

A Study on the Optical Efficiency of Solar Concentrators Based on Ray Tracing Method

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Abstract. This paper calculates the solar altitude angle, solar azimuth angle, and normal direct solar radiation. Subsequently, utilizing geometric optics and ray propagation theory, this study employs vector methods combined with the law of light reflection to compute the cosine efficiency. By characterizing the equivalent shadow shading using the topological properties of connected regions, the approximate shadow shading efficiency is determined through the element method. Finally, based on the principles of ray tracing, this paper derives the Gaussian flux density distribution formula for the collector truncation efficiency and calculates the truncation efficiency. Ultimately, through iterative calculations and averaging for different times and locations, this paper successfully obtains the optical efficiency and annual average output thermal power of the solar concentrator field at various time points. The annual average optical efficiency, annual average cosine efficiency, annual average shadow shading efficiency, and annual average truncation efficiency are found to be 0.64, 0.76, 0.96, and 0.98, respectively. Consequently, the annual average output thermal power and the annual average output thermal power per unit area are determined to be 38.93 and 0.62, respectively.

Keywords: Vector Analysis, Ray Tracing Method, Element Method, Optical Efficiency.

1. Introduction

In recent years, global climate degradation has heightened the importance of ecological preservation. Renewable energy sources, notably solar power, have gained international prominence [1-3]. Solar thermal power generation, recognized for its high efficiency, is a focal point in the current energy transition. The '13th Five-Year Plan' outlines robust support for its development and demonstration projects [4-6].

Tower-type solar thermal power generation, a cornerstone of future solar energy, captures real-time solar radiation by reflecting and concentrating it onto collectors. These systems can achieve concentration ratios of 300 to 1200, operate at high temperatures, and exhibit remarkable efficiency, garnering increasing domestic attention.

This paper, in the context of the circular solar concentrator located at the center of the mirror field for tower-type generation, focuses on researching the calculation method for its optical efficiency at different times when the solar concentrator size is $6\text{m} \times 6\text{m}$, and the installation height is 4m.

2. Calculation Derivation of Optical Efficiency

2.1. Calculation of Solar Altitude Angle and Solar Azimuth Angle

According to geographical definitions, the solar azimuth angle represents the solar angle in the horizontal direction and is typically defined as the angle measured clockwise from the north along the horizon. The solar altitude angle, on the other hand, refers to the angle between the direction of solar radiation incident on a specific location on Earth and the horizontal plane. Diagrams illustrating the solar altitude angle and solar azimuth angle is shown in Fig 1 and 2.

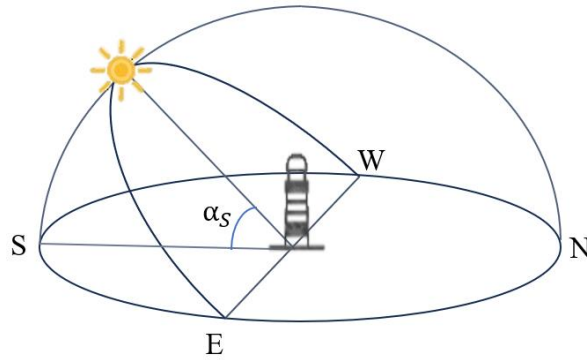


Figure 1. Illustration of Solar Altitude Angle

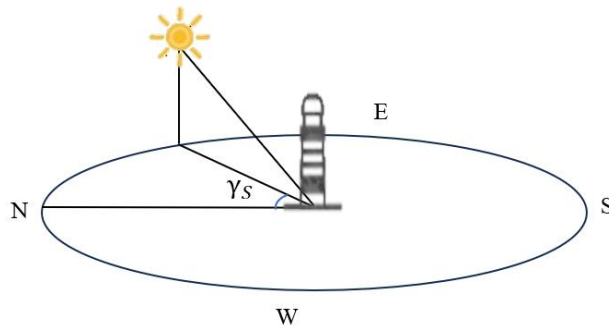


Figure 2. Illustration of Solar Azimuth Angle

Next, this paper will solve for these two parameters.

Known formulas [7-8]

$$\sin \delta = \sin \frac{2\pi D}{365} \sin \left(\frac{2\pi}{360} \times 23.45 \right) \quad (1)$$

By substituting the number of days D counted from the vernal equinox as day 0, calculate the solar declination angle δ .

Known formulas,

$$\omega = \frac{\pi}{12} (ST - 12) \quad (2)$$

Substitute the local time ST and calculate the solar hour angle ω .

$$\sin \alpha_s = \cos \delta \cos \varphi \cos \omega + \sin \delta \sin \varphi \quad (3)$$

$$\cos \gamma_s = \frac{\sin \delta - \sin \alpha_s \sin \varphi}{\cos \alpha_s \cos \varphi} \quad (4)$$

By substituting the above two parameters into equations (3) and (4), calculate the solar altitude angle α_s and solar azimuth angle γ_s at the (D, ST) moment. The visual results generated using Mathematica programming are shown in Fig 3 and 4.

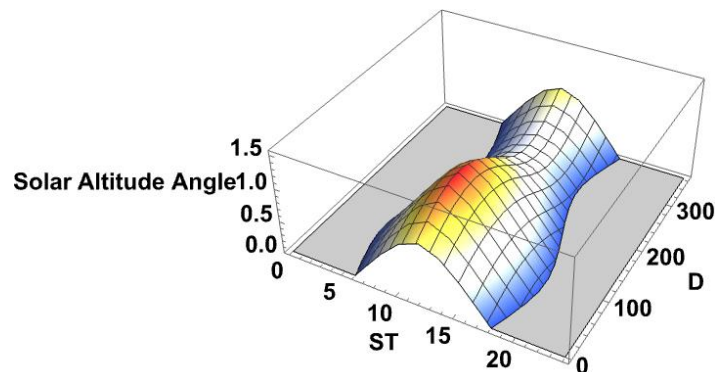


Figure 3. Graphs of Solar Altitude, D, and ST

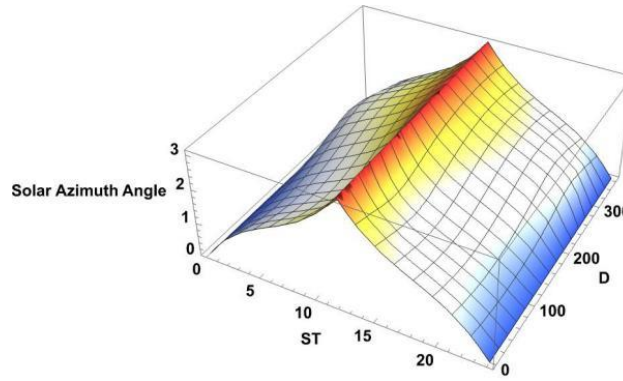


Figure 4. Graphs of Solar Azimuth, D, and ST

2.2. Calculation of Direct Normal Irradiance (DNI)

When solar radiation passes through the Earth's atmosphere, some of it is absorbed or scattered by air molecules, water vapor, aerosols, and clouds. The solar radiation that directly reaches the Earth's surface is called Direct Normal Irradiance (DNI), and its calculation formula is as follows [9]:

$$DNI = G_0 \left(a + b e^{-\frac{c}{\sin \alpha_s}} \right) \quad (5)$$

In the equation, the three parameters a , b , and c are determined by the local altitude (H).

$$\begin{cases} a = 0.4237 - 0.00821(6 - H)^2 \\ b = 0.5055 - 0.00595(6.5 - H)^2 \\ c = 0.2711 + 0.01858(2.5 - H)^2 \end{cases} \quad (6)$$

By substituting the solar constant $G_0 = 1.366 \text{ KW/m}^2$ and solar altitude angle into equation (5), you can calculate the Direct Normal Irradiance (DNI).

2.3. Calculation of Optical Efficiency η

Based on the annual average optical efficiency calculation method proposed by Collado et al. using the Cell-Wise approach [8-10], it can be observed that optical efficiency is not only related to the assumed mirror reflectance of 0.92 in this study but also involves the calculation of four component efficiencies, this paper will proceed to solve for these four parameters individually.

2.3.1 Calculation of Atmospheric Transmittance η_{at}

Atmospheric transmittance represents the proportion of energy from a specific light wavelength that passes through Earth atmosphere. Therefore, the initial step involves calculating the distance d_{HR} .

This study assumes that each solar concentrator within the solar array has dimensions of $6\text{m} \times 6\text{m}$ and is installed at a height of 4m . Consequently, the diagonal of each concentrator, denoted as L , measures $6\sqrt{2}\text{m}$, and the angle between the mirror width and the diagonal is represented as $\rho_0 = 45^\circ$. We establish a three-dimensional Cartesian coordinate system $O\text{-}xyz$ for spatial positioning and geometric calculations of each solar concentrator, with the tower's center axis at a height of 4m serving as the origin (O). Distances between points are calculated using the distance formula:

$$d_{HR} = \sqrt{x^2 + y^2 + 80^2} \quad (7)$$

Substitute the obtained d_{HR} into equation (8).

$$\eta_{at} = 0.99321 - 0.0001176d_{HR} + 1.97 \times 10^{-8} \times d_{HR}^2 \quad (8)$$

This will yield the value of atmospheric transmittance η_{at} .

2.3.2 Calculation of Cosine Efficiency η_{cos}

When light is perpendicular to a surface, it maximally absorbs energy. When not perpendicular, energy absorption decreases, described by cosine loss. It quantifies the reduction in received energy due to non-parallel alignment with incoming sunlight and the mirror aperture's normal direction.

$$\text{Cosine Efficiency } \eta_{cos} = 1 - \text{Cosine Loss} \quad (9)$$

In this paper, the sun is denoted as point S, the center of the collector as point T, and it is defined that the unit vector in the direction of the line connecting a certain point M on the solar concentrator and point T is represented as $\vec{t}$. Similarly, the unit vector in the direction of the line connecting a certain point on the solar concentrator and point S is represented as $\vec{s}$, these two vectors, derived using the parallelogram law of vectors, result in the sum vector, which is the normal vector of the solar concentrator. A unit vector in the same direction as this normal vector is designated as $\vec{f}$, A schematic diagram illustrating this is shown in Fig 5.

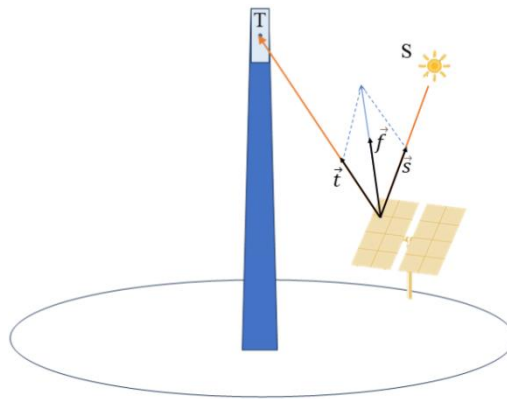


Figure 5. Process of Calculating Cosine Efficiency with Annotated Diagrams

Using the formula for vector dot product operation.

$$\eta_{cos} = \cos \angle \vec{s}, \vec{f} \geq \vec{s} \cdot \vec{f} \quad (10)$$

In the spatial Cartesian coordinate system established earlier, the coordinates of the vectors are as follows:

$$\vec{s} = (-\cos\alpha_s \sin\gamma_s, -\cos\alpha_s \cos\gamma_s, \sin\alpha_s) \quad (11)$$

$$\vec{t} = \left(\frac{-x}{d_{HR}}, \frac{-y}{d_{HR}}, \frac{80}{d_{HR}} \right) \quad (12)$$

Therefore,

$$\vec{f} = \frac{\vec{s} + \vec{t}}{|\vec{s} + \vec{t}|} = \frac{\left(\frac{-x}{d_{HR}} - \cos\alpha_s \sin\gamma_s, \frac{-y}{d_{HR}} - \cos\alpha_s \cos\gamma_s, \frac{80}{d_{HR}} + \sin\alpha_s \right)}{\left(\sqrt{\frac{2\cos\alpha_s(x\sin\gamma_s + y\cos\gamma_s) + 160\sin\alpha_s}{d_{HR}}} + 2 \right)} \quad (13)$$

Subsequently, the expression for cosine efficiency Equation (14) is obtained.

$$\eta_{cos} = \sqrt{\frac{1 + \cos\alpha_s(x\sin\gamma_s + y\cos\gamma_s) + 80\sin\alpha_s}{2d_{HR}}} \quad (14)$$

2.3.3 Calculation of Shadow Shading Efficiency η_{sb}

The paper indicates that the shadow shading loss is considered to consist of the following three parts.

1) The shadow projection of the tower covers the solar concentrator, leading to shading, no light is reflected from the shaded area to the collector center.

2) The shadow projection of solar concentrator V covers solar concentrator U, causing shading, and it cannot reflect light to the collector center.

3) Light reflected by solar concentrator U is blocked by solar concentrator V, preventing it from reflecting light to the collector center.

To simplify subsequent calculations, the paper defines the acute angle between the mirror surface and the xOy plane as the pitch angle α , as shown in Fig 6; The angle formed with the yOz plane is referred to as the heading angle β , as shown in Fig 7.

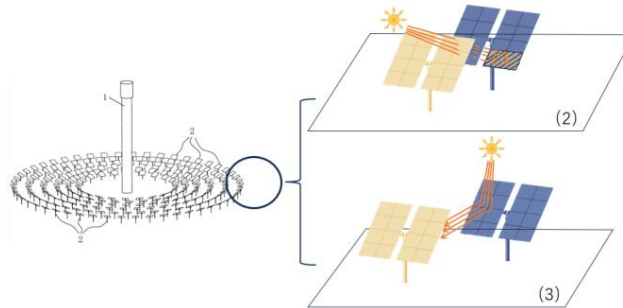


Figure 6. Illustrations of the Effects of Shading (2) and (3)

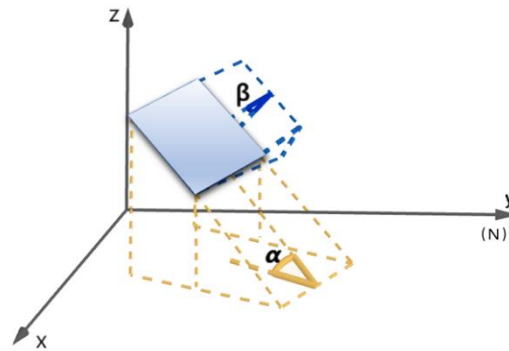


Figure 7. Pitch Angle and Heading Angle

The text mentions that with the known installation height H_0 as 4m, solve the triangle to obtain some results or calculations.

$$\cos\alpha = \frac{d_{HR}\sin\alpha_s + H_0}{2d_{HR}\cos\langle\vec{s}, \vec{f}\rangle} \quad (15)$$

$$\cos\beta = -\sin\left(\beta - \frac{\pi}{2}\right) = -\frac{x + d_{HR}\cos\alpha_s\sin\gamma_s}{\sqrt{(d_{HR}\cos\alpha_s\sin\gamma_s + x)^2 + (d_{HR}\cos\alpha_s\cos\gamma_s + y)^2}} \quad (16)$$

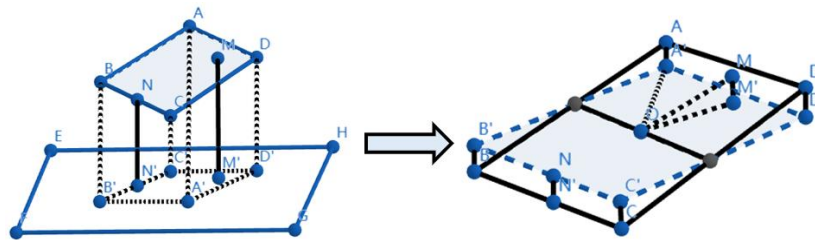


Figure 8. Illustration of Mirror Projection onto a Horizontal Plane

Draw a schematic diagram of the ground and the mirror plate as shown in Fig 8. From the diagram, we can determine the coordinates of the projected points A', B', C', D' on the xOy plane for the vertices of the solar concentrator. According to the principles of vertical projection, the following coordinate relationships exist:

$$(x_{X'}, y_{X'}) = (x_X, y_X) \quad (17)$$

Where $X = A, B, C, D$.

For any point $M \in$ plane $ABCD$, a perpendicular point $M' \perp$ plane $A'B'C'D'$, According to the principles of vertical projection, it can be determined that,

$$\begin{cases} M \in \text{Int}(ABCD) \Leftrightarrow M' \in \text{Int}(ABCD) \\ M \in \partial(ABCD) \Leftrightarrow M' \in \partial(ABCD) \\ M \in \text{Ext}(ABCD) \Leftrightarrow M' \in \text{Ext}(ABCD) \end{cases} \quad (18)$$

Where $\text{Int}, \partial, \text{Ext}$ respectively represent the interior, boundary, and exterior of the rectangular solar concentrator ABCD. So, the equivalence of point M being inside the rectangular solar concentrator ABCD can be expressed as follows Equation (19) being satisfied.

$$\begin{cases} |x_{M'} - x| \leq 3\cos\beta\cos\alpha \\ |y_{M'} - y| \leq 3\sin\beta\cos\alpha \end{cases} \quad (19)$$

By considering the direction of incident light in relation to vector $\vec{s}$, the point direction equation for incident light at any point M on the solar concentrator can be obtained as Equation (20).

$$\frac{x-x_M}{\cos\alpha_s\sin\gamma_s} = \frac{y-y_M}{\cos\alpha_s\cos\gamma_s} = \frac{z-z_M}{-\sin\alpha_s} \quad (20)$$

The point direction equation for reflected light at any point M on the solar concentrator can be easily obtained as Equation (21).

$$\frac{x-x_M}{x_M} = \frac{y-y_M}{y_M} = \frac{z-z_M}{z_M-80} \quad (21)$$

Next, the paper will proceed to calculate and analyze three types of shading situations in succession.

a) The Shading Effect of the Tower on the Solar Concentrator

According to the definition of the coordinate system, the equation of the plane containing the tower is $y = 0$. In this paper, by simultaneously equating the tower plane equation with the incident light equation, the coordinates of the intersection point M' are obtained. First, by equating the tower plane equation with the incident light equation, we obtain Equation (22).

$$\begin{cases} y = 0 \\ \frac{x-x_M}{\cos\alpha_s\sin\gamma_s} = \frac{y-y_M}{\cos\alpha_s\cos\gamma_s} = \frac{z-z_M}{-\sin\alpha_s} \end{cases} \quad (22)$$

The coordinates of point M' are solved for, resulting in Equation (23).

$$\begin{cases} x_{M'} = x_M - y_M \tan\gamma_s \\ y_{M'} = 0 \\ z_{M'} = z_M + y_M \frac{\tan\gamma_s}{\cos\gamma_s} \end{cases} \quad (23)$$

At this point, the analysis is conducted to determine whether the tower obstructs the solar concentrator by transforming the relative position relationships into coordinate relationships.

Because the tower and the collector studied in this paper have radii of 3.5m, a tower height of 80m, and a collector height of 8m, the coordinates of the top center of the collector should be (0, 0, 88-4), i.e., (0, 0, 84). If the point M' of intersection between the incident light ray and the tower plane is inside the tower, it indicates that the tower obstructs the incident light ray, thus causing a shading effect. M' being inside the tower implies that its coordinates satisfy the constraints of the tower radius and tower height. Therefore, in this paper, the equivalent characterization of the tower causing shading is obtained as Equation (24).

$$\begin{cases} -3.5 \leq x_{M'} \leq 3.5 \\ 0 \leq z_{M'} \leq 84 \end{cases} \quad (24)$$

b) Shading of incident light Rays Between Solar Concentrators

In this paper, we first define $\vec{f}_1 = \vec{t} + \vec{s}$, Therefore, $\vec{f}_1$ represents the normal vector of the solar concentrator's surface, denoted as $\vec{f}_1 = (A, B, C)$, the paper also denote the coordinates of another solar concentrator's center as $(x', y', 0)$, This results in the point-normal equation for the surface of the other solar concentrator as Equation (25).

$$\vec{f}_1 \cdot (x - x', y - y', z - 0) = 0 \quad (25)$$

By simultaneously equating it with the incident light equation (Equation 20), the coordinates on the xOy plane corresponding to point M' are obtained as follows,

$$\begin{cases} x_{M'} = x = \tan\gamma_s(y - y_M) + x_M \\ y_{M'} = y = \frac{A(\tan\gamma_s \cdot y_M - x_M + x') + By' - C(y_M \frac{\tan\alpha_s}{\cos\gamma_s} + z_M)}{A \tan\gamma_s + B - C \frac{\tan\alpha_s}{\cos\gamma_s}} \end{cases} \quad (26)$$

Where $A = \frac{-x'}{d_{HR}} - \cos\alpha_s \cos\gamma_s$, $B = \frac{-y'}{d_{HR}} - \cos\alpha_s \cos\gamma_s$, $C = \frac{80}{d_{HR}} + \sin\alpha_s$.

Since M being inside the rectangular solar concentrator is equivalent to Equation (19) being valid, this paper verifies whether the coordinates of M' in Equation (26) satisfy Equation (19). If they do, it indicates that M is inside the rectangular solar concentrator, thereby suggesting that another solar concentrator obstructs the path of the incident light to point M .

c) Shading of Reflected Light Rays Between Solar Concentrators

To investigate the shading effect of reflected light rays between solar concentrators, this paper further combines the equation for reflected light (Equation 21) with another mirror equation (Equation 25). The coordinates on the xOy plane obtained from this are as follows:

$$\begin{cases} x_{M'} = x = \frac{x_0}{y_0}(y - y_M) + x_M \\ y_{M'} = y = \frac{A(\frac{x_0}{y_0} \cdot y_M - x_M + x') + By' - C(\frac{8y_M}{y_0} - z_M)}{A \frac{x_0}{y_0} + B - C \frac{80}{y_0}} \end{cases} \quad (27)$$

Similarly to the calculation of the shading effect of solar concentrators on incident light rays in (b), verify whether the coordinates of M' in Equation (24) satisfy Equation (19). If they do, it indicates that M is inside the rectangular solar concentrator, thereby suggesting that another solar concentrator obstructs the path of the incident light to point M .

Finally, by utilizing Equation (28), the shadowing occlusion efficiency is determined.

$$\text{The Shadowing Efficiency } \eta_{sb} = 1 - \text{The Shadowing Loss} \quad (28)$$

2.3.4 Calculation of the Solar Collector Truncation Efficiency η_{trunc}

The truncation efficiency is defined as the ratio of the solar radiation absorbed on the absorber to the solar radiation reflected by the solar concentrator. Literature [11] suggests using a circular normal distribution to represent the flux density distribution on the absorber surface, as expressed in Equation (29). [12]

$$\eta_{trunc} = \frac{1}{2\pi\sigma_{tot}^2} \iint_D \exp\left(-\frac{x^2+y^2}{2\pi\sigma_{tot}^2}\right) dx dy \quad (29)$$

Here, x and y are the coordinates in the plane perpendicular to the surface of the solar collector, D is the area of the plane that intersects the collector's surface perpendicularly, and σ_{tot} is the total standard deviation of the flux distribution, which can be obtained using the following formula:

$$\sigma_{tot} = \sqrt{d_{HR}^2(\sigma_{sun}^2 + \sigma_{bq}^2 + \sigma_{ast}^2 + \sigma_t^2)} \quad (30)$$

Where, σ_{sun} represents the half-cone angle of the solar light cone when considering sunlight as having angular spread. This half-cone angle is also known as the standard deviation of the sun in a circular Gauss shape, and its value is 2.51 mrad. σ_{bq} represents the standard deviation of the optical beam quality associated with the slope of the reflecting surface. It can be calculated using the following formula (31):

$$\sigma_{bq}^2 = (2\sigma_s)^2 \quad (31)$$

Where, σ_s represents the slope error, with an approximate value of 0.94mrad. σ_t stands for the tracking error in the aiming precision of the solar concentrator, assumed to be 0.63mrad. The defocusing effect is considered with the standard deviation σ_{ast} and can be calculated using the following equation (32):

$$\sigma_{ast} = \frac{\sqrt{0.5(H_t^2 + W_s^2)}}{4d_{HR}} \quad (32)$$

Where, H_t and W_s are the image sizes on the tangential and sagittal planes at the receiver position, and in the rectangular solar concentrator, they have the following equations (33) and (34):

$$H_t = \sqrt{A}|1 - \cos < \vec{s}, \vec{f} >| \quad (33)$$

$$W_s = \sqrt{A}|\cos < \vec{s}, \vec{f} > - 1| \quad (34)$$

Where A represents the surface area of the rectangular solar concentrator.

3. Model Solution and Visualization Process

Combining the discussions on the "elemental process" above, this paper employs Matlab to perform calculations and analyses on each elemental unit. The daily parameter values are obtained by averaging data from 9:00, 10:30, 12:00, 13:30, 15:00 each day. These four parameters individually Programming is employed to calculate the parameter values for each moment and each day, and the results are presented in the form of visualizations. As an example, the visualization of the parameter values at noon (12:00) is shown in Fig 9-12.

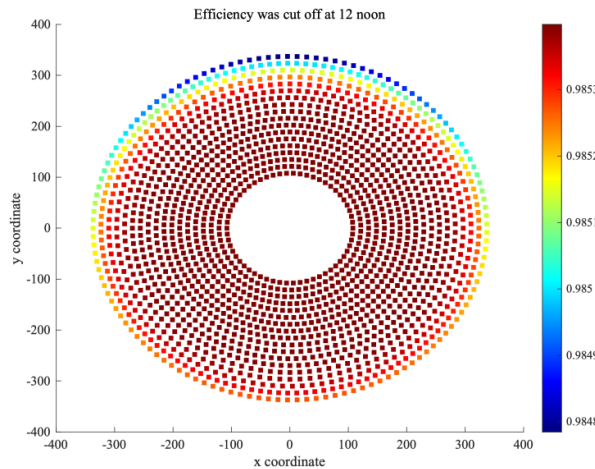


Figure 9. Efficiency was cut off

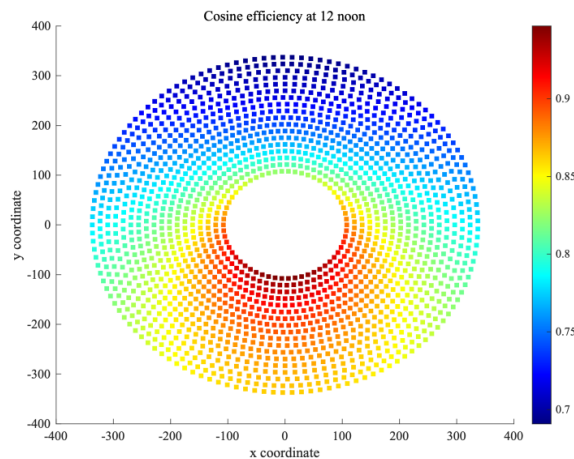


Figure 10. Cosine efficiency

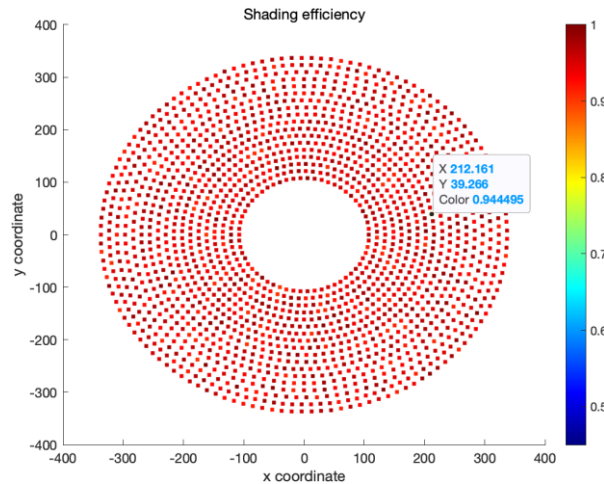


Figure 11. Shading efficiency

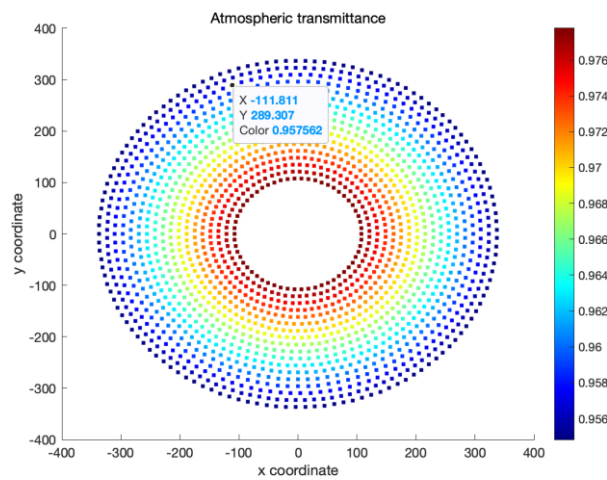


Figure 12. Atmospheric transmittance

4. Conclusion

This paper presents a comprehensive and in-depth study of the optical efficiency in a heliostat field based on ray-tracing methods. Firstly, the paper systematically analyzes the various components of optical efficiency and proposes corresponding computational models. In particular, regarding the calculation of shadow efficiency η_{sb} , the paper not only provides a detailed description of the impacts caused by mutual shading between the tower and heliostats, but also introduces the concepts of azimuthal angle and zenith angle, employing the microelement method for precise calculations. Additionally, for the truncation efficiency of the collector η_{trunc} , the paper employs **HFLCAL** model for calculation and provides specific mathematical expressions.

By using Matlab programming, this paper solved and visualized the model, obtaining numerical values for various parameters such as the average optical efficiency, average cosine efficiency, average shadow obstruction efficiency, average truncation efficiency, and the average output thermal power per unit area of the mirror for each 21st day of the month. In summary, these results are not only theoretically significant but also provide strong support for the practical design of solar heliostat fields.

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