Optimization Design of Heliostat Field Arrangement Based on Objective Optimization Model

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Abstract. Tower solar energy achieves high efficiency solar heat conversion by virtue of the advantages of concentrating light and collecting heat. In this paper, a spatial model of the sun's incident light collection and the mirror's reflected light collection is established. The sun's orientation at different times is investigated and the optical efficiency of each heliostat is obtained. The average of the optical efficiency of the heliostat field in the case of the collection of incoming solar rays is obtained as the optical efficiency of the heliostat field under the irradiation of the sun cone. Firstly, based on the latitude and longitude of the heliostat field, the altitude Angle and azimuth Angle of the sun at different times are calculated. Secondly, according to the position coordinates of heliostatic center and absorption tower in geodetic coordinate system, the normal direct radiation irradiance DNI and atmospheric transmittance $\eta_{at}$ of all heliostatic fields are determined $\theta$. Subsequently, a number of mirror coordinate systems are established to calculate the direction vector of the sun incident light and the direction vector of the mirror center pointing to the collector center, and obtain the incidence Angle, mirror normal vector, helioscope pitch Angle, helioscope azimuth Angle and cosine efficiency $\eta_{cos}$ of each helioscope. According to this, the ray tracing method is used to calculate the shadow blocking efficiency $\eta_{sh}$ of the mirror. Finally, the HFLCAL model is used to calculate the collector truncation efficiency $\eta_{trunc}$, and the average annual optical efficiency of the heliostat field is 0.313067, the average annual output work and heat rate is 19.266981MW and the average annual output power per unit mirror area is 0.306701kw/m².

Keywords: Heliostat Field, Optical Efficiency, Optimization Arrangement.

1. Introduction

Tower solar thermal power generation is a new type of clean energy technology with potential, which can provide reliable power supply for China to achieve the goal of "carbon peak" and "carbon neutrality", reduce the dependence on traditional fossil energy, and promote the transformation and sustainable development of energy structure.

The heliostat field is composed of a large number of heliostats, a cylindrical collector with a height of 8m and a diameter of 7m installed on the top of the 80m high absorption tower [1], and the plant around the absorption tower. Helioscope is the basic component of the tower solar power station to collect solar energy, its base is composed of longitudinal axis and horizontal axis, the intersection of the two axes is the center of the helioscope, the height of the center from the ground is called the installation height of the helioscope. The heliostatic mirror is composed of a mirror with a mirror side length of 6m×6m and a base with a longitudinal rotating shaft and a horizontal rotating shaft, and its installation height is 4m. The sunshineson the heliostat, and the heliostat concentrates the reflection of the sun to the collector installed on the top of the absorption tower in the mirror field, making it reach a very high temperature, and then converts the heat energy into electrical energy through the thermal energy conversion device.

Now it is planned to build a circular helioscope field in a circular area with a radius of 350m and the center is located at 98.5° E, 39.4° N, 3000m above sea level, known helioscope position, size, installation height and absorption tower position, to establish a model of shadow shielding efficiency,
cosine efficiency, collector truncation efficiency and helioscope field output thermal power. So as to obtain the average annual optical efficiency of the heliostat field, the average annual thermal output power, and the average annual thermal output power per unit mirror area.

2. The model is established and solved

2.1. Calculation of heliostat field output thermal power

The thermal output power of a heliostat $E_{\text{field}}$ refers to the ability to concentrate solar energy through a heliostat and convert it into heat. The formula is:

$$E_{\text{field}} = DNI \cdot \sum_{i} A_i \eta_i$$

(1)

DNI is normal direct radiation irradiance [2]; $A_i$ is the daylighting area of the I-side heliostat; $\eta_i$ is the optical efficiency of the I-th mirror.

The optical efficiency of heliostat is:

$$\eta = \eta_{sb} \eta_{\cos} \eta_{at} \eta_{\text{trunc}} \eta_{\text{ref}}$$

(2)

Where $\eta_{sb}$ is the shadow blocking efficiency, $\eta_{\cos}$ is the cosine efficiency, $\eta_{at}$ is the atmospheric transmittance, $\eta_{\text{trunc}}$ is the collector truncation efficiency, $\eta_{\text{ref}}$ is the mirror reflectivity [3]. The constant of mirror reflectance $\eta_{\text{ref}}$ is 0.92.

2.2. Normal direct radiation irradiance is calculated

Normal direct radiation irradiance refers to the solar radiation energy received in unit area and unit time on the plane perpendicular to the sun rays on the earth, the formula is:

$$DNI = G_0[a + b \exp(-\frac{c}{\sin \alpha_s})]$$

$$a = 0.4237 - 0.0082(6 - H)^2$$

$$b = 0.5055 + 0.00595(6.5 - H)^2$$

$$c = 0.2711 + 0.01858(2.5 - H)^2$$

(3)

Where, $G_0$ is the solar constant, its value is 1.366kw/m$^2$, H is the altitude, $\alpha_s$ is the sun height Angle. The solar altitude Angle $\alpha_s$ refers to the Angle between the sun's rays and the horizon. It is used to describe how high the sun is in the sky. The relevant formula is

$$\sin \alpha_s = \cos \delta \cos \varphi \cos \omega + \sin \delta \sin \varphi$$

(4)

Where $\varphi$ [4] is the local latitude and the north latitude is positive, $\varphi = 39.4\,\text{deg}$ can be obtained from the question, $\delta$ is the sun declination Angle, $\omega$ is the sun hour Angle.

The sun declination Angle $\delta$ [5] refers to the Angle of the sun relative to the Earth's equatorial plane. It is an important parameter used in astronomy to describe the position of the Sun on the celestial sphere. the formula is:

$$\sin \delta = \sin \frac{2\pi D}{365} \sin \left(\frac{2\pi}{360} 23.45\right)$$

(5)

Where $D$ is the number of days starting from the vernal equinox as the 0 day, for example, if the vernal equinox is March 21, then April 1 corresponds $D = 11$. 
The solar hour Angle $\omega$ is the Angle of the sun relative to the meridian of the observer's location. The relevant formula is:

$$\omega = \frac{\pi}{12} (ST - 12)$$

(6)

Where ST is the local time.

2.3. Atmospheric transmittance calculation

Atmospheric transmittance is the proportion of energy that sunlight passes through the atmosphere and reaches the ground. Atmospheric transmittance depends on the absorption, scattering and refraction of gases, particles and water vapor in the atmosphere [6]. Its expression is:

$$\eta_{at} = 0.99321 - 0.0001176 d_{HR} + 1.97 \times 10^{-8} \times d_{HR}^2 (d_{HR} \leq 1000)$$

(7)

$d_{HR}$ indicates the distance from the center of the mirror to the center of the collector (unit: m).

2.4. Calculation of shadow occlusion efficiency

The loss caused by heliostat A being in the shadow of heliostat B and not being able to accept sunlight for reflection is called shadow loss [7]; The solar rays reflected by heliostat A cannot propagate to the collector due to the occlusion of heliostat B, which is called the occlusion loss, as shown in Fig. 2.

In solving the shadow occlusion loss, since the field radius of the heliofix is too small compared with the half Angle of the sun cone, the sun beam received by each heliofix can be approximately parallel light.

The mirror normal vector of heliostatic mirror A and B is calculated respectively, the coordinate system is converted, the conditions of shadow and occlusion are obtained, and the shadow occlusion
loss can be obtained by judging the proportion of the shadow occlusion area of each mirror in the total area, so as to obtain the shadow occlusion efficiency.

A heat collecting tower is established at the center of the circle, with the base of the heat collecting tower as the origin. Due east direction is the positive direction of x axis, due north direction is the positive direction of y axis, and perpendicular to the ground upward direction is the positive direction of z axis. It is called heliostat field coordinate system [8].

The heliostat field arrangement is shown in Fig. 3:

![Heliostat diagram](image)

**Figure 3. Heliostat diagram**

Let the collector coordinate be O (0, 0, h), where h is the height of the difference between the collector and the surface of the mirror field. M as the origin to establish the mirror coordinate system [9] \( X_m, Y_m, Z_m \), the \( Z_m \) axis and the mirror center normal overlap, the direction upward, the \( X_m \) axis and the horizontal axis of the heliostatic mirror overlap, the \( Y_m \) axis perpendicular to the \( X_m \) axis direction upward, the \( X_m, Y_m \) axis are in the mirror. This time, the center point of the heliostat mirror is set on the central axis of the base, and the center coordinate of the heliostat A mirror is expressed as \( O_m (x_m, y_m, z_m) \), \( x_m \) is the horizontal coordinate of the heliostat in the mirror field plane, \( y_m \) is the longitudinal coordinate of the heliostat in the mirror field plane, and \( z_m \) is the height difference between the center of the mirror and the net long plane.

The unit normal vector \( \vec{S} \) of the light rays pointing to the collector at the center of the heliostat mirror is:

\[
\vec{S} = \frac{O - O_m}{|O - O_m|} = \frac{(-x_m, -y_m, h - z_m)}{\sqrt{x_m^2 + y_m^2 + (h - z_m)^2}}
\]  

(8)

The height Angle of the sun position is \( \alpha_s \), the azimuth Angle of the sun position is \( \gamma_s \), the unit normal vector obtained by the sun light at different times, assuming that the unit vector obtained by the incident light is expressed as \( \vec{S}_i = (x_i, y_i, z_i) \).

It can be seen that the calculation formula in the mirror field coordinate system is:

\[
\begin{align*}
x_i &= \cos(\alpha_i) \cos(\gamma_s - 90^\circ) \\
y_i &= \cos(\alpha_i) \sin(\gamma_s - 90^\circ) \\
z_i &= \sin(\alpha_i)
\end{align*}
\]  

(9)

According to the law of light reflection, the Angle of reflection is equal to the Angle of incidence, and the mirror normal vector of the heliostat can be obtained:
\[ s_n = \frac{s_i - s_j}{s_i - s_1} \] (10)

From this calculation, the pitch Angle \( \theta_z \) and azimuth Angle \( \theta_s \) of the heliostat can be obtained.

\[
\begin{align*}
\tan(\theta_z) & = \frac{\sin(\alpha_s) \cdot m + h}{\sqrt{x_A^2 + y_A^2 + m^2 \cdot \cos^2(\alpha_s) - 2 \cos(\alpha_s) \cdot m \cdot (x_A \cdot \sin(\gamma_s) - y_A \cdot \cos(\alpha_s))}} \\
\sin(\theta_s) & = \frac{x_A - \cos(\alpha_s) \cdot \sin(\gamma_s) \cdot m}{\sqrt{x_A^2 + y_A^2 + m^2 \cdot \cos^2(\alpha_s) - 2 \cos(\alpha_s) \cdot m \cdot (x_A \cdot \sin(\gamma_s) - y_A \cdot \cos(\alpha_s))}} \\
\end{align*}
\] (11)

Known: \( m = \sqrt{x_A^2 + y_A^2 + h_0^2} \)

In the mirror coordinate system, each heliostat is divided into \( n \times n \) lattice, and the projection point coordinates of each point on the shadoed heliostat mirror in the direction of the incident light are calculated [10].

Using the ray tracing method to calculate the shadow of the heliofixor on the ground, the direction cosine of the incident light is:

\[
\begin{align*}
\cos \alpha_i & = \cos A \cos \alpha \\
\cos \beta_i & = \sin A \sin \alpha \\
\cos \gamma_i & = \sin \alpha \\
\end{align*}
\] (12)

Note: \( \alpha \) is the sun altitude Angle, \( A \) is the sun azimuth Angle.

The coordinates of the obtained mirror tracking points in the ground coordinate system are:

\[
\begin{align*}
x_{mg} & = -\sin A_H x_m - \cos A_H \cos E_H y_m + X_{gh} \\
y_{mg} & = \cos A_H x_m - \sin A_H \cos E_H y_m + Y_{gh} \\
z_{mg} & = \sin E_H y_m + Z_{gh} \\
\end{align*}
\] (13)

The coordinates of the intersection of the incident light with the ground \( Z_g = 0 \) are:

\[
\begin{align*}
x_g & = x_{mg} - z_{mg} c \tan \alpha \cos A \\
y_g & = y_{mg} - z_{mg} c \tan \alpha \sin A \\
\end{align*}
\] (14)

The coordinates of the shadow heliostat in the coordinate system \( X_OY_OZ_O \) are:

\[
\begin{align*}
X_{so} & = X_{ig} - X_{og} \\
Y_{so} & = Y_{ig} - Y_{og} \\
Z_{so} & = Z_{ig} - Z_{og} \\
\end{align*}
\] (15)

The coordinates of the shadow heliostat in the incident coordinate system are:

\[
\begin{align*}
X_i & = -X_{so} \sin A + Y_{so} \cos A \\
Y_i & = -\sin \alpha (X_{so} \cos A + Y_{so} \sin A) + Z_{so} \cos A \\
Z_i & = \cos \alpha (Y_{so} \sin A + X_{so} \cos A) + Z_{so} \sin \alpha \\
\end{align*}
\] (16)

The projection of the center of the mirror on the mirror along the direction of the incident light is:
\[ \begin{align*}
X_{is} &= X_i \\
Y_{is} &= Y_i \\
Z_{is} &= Z_i - Z_m / \cos \theta
\end{align*} \] (17)

By using coordinate transformation:
\[ Z_{so} = \sin E_H (X_{so} \cos A_H + Y_{so} \sin A_H) + Z_{so} \cos E_H \] (18)

According to the coordinate transformation, the projection coordinates of the center of the mirror along the direction of the incident light on the Shadowed mirror are:
\[ \begin{align*}
X_{mi} &= X_{is} \cos (A_H - A) + Y_{is} \sin \alpha \sin (A_H - A) - Z_{is} \cos \alpha \sin (A_H - A) \\
Y_{mi} &= -X_{is} \cos E_H \sin (A_H - A) + Y_{is} \cos E_H \sin \alpha \cos (A_H - A) + \sin E_H \cos \alpha \\
Z_{mi} &= \cos E_H \cos \alpha \cdot \cos (A_H - A) - \sin E_H \sin \alpha
\end{align*} \] (19)

The conditions for the presence of shadow are:
\[ Z_i > 0 \text{ and } \begin{cases} |X_{mi}| < h_{so} \\ |Y_{mi}| < h_l \end{cases} \] (20)

Generate a two-dimensional all-1 matrix with \( n \) rows and \( n \) columns, then the coordinates of the mirror's \( k \) row \( l \) column (\( 0 < k < n \), \( 0 < l < n \)) are:
\[ \begin{align*}
X_{mdl} &= \frac{h_{so}}{n} (l - \frac{n}{2} - 0.5) \\
Y_{mdl} &= \frac{h_l}{n} (0.5 + \frac{n}{2} - k)
\end{align*} \] (21)

The number of shadow points of the whole heliostat field is obtained, and its ratio to the total number of tracking points of the whole heliostat field is the shadow loss.

2.5. Cosine efficiency calculation

The physical meaning of cosine efficiency is the ratio of the actual area of the heliostat and the area of the heliostat, that is, the cosine value of the incident Angle of the incident sunlight.

As shown in Fig 4, the variation of \( E_h \) in the pitch of the mirror will lead to the cosine loss of sunlight, while the incidence Angle of the solar light is determined by the position of the mirror center and the position of the collector.

![Figure 4. Cosine efficiency diagram](image)

Cosine efficiency of a single heliostat:
\[ \eta_{\cos} = \cos \theta = \vec{i} \cdot \vec{n} \] (22)
\( \vec{n} \) is a mirror normal vector.
\( \vec{i} \) is the unit vector in the opposite direction of the incident light, expressed as:
\[
\vec{i} = [-\cos(\alpha) \sin(\gamma), -\cos(\alpha) \cos(\gamma), -\sin(\alpha)]
\]

The cosine efficiency of a single heliostat determines the effective daylighting area of the heliostat. With the change of the sun's position, the optical efficiency of each heliostat is not a certain value and the optical efficiency of different heliostat may not be the same. Therefore, the optical efficiency of each heliostat under different sun positions should be calculated, and the optical efficiency of the heliostat field should be replaced by the average optical efficiency of all heliostat [10].

### 2.6. Collector cutoff efficiency calculation

Truncation efficiency refers to the scattering of light, the shape of the solar beam, the size and shape of the heliostat, and other factors, so that the sunlight reflected by the heliostat cannot be completely absorbed by the collector at the top of the absorption tower, and the energy loss caused by privacy. In order to reduce the amount of calculation, this paper will use the HFLCAL model to calculate the truncation efficiency of the collector.

The calculation formula is as follows:
\[
\left\{ \begin{align*}
\eta_{\text{int}} &= \frac{1}{2\pi \sigma_{\text{tot}}^2} \int \int \exp \left(-\frac{x^2 + y^2}{\sigma_{\text{tot}}^2}\right) \, dx \, dy \\
\sigma_{\text{tot}} &= \sqrt{d^2 (\sigma_{\text{sum}}^2 + \sigma_{\text{bq}}^2 + \sigma_{\text{at}}^2 + \sigma_{\text{track}}^2)}
\end{align*} \right.
\]

Where, \( \sigma_s \) is the standard deviation of the slope error; \( H, W \) are the size of the light spot on the collector in the meridian direction and the arc direction (the meridian direction is the direction of the longitude on the earth's surface, and the arc direction is the direction of the shortest path connecting the two points.); LH and LW are the height and width of the heliostat; \( f \) is the axial focal length of the heliostat; \( \omega \) is the incidence Angle of light. \( \sigma_{\text{sum}} = 2.51 \text{mrad} \) data provided by PSA, known from the \( \sigma_s = 0.94 \text{mrad}, \sigma_{\text{track}} = 0.63 \text{mrad} \) data obtained from the SENER heliostat test. (The heliostats calculated in this paper are all rectangular flat mirrors, the ratio of axial focal length to reflection distance is \( \cos \omega \)).

### 3. Conclusion

According to the position distribution data of heliostat field studied in this paper, the annual average optical efficiency of the heliostat field is \( \eta = 0.313067 \), the annual average cosine efficiency is \( \eta_{\cos} = 0.496663 \), the annual average shadow occlusion efficiency is \( \eta_{\text{sb}} = 0.914202 \), the annual average truncation efficiency is \( \eta_{\text{trunc}} = 0.773454 \), the annual average output power is \( E_{\text{field}} = 19.266957\text{MW} \), the average output thermal power per unit area of the mirror is 0.306701KW/m².

In this paper, based on the heliostat field optical efficiency formula, the formula of shadow blocking efficiency, cosine efficiency, collector truncation efficiency modeling, the numerical solution obtained by the model is in line with the experimental results of the references, more accurate to find the heliostat field of the average annual optical efficiency, the average annual output thermal power, and the average annual output thermal power per unit mirror area.

But in the calculation of truncation efficiency, approximate equal to overflow efficiency, there is an error between the two; When calculating the optical efficiency of heliostat field, the sunlight received by each heliostat is approximately regarded as parallel light. In fact, the solar beam is a light cone shape with a certain beam broadening. In real life, the environment, weather and many other factors, coupling may occur.
References


