc} \cdot \eta_{ref} \tag{37}$$

G_0 Is the solar constant, the value is 1.366 kW/m^2 , H is the altitude (unit: km). To calculate the normal direct irradiance DNI:

$$\begin{aligned} \text{DNI} &= G_0[a + b \exp(-\frac{c}{\sin \alpha_s})], \\ a &= 0.4237 - 0.00821(6 - H)^2, \\ b &= 0.5055 + 0.00595(6.5 - H)^2, \\ c &= 0.2711 + 0.01858(2.5 - H)^2, \end{aligned} \tag{38}$$

Average output thermal power of mirror field:

$$E_{field} = \text{DNI} \cdot \sum_i^k A_i \cdot \eta_i \tag{39}$$

2.7. Result analysis

Obtained by MATLAB mathematical operation:

Table 1. Average optical efficiency and output power on the 21st day of each month

Date	Average optics efficiency	Average cosine efficiency	Average shading efficiency	Average truncation efficiency	Average output thermal power per unit area (kW/m ²)
Jan 21 st	0.5890	0.7206	0.9401	0.9850	0.4420
Feb 21 st	0.6037	0.7407	0.9342	0.9859	0.5042
Mar 21 st	0.6283	0.7611	0.9411	0.9889	0.5689
Apr 21 st	0.6532	0.7791	0.9535	0.9895	0.6274
May 21 st	0.6651	0.7890	0.9536	0.9935	0.6528
Jun 21 st	0.6667	0.7921	0.9532	0.9916	0.6581
Jul 21 st	0.6650	0.7889	0.9536	0.9935	0.6526
Aug 21 st	0.6471	0.7784	0.9453	0.9900	0.6155
Sept 21 st	0.6270	0.7601	0.9407	0.9886	0.5658
Oct 21 st	0.6009	0.7382	0.9337	0.9855	0.4958
Nov 21 st	0.5949	0.7189	0.9497	0.9852	0.4462
Dec 21 st	0.5855	0.7119	0.9457	0.9859	0.4128

Table 2. Year average optical efficiency and output power table

Average annual optical efficiency	Annual mean cosine efficiency	Average annual shading efficiency	Average annual truncation efficiency	Average annual thermal power output (MW)	Average annual thermal output per unit area (kW/m ²)
0.6272	0.7566	0.9454	0.9886	965857.5	0.5535

3. Conclusion

In this paper, a model of optical efficiency is established considering many factors such as shadow blocking efficiency and collector truncation efficiency, which ensures a detailed analysis of the source of energy loss. At the same time, ray tracing and intelligent optimization algorithm are used to

simulate the propagation mode of light in the heliostatic field, which is helpful to understand the loss of light and establish the collector truncation efficiency model. Quantitative evaluation of light loss, analysis of light behavior and energy loss sources. Finally, by applying the simulated annealing algorithm in the intelligent optimization algorithm, the key parameters of the heliostat field are optimized. As shown in Table 1 and Table 2, the changes of each optical efficiency and output power have obvious seasonality. The closer the time is to summer, the higher the efficiency and the greater the power, among which the average annual output thermal power per unit area is a typical indicator. At the same time, the truncation efficiency and shadow occlusion efficiency do not change much. The reason for this result is that the power plant is located at the north latitude of 39.4° , to the north of the Tropic of Cancer. In summer, the sun is near the Tropic of Cancer, and the average daily solar altitude Angle of the power plant is the largest, which results in the reduction of cosine loss, the reduction of shadow occlusion, the increase of average optical efficiency and the increase of output power. The model has wide applicability and can be applied to different geographical and climatic conditions. As long as it is properly adjusted and optimized according to specific geographical location and meteorological data, it can be applied to different regions and environments. At the same time, the calculation and optimization methods of multiple optical efficiency indexes and output power indexes provided by this model are of guiding significance for the design and operation of solar thermal power stations. By adjusting and optimizing the parameters, the optical efficiency and output power of the solar power station can be improved, and the economy and sustainability of the solar thermal power station system can be improved.

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