

Research on the Layout of Tower-type Concentrated Solar Power Plants Based on the Ray Tracing Method

Zhongming Zhang^{1,*†}, Zihan Zhao^{2,†}, Xinyu Li^{1,†}

¹ School of Energy and Electrical Engineering, Qinghai University, Xining, China, 810016

² School of Civil and Hydraulic Engineering, Qinghai University, Xining, China, 810016

* Corresponding Author Email: 17538320307@163.com

† These authors contributed equally.

Abstract. Tower-type solar thermal power plants are favored by the industry for their high conversion efficiency and good commercial prospects. This paper focuses on the study of the optical energy transfer efficiency and thermal power output of the heliostat field in a tower-type solar thermal power plant. Firstly, based on the motion model of the sun and the earth, the ray tracing method is applied to investigate the path of mirror-reflected rays. Mathematical models are established for the optical efficiency, shading efficiency, cosine efficiency, truncation efficiency, and unit area thermal power output of the heliostat field. Based on these models, mathematical models for the optical energy transfer efficiency and thermal power output of the heliostat field are constructed, and the annual average thermal power output per unit mirror area of the solar thermal power plant is determined. Next, taking the maximum annual average thermal power output per unit mirror area of the heliostat field as the objective function, penalty functions are introduced to constrain the decision variables such as the height, width, and installation height of the heliostats. Finally, the objective function and penalty functions are incorporated into the Particle Swarm Optimization algorithm (PSO) for multiple iterations, yielding the maximum annual average thermal power output per unit mirror area of the heliostat field.

Keywords: Tower-type Concentrated Solar Power Plant, Shading Efficiency, Cosine Efficiency, Ray Tracing Method, Particle Swarm Optimization Algorithm.

1. Introduction

As the economy continues to develop, countries around the world are facing energy crisis issues. Traditional fossil fuels such as coal and oil, as non-renewable energy sources, are experiencing a gradual depletion [1], leading to serious environmental problems. Solar energy, characterized by its renewability, accessibility, and environmental friendliness [2], is an ideal renewable and clean energy source for a future low-carbon society. Tower solar thermal power generation technology plays a significant role in the application of solar energy, and it serves as an effective support for the new energy system with advantages such as thermal storage, load regulation, high collection efficiency, high thermal conversion efficiency, and continuous power generation to ensure the safe and stable operation of the power grid [3]. Therefore, tower solar thermal power generation technology is considered as a new energy technology that can replace conventional fossil fuels and is environmentally friendly [4]. It has received attention worldwide and has been actively researched and promoted. The study of solar thermal transfer efficiency in tower solar thermal power plants is of great value. In this paper, tower solar thermal power plants are taken as the research object. By establishing a model to calculate the power generation efficiency of the power plants, the particle swarm optimization algorithm is used to optimize the arrangement of heliostats, aiming to achieve optimal power generation efficiency and promote the development of the new energy industry. This research provides valuable references for the treatment aspect of tower solar thermal power generation technology. The data used in this paper is from the 2023 National College Student Mathematical Modeling Competition hosted by the Higher Education Press.

2. Earth Orientation Model

With the Earth's center as the origin, the xOy plane is the plane of the equator, and the direction towards the North Pole is the positive direction of the z -axis. Fixing a location P , the plane of the meridian at noon at that location is the xOz plane. A spatial rectangular coordinate system is established as shown in Fig 1.

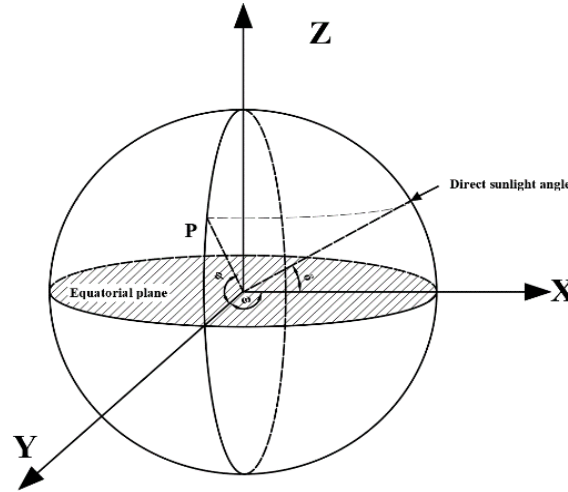


Figure 1. Diagram of the Sun shining on the Earth

The Earth completes one rotation in t^* hours. When point P reaches the xOz plane at noon, the corresponding hour angle at any given time $t (0 \leq t \leq t^*)$ is as follows.

$$\omega = \frac{\pi}{12}(t - 12) \quad (1)$$

For point P , let its latitude be $\varphi \left(-\frac{\pi}{2} \leq \varphi \leq \frac{\pi}{2} \right)$, where positive values represent the northern hemisphere and negative values represent the southern hemisphere.

To simplify the model, assume that the declination angle δ remains constant throughout the day.

$$\sin \delta = \sin \frac{2\pi D}{T^*} \sin \delta_0 \quad (2)$$

Where T^* represents the number of days in a complete year, δ_0 denotes the latitude of the Tropic of Cancer, and D is the number of days calculated from March 21st as the reference time.

The solar elevation angle at point P is defined as the angle between the direction of the incoming sunlight and the local horizontal plane at point P , as shown in Fig 2. Denoted by α_s , it is satisfied that

$$\sin \alpha_s = \cos \varphi \cos \delta \cos \omega + \sin \varphi \sin \delta \quad (3)$$

The solar azimuth angle at point P is defined as the angle measured in a clockwise direction from the north direction at point P to the direction of the incoming sunlight.

$$\cos \gamma_s = \frac{\sin \delta - \sin \alpha_s \sin \varphi}{\cos \alpha_s \cos \varphi} \quad (4)$$

Denoted by γ_s , it represents the solar azimuth angle at point P [5].

3. Shadow obstruction efficiency mode

The shadow obstruction loss can be divided into three parts: (1) the shadow loss caused by the absorption tower on the heliostat field; (2) the shadow loss incurred by the rear-row heliostat due to the sun being blocked by the front-row heliostat; (3) the loss caused by the reflection of solar radiation from the rear-row heliostat being blocked by the front-row heliostat. Below, we will calculate the shadow loss for each part.

3.1. The obstruction of the heliostat field by the absorption tower

Due to the variations in the solar altitude and azimuth angles at different times, the orientation and length of the shadow cast by the absorption tower also differ, which results in obstruction to the mirrors in the heliostat field. This is illustrated in Fig 2.

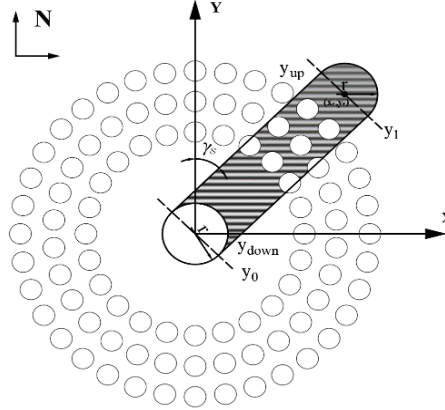


Figure 2. Shadow obstruction of the heliostat field caused by the shadow cast by the absorption tower

Based on geometric knowledge, the shadow boundary can be described when the solar altitude angle is α_s and azimuth angle is γ_s [6].

$$\begin{cases} y_{up} = \frac{x}{\tan \gamma_s} + \frac{r}{\sin \gamma_s} \\ y_{down} = \frac{x}{\tan \gamma_s} - \frac{r}{\sin \gamma_s} \\ y_0 = -\tan \gamma_s x \\ y_1 = -\tan \gamma_s x + \frac{l}{\cos \gamma_s} \\ (x_r, y_r) = ((l+r) \sin \gamma_s, (l+r) \cos \gamma_s) \end{cases} \quad (5)$$

Here, r represents the radius of the absorption tower and l represents the length of the tower's shadow at that moment. Therefore, all mirrors within the aforementioned range should be removed first.

3.2. Calculation of occultation ratio of heliostat using ray tracing method

The occlusion effect of heliostats is illustrated in Fig 3. The process of calculating the shadow loss caused by the heliostats is to determine whether a point on mirror A, along the direction opposite to the incident or reflected light, will fall on mirror B, and calculate the coordinates of that point on mirror B [7].

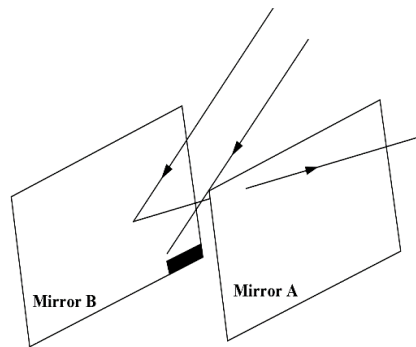


Figure 3. Schematic diagram of occlusion effect of heliostats

Divide the surface of mirror A into 10×10 equal square regions, with the center of each square representing the point of light incidence within the region. In the coordinate system of mirror A, given

a point H_1 , we seek the coordinates of a point H_2 where the ray passes and falls into mirror B, and determine whether H_2 is inside mirror B. This allows us to identify the occluded region of mirror A by mirror B.

Given $H_1''(x_1'', y_1'', z_1'')$ in the coordinate system of mirror B, we can obtain the vector of the incident ray $V_H(a, b, c)$, and solve for $H_2(x_2, y_2, 0)$ in the coordinate system of mirror B.

The equation of the line that contains the incident ray can be expressed in the form of parametric equations.

$$\frac{x_2 - x_1''}{a} = \frac{y_2 - y_1''}{b} = \frac{-z_1''}{c} \quad (6)$$

It can be solved that [8]

$$\begin{cases} x_2 = \frac{cx_1'' - az_1''}{c} \\ y_2 = \frac{cy_1'' - bz_1''}{c} \end{cases} \quad (7)$$

The number of obscured points on the mirror out of 100 points is denoted as S . The ratio of S to the total number of points on the mirror represents the shadow occlusion loss, denoted as η_{shadow} .

$$\eta_{sb} = 1 - \eta_{shadow} = 1 - \frac{S}{100} \quad (8)$$

In the above equation, η_{sb} represents the shadow occlusion efficiency of a single heliostat mirror.

Assuming that the total number of heliostat mirrors in the entire heliostat field is n , the shadow occlusion loss for each heliostat mirror at the five-time points of 9:00, 10:30, 12:00, 13:30, and 15:00 on the 21st day of each month is calculated, considering that they are not occluded by absorbing towers. Let $\overline{\eta_{sb,m}}$ represent the average shadow occlusion efficiency of the heliostat field on the 21st day of the m -th month, which can represent the average shadow occlusion efficiency of the heliostat field for that month.

$$\overline{\eta_{sb}} = \frac{\sum_{m=1}^{12} \overline{\eta_{sb,m}}}{12} \quad (9)$$

Where $\overline{\eta_{sb}}$ represents the annual average shadow occlusion efficiency of the heliostat field.

3.3. Truncation efficiency model

The truncation effect, also known as the spillage effect, occurs because sunlight is not parallel rays. Therefore, after being reflected by the heliostat mirrors, it is emitted in a conical shape. The half-cone angle α_0 is 4.65×10^{-3} rad. The process of light diffusion is shown in Fig 4.

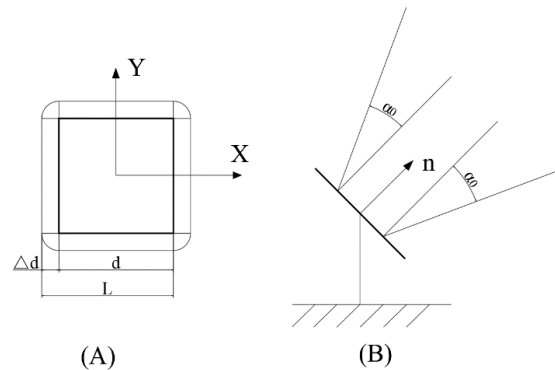


Figure 4. Schematic diagram of light cone diffusion

Through calculation, it can be determined that the truncation effect is mainly manifested as loss of light rays in the xOy plane, namely.

$$\eta_{tunc} = \frac{E_{rec}}{E_{ref} - E_{sha}} = \frac{E_{rec}}{E_{ref}(1 - E_{sha})} = \frac{E_{rec}}{\eta_{sb} E_{ref}} \quad (10)$$

Among them, E_{rec} represents the energy received by the receiver, E_{ref} represents the energy lost due to mirror total reflection, and E_{sha} represents the energy lost due to shadow obstruction.

3.4. Cosine efficiency model

Define θ_i as the solar incidence angle, which is the angle between the solar incident light direction and the normal direction of the earth's surface. Therefore, assuming that the direction vector of the incident light is $\mathbf{V}_0$ and the normal vector of the sun-tracking mirror is $\mathbf{n}$, the paper have:

$$\eta_{cos} = \cos \theta_i = |\mathbf{V}_0 \cdot \mathbf{n}| = |\cos \alpha_s \sin \gamma_s \sin \alpha \cos \beta + \cos \alpha_s \cos \gamma_s \sin \alpha \sin \beta - \sin \alpha_s \cos \alpha| \quad (11)$$

3.5. Optical efficiency model

The optical efficiency η of a single sun-tracking mirror is influenced by multiple factors and can be expressed by the following formula [9]:

$$\eta = \eta_{sb} \eta_{cos} \eta_{at} \eta_{trunc} \eta_{ref} \quad (12)$$

Where η_{sb} represents the shadowing and blocking efficiency of the sun-tracking mirror, η_{cos} is the cosine efficiency, η_{trunc} is the collector truncation efficiency, η_{ref} is the mirror reflectance, and η_{at} is the atmospheric transmittance [10].

The optical efficiency η of each mirror is calculated based on the aforementioned factors, and there are n sun-tracking mirrors in total. By plugging in the values of η_n for each mirror at the five-time points of 9:00, 10:30, 12:00, 13:30, and 15:00, as obtained from the formula, the following equation is derived. The average optical efficiency of the sun-tracking mirror field on the 21st day of the m month $\bar{\eta}_m$ is then calculated, where $\bar{\eta}_m$ represents the average optical efficiency of the field for the given month.

To obtain the annual average optical efficiency $\bar{\eta}$ of the sun-tracking mirror field, the value of $\bar{\eta}_m$ can be substituted into the following equation to calculate the result.

$$\bar{\eta} = \frac{\sum_{m=1}^{12} \bar{\eta}_m}{12} \quad (13)$$

3.6. Thermal power model

The output thermal power E_{field} of the sun-tracking mirror field is given by the following equation.

$$E_{field} = DNI \sum_{i=1}^n A_i \eta_i \quad (14)$$

Where n is the total number of sun-tracking mirrors, A_i is the collecting area of the i -th mirror, η_i is the optical efficiency of the i -th mirror. DNI refers to the solar radiation energy received per unit area per unit time on a plane perpendicular to the solar rays on Earth and can be approximated by the following equation.

$$DNI = G_0 \left[a + b \exp \left(-\frac{c}{\alpha_s} \right) \right] \quad (15)$$

Where G_0 represents the solar constant, a , b , and c are functions related to the altitude [11].

Calculate the output thermal power of the sun-tracking mirror field at five time points, namely 9:00, 10:30, 12:00, 13:30, and 15:00 on the 21st day of the m -th month. Taking the average of these values provides an estimation for the average output thermal power of the sun-tracking mirror field in the m -th month $\bar{E}_{field,m}$.

To calculate the annual average output thermal power of the sun-tracking mirror field $\bar{E}_{field}$, substitute $\bar{E}_{field,m}$ into the following formula.

$$\bar{E}_{field} = \frac{\sum_{m=1}^{12} \bar{E}_{field,m}}{12} \quad (16)$$

Divide the average output thermal power of the sun-tracking mirror field in the m -th month $E_{field,m}$ by the total mirror surface area to obtain $\overline{E_{s,m}}$. Consequently, the annual average output thermal power per unit area of the mirror $\overline{E_s}$ can be determined using the following equation.

$$\overline{E_s} = \frac{\sum_{m=1}^{12} \overline{E_{s,m}}}{12} \quad (17)$$

The annual average output thermal power of the sun-tracking mirror field can be obtained and is shown in Table 1.

Table 1. Annual average optical efficiency and output power table

AAOE ¹	AACE ²	AASOE ³	AATE ⁴	AATPO ⁵ (MW)	AATPOA ⁶ (kW/m ²)
45%	76%	91%	75%	27.57	0.44

4. Model of layout design optimization for sun-tracking mirror field based on particle swarm optimization algorithm

The objective of this paper is to maximize the annual average output power per unit mirror area. As indicated by the solution process above, $\overline{E_s}$ mean is given by Equation.

$$\overline{E_s} = \frac{\sum_{m=1}^{12} \overline{E_{s,m}}}{12} = \frac{\sum_{m=1}^{12} \frac{\overline{E_{field,m}}}{dwn}}{12} = \frac{\sum_{m=1}^{12} \frac{DNI \sum_{i=1}^n A_i \eta_i}{dwn}}{12} \quad (18)$$

Where d is the mirror height and w is the mirror width.

Based on this, the objective function is formulated and the constraints are determined according to the actual situation. Due to the complexity of the solution process, it is difficult to express the objective function as a simple function of decision variables. Therefore, it is necessary to recombine the obtained model mentioned above for further computation.

To utilize the PSO optimization algorithm for solving the objective function, the penalty function to be optimized is defined as follows:

$$y = 100\overline{E_s} - 10 \left| \overline{E_s} wd \frac{3500}{1000} - 60 \right| + (h-2)(6-h) + (w-2)(8-w) + (d-2)(8-d) + \left(h - \frac{d}{2} \right) \quad (19)$$

To meet the requirements, the constraints are determined and the optimization problem of $\min y$ is solved.

The fitness curve during the PSO algorithm process is shown in Fig 5. It can be observed that the curve tends to converge after 30 iterations.

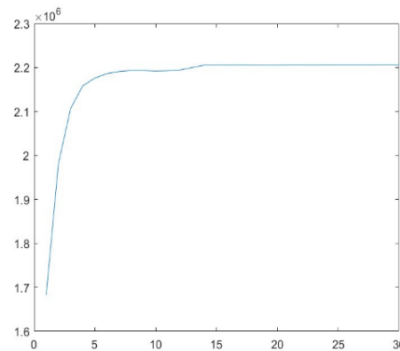


Figure 5. Fitness curve

¹ AAOE: Annual average optical efficiency.

² AACE: Annual average cosine efficiency.

³ AASOE: Annual average shadow occlusion efficiency.

⁴ AATE: Annual average truncation efficiency.

⁵ AATPO: Annual average thermal power output.

⁶ AATPOA: Annual average thermal power output per unit area mirror.

For reference, Figure 6 displays only the efficiency diagrams after optimizing the layout on December 21st at 15:00.

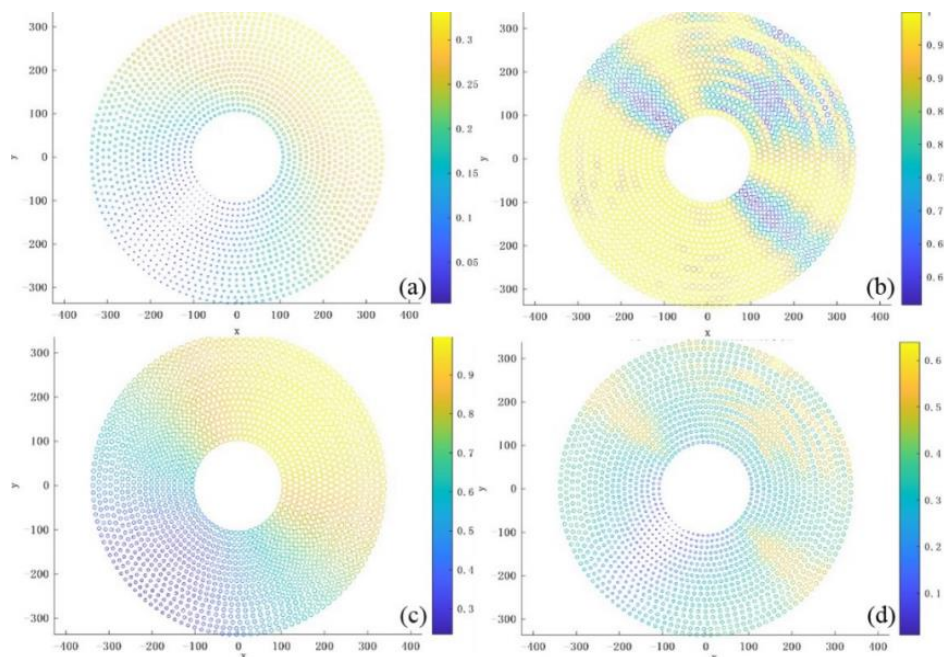


Figure 6. Shows the efficiency diagrams at 15:00 on December 21st, including (a) overall optical efficiency, (b) shading efficiency, (c) cosine efficiency, and (d) truncation efficiency

5. Conclusion

In this paper, the influence of the layout of a heliostat field on the energy transmission efficiency and output thermal power of a solar thermal power plant is studied. Mathematical models of the optical efficiency of the heliostat field, shading efficiency, cosine efficiency, truncation efficiency, and unit area output thermal power are established based on the trajectory model of the sun and the earth and the ray tracing method. The relationship between the above models and the energy transmission efficiency and output thermal power of tower-type solar thermal power plants is studied to obtain the values of the energy transmission efficiency and output thermal power of the tower-type solar thermal power plant. Furthermore, this paper introduces the objective function and penalty function and incorporates them into the particle swarm optimization algorithm. After multiple iterations, the maximum annual average output thermal power per unit area of the heliostat field is obtained. The optimization problem of the output thermal power of the tower-type solar thermal power plant is solved by designing the layout of the heliostat field. The ray tracing method used in this paper can more accurately simulate the physical propagation process of light rays and perform path analysis, making the simulation results more accurate. The global search ability of the particle swarm optimization algorithm is strong. The group behavior of the particle swarm allows the algorithm to explore multiple search space positions simultaneously and has high convergence and global optimization capabilities. The study of the layout design of a heliostat field for tower-type solar thermal power plants plays an important role in how to efficiently use solar energy resources, reduce construction and operation costs, and improving power generation efficiency.

References

- [1] Ma, W., International status and development trend of solar power generation industry. *Solar Energy*, 2020 (01), 5-12.
- [2] Li, H., Modeling and simulation study of tower-type solar thermal power generation with heat transfer and storage system. (Master's thesis, North China Electric Power University, 2020).

- [3] Qin, H., Overall layout and optimization of power generation area in the tower-type solar thermal power station. *Electric Power Survey & Design*, 2023.09.013.
- [4] Xu, L., Hou, X., Liu, K., Tower-type molten salt solar thermal power generation technology. *Energy Research & Information*, 2020, 36 (03), 135-142.
- [5] Cai, Z., Solar shadow positioning. *Mathematical Modeling and its Applications*, 2015, 4 (4), 25-33.
- [6] Liu Shi, Zou Guoxing, Zhang Guyu, et al. Design of a Freeform Surface Concentrator for Solar Simulators [J]. *Acta Optica Sinica*, 2023, 43 (06): 297-306.
- [7] Li Xiaolu, Hua Kaicheng, Lu Kexin, et al. Research on Energy-saving Technology for Highway Tunnels Based on Ray Tracing [J]. *Transportation Energy Conservation and Environmental Protection*, 2023, 19 (02): 122-129.
- [8] Zhang, P., Xi, Z., Hua, W., et al., Calculation method of optical efficiency for tower-type solar thermal mirror field. *Technology and Market*, 2021, 28 (06), 5-8.
- [9] Zhang, P., Xi, Z., Hua, W., et al. (2021). Calculation Method of Optical Efficiency for Tower-Type Solar Thermal Mirror Field. *Technology and Market*, 28 (06), 5-8. Fang, M., Lu, J., Jiang, Y., Modeling of energy transmission efficiency of heliostat and mirror field layout design. *Journal of Solar Energy*, 2021, 42 (01), 112-116.
- [10] Liu, J., Modeling, and simulation of optical efficiency of the tower-type solar thermal power station and optimization of heliostat field layout. (Master's thesis, Lanzhou Jiaotong University, 2022.001089.
- [11] Du, Y., et al., Analysis of the influence of different focusing strategies of heliostats in the tower-type solar thermal power station. *Journal of Power Engineering*, 2020, 40 (5), 426-432.