Modelling and Simulation of Quadrotor Using Model-Based Design

Di Zhang[1,a], Jing Zhang[2,b]

[1]Aeronautical School, Beijing Polytechnic, Beijing, China

[2]Dept. of Student Affairs, The Tourism College of Changchun University, Changchun, China
[a]zhangdi@bpi.edu.cn, [b]zhangjing202204@126.com

Abstract. In the recent years, the control of Unmanned Aerial Vehicles (UAV) has become one of the most interesting fields of research, especially for quadrotor aircraft due to the simple structure, superior maneuverability and low cost. This paper presents a Model-Based approach to develop the mathematical model of quadrotor and establish a control method to meet the target of the actual flight. It employs Simulink to design the Proportional Integral Derivative (PID) controller for altitude and attitude loop of quadrotor, and uses Stateflow to implement control logic method for different flight conditions and states.

Keywords: model-based design, modelling, simulation, quadrotor.

1. Introduction

Quadrotor aircraft has been widely used in aerial photography, surveying, power line inspection, urban management and other fields. Since the 1990s, with the development of the Micro Electro Mechanical System (MEMS) technology, a single rotor helicopter has been the favorable platform for MAV development. However, the disadvantages such as complexity, instability and limited payload slowed down its development. While the multi-rotor vehicle, especially the quadrotor vehicle, is studied in more research experiments. Due to the excellent capabilities of small size, low cost and simple manufacture, quadrotor can be widely used in military reconnaissance, meteorological survey, environmental monitoring and air transportation in emergency and other aspects [1]. The implementation and design of the quadrotor flight control system is the key point to the autonomous flight of the aircraft.

Recently, Model-Based design is a modern design methodology that enables faster, more cost-effective development of dynamic systems. It can shorten the design cycles and reduce the development cost due to the usage of Model-Based design in quadrotor flight system. Model-Based design is a concept where mathematical process models are used to simulate and verify system performance before building physical prototypes. Model-Based software design simplifies conventional development using an intuitive block diagram environment and automatic code generation. It can be argued that this method reduces coding errors and the need for programming skills, thus enabling the engineer to focus on his or her area of expertise [2].

This paper presents a Model-Based approach to develop the aerodynamic model of quadrotor aircraft, and derive the flight control algorithm. Modelling and simulation techniques are combined to accelerate the development.

2. Quadrotor Model

2.1 Mathematical Model

Quadrotor relies on four propellers to provide power for flight. Among the four propellers, the two opposite propellers rotate in the same direction, and the two adjacent propellers rotate in opposite directions in order to offset the anti-torque generated during the rotation of the motor, which can ensure the controllability of the attitudes [3]. Rotor wing, which relies on the change of each motor speed to achieve flight attitude control, is controlled by the motor. In the figure 1, the rotor 1 on the left and the rotor 3 on the right counterclockwise, the rotor 2 on the right side and the rotor 4 on the left side clockwise. When hovering, the rotate speed of four rotors should be equal to offset the torsion moment. Amount at the same time to increase or decrease the speed of four rotors, will cause rising or falling movement. Increase the revolving speed of two adjoining rotors, while an equal amount to reduce the speed of the other two adjoining rotors, to produce pitch and roll motion. Increase the revolving speed of the rotor of a group, while an equal amount to reduce the speed of the rotor of another group, to produce yaw motion [4].

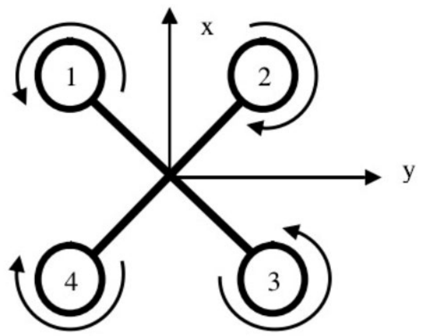

Figure 1. The rotation direction of the x-quadrotor

The model analyzed in this paper assumes the following [5]:

The structure is rigid;

The structure is symmetrical;

The center of gravity and the body fixed frame origin coincident;

The propellers are rigid;

Thrust and drag are proportional to the square of propeller's speed.

Quadrotor operates with six degrees of freedom: three degrees determines translator motion and three other degrees determines rotational motion [6]. Position of the quadrotor in the inertial frame is determined by the ξ vector:

$$\xi = (x, y, z)^T \tag{1}$$

Angle position is determined by the η vector:

$$\eta = (\phi, \theta, \psi)^T \tag{2}$$

Angular velocity vector is:

$$v = (p, q, r)^T \tag{3}$$

Conversion from the base frame to the inertial is performed by the rotation matrix R. Conversion from the angular velocity in the base frame to the same velocity in the inertial frame is performed by the matrix W. Mathematical model of the quadrotor can be presented as:

$$\begin{cases} \ddot{x} = \frac{\cos\phi \sin\theta \cos\psi + \sin\phi \sin\psi}{m} U_1 \\ \ddot{y} = \frac{\cos\phi \sin\theta \sin\psi + \sin\phi \cos\psi}{m} U_1 \\ \ddot{z} = -g + \frac{\cos\phi \cos\theta}{m} U_1 \end{cases} \tag{4}$$

$$\begin{cases} \ddot{\phi} = \dot{\theta}\dot{\psi}(\frac{I_y - I_z}{I_x}) + \frac{I_{rotor}\theta\dot{\Omega}_g}{I_x} + \frac{l}{I_x} U_2 \\ \ddot{\theta} = \dot{\phi}\dot{\psi}(\frac{I_z - I_x}{I_y}) + \frac{I_{rotor}\phi\dot{\Omega}_g}{I_y} + \frac{l}{I_y} U_3 \\ \ddot{\psi} = \dot{\phi}\dot{\theta}(\frac{I_x - I_y}{I_z}) + \frac{l}{I_z} U_4 \end{cases} \tag{5}$$

The model inputs are:

$$\begin{cases} U_1 = b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \\ U_2 = b(\Omega_2^2 - \Omega_4^2) \\ U_3 = b(\Omega_1^2 - \Omega_3^2) \\ U_4 = d(\Omega_2^2 - \Omega_1^2 + \Omega_3^2 - \Omega_4^2) \end{cases} \tag{6}$$

Where d is the drag coefficient, l is the distance between the motors and the center of gravity and I_{rotor} is the rotor inertia, Ω_i is the angular speed produced by the i^{th} motor, and b is the thrust coefficient [7].

The model is used to write the system in state-space form $\dot{X} = AX + BU$ with U input vector and X state vector as below:

$$X = [\phi, \dot{\phi}, \theta, \dot{\theta}, \psi, \dot{\psi}]^T$$
$$U = [U_2, U_3, U_4]^T \tag{7}$$

2.2 Control Model

The control model of the quadrotor contains two control parts, the first is for the positions control and the second is for the angles control as show in figure 2.

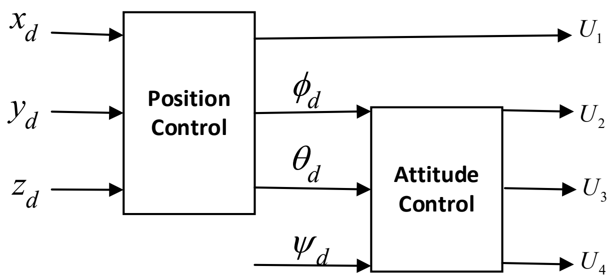

Figure 2. The control logic model of quadrotor

3. Introduction Simulation Results

Simulations are carried out in MATLAB Simulink to analyze stability and performance of the proposed controller. In the Figure 3, the control block diagram is shown. In Figure 5, the control input for pitch is set to 1 rad. From simulation result, it can be seen that output tracks the desired value within 2 seconds. In Figure 6, the desired altitude is given as 5m to the quadrotor in Simulink and following graph is obtained. The simulation model tracks the set point in 1 second, but there is steady state error of 0.1m.

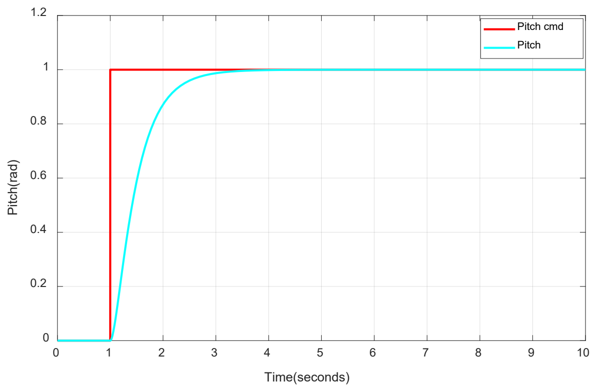

Figure 3. Pitch response of quadrotor

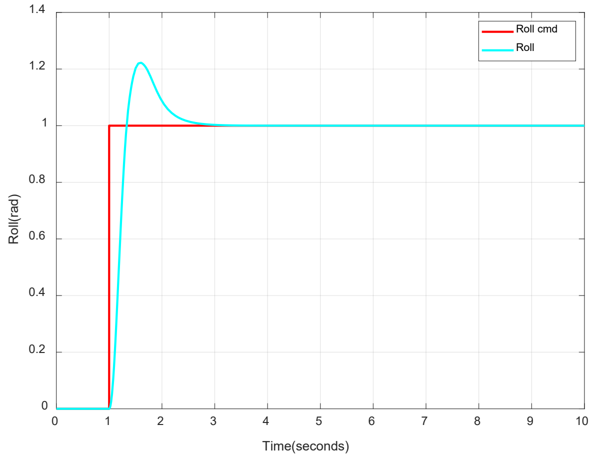

Figure 4. Roll response of quadrotor

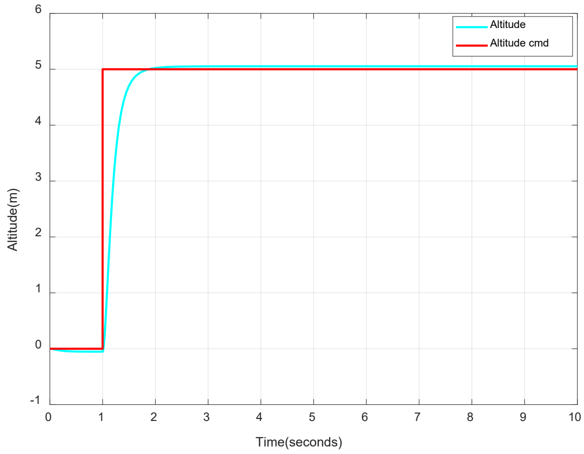

Figure 5. Altitude response of quadrotor

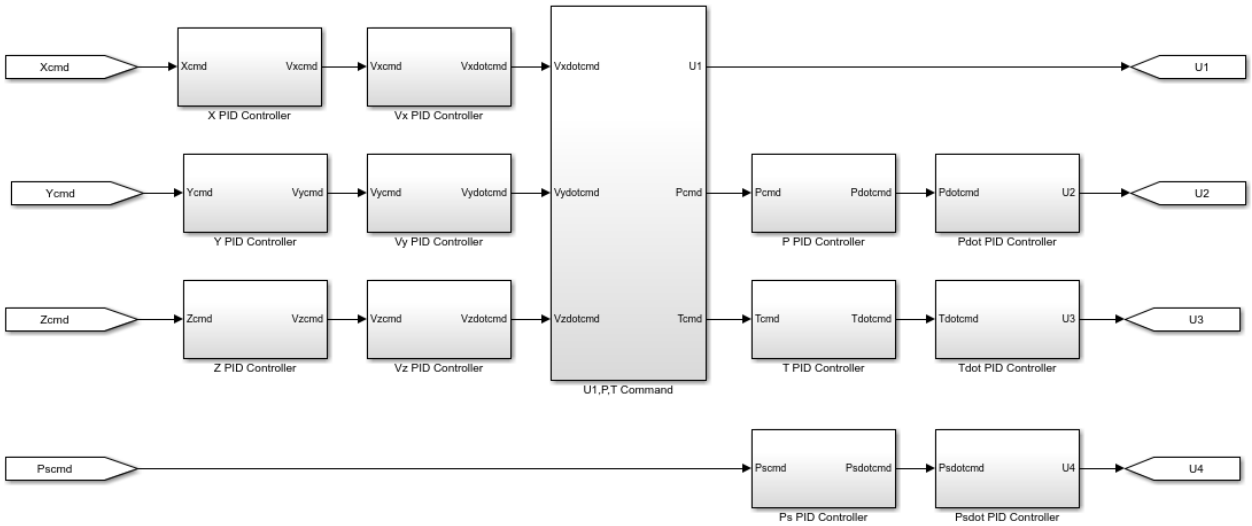

Figure 6. The simulation control model of quadrotor

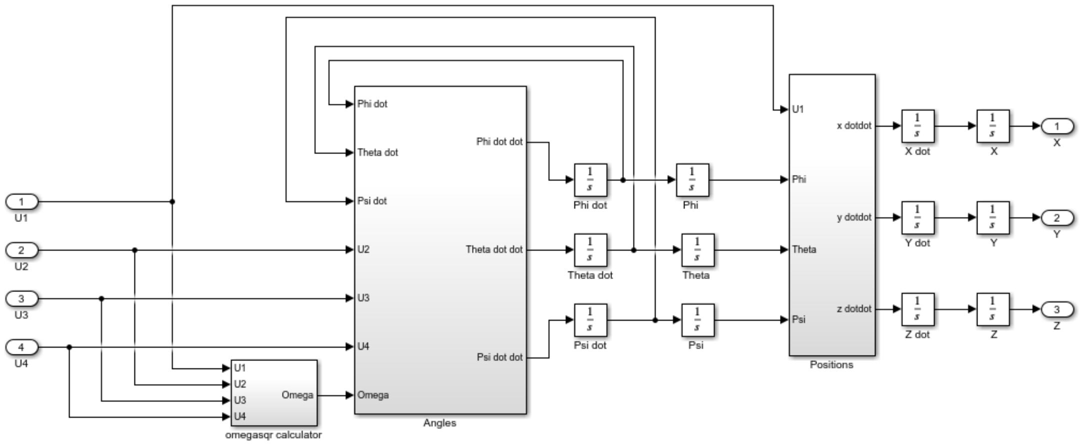

Figure 7. The simulation model of quadrotor

4. Summary

The mathematical model and control model of quadrotor is done using Model-Based design approach. Full dynamic model of quadrotor is considered while designing PID controller and simulations are carried out in MATLAB Simulink environment. Simulation results show that the PID controller provides satisfactory results in tracking the desired reference values. The Model-Based design approach is proved to be feasible and effective. Embedded codes can be automatically generated from Simulink models, and numeric and HIL simulations are conducted to verify the control algorithm. Simulation and field trial results show that the simulations accurately predict the real-life behavior and the code generation process is effective.

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