

# Reconstructed doppler cycle slip detection method based on inter satellite difference

Zhiqiang Zhang, Chenglin Cai, Qian Wu, Wei Li And Yuzhen Deng\*

School of automation and electronic information, Xiangtan University, Xiangtan, China

\*Corresponding author e-mail:1370132582@qq.com

**Abstract.** Aiming at the problem of single frequency cycle slip detection in GNSS location, a new cycle slip detection method is proposed. This method uses the position and speed of the satellite and the position and speed of the receiver to calculate the Doppler frequency shift, then uses the integral value of the Doppler frequency shift to make a difference with the station satellite distance, and uses the inter satellite difference method to eliminate the receiver clock error, which can accurately detect the small cycle slip of one cycle. This method is not affected by the coordinates of the receiver, and can be realized only by the pseudo range single point positioning result.

**Keywords:** Single frequency; Cycle slip detection; Reconstructed Doppler; Inter satellite difference.

## 1. Introduction

Single frequency precise single point positioning requires only a single single frequency receiver to achieve high-precision positioning, which is widely used. However, in the process of single frequency precise single point positioning, it is inevitable that cycle jump will occur, resulting in positioning error or even divergence. For single frequency observation signals, the processing strategy of epoch difference is usually adopted. The common methods are phase pseudo range combination method and Doppler integration method. The phase pseudo range combination method includes pseudo range noise, and generally can only detect large cycle slips. The Doppler integration method is greatly affected by the sampling rate, with an error of about 2cm / s. This paper proposes an inter satellite differential reconstruction Doppler method based on ephemeris calculation, which uses the previous epoch positioning solution or pseudo range positioning solution, combined with the precise ephemeris to calculate the Doppler frequency shift value for integration, and then uses the inter satellite differential method to eliminate the receiver clock error, It can truly reflect the change of carrier phase. The experiment shows that one cycle slip can be detected under the conditions of 1s and 30s sampling rate.

## 2. Single frequency cycle slip detection method

### 2.1 Phase pseudorange combination method

The observation equation of non differential single frequency precision single point positioning is:

$$P = \rho_r^s + c \cdot (dt_r - dt^s) + d_{trop} + I_p + d_{rel} + d_{mul}^P + \varepsilon_p \quad (1)$$

$$\phi = \rho_r^s + c \cdot (dt_r - dt^s) + d_{trop} + I_\phi + d_{rel} + d_{mul}^\phi + \lambda \cdot N + \varepsilon_\phi \quad (2)$$

In the formula,  $P$  is the pseudo range observation value;  $\phi$  is the carrier phase observation value;  $\rho_r^s$  is the geometric distance from the satellite to the receiver;  $c$  is the speed of light;  $dt_r$  is the clock error of the receiver;  $dt^s$  is satellite clock error;  $d_{trop}$  is tropospheric delay;  $I_p$  And  $I_\phi$  tropospheric delay of carrier phase and pseudo range observations respectively;  $d_{rel}$  is Relativistic effect;  $\lambda$  is the wavelength of the carrier  $L_1$ ;  $N$  Is the integer ambiguity of the carrier  $L_1$ ;

Subtract Formula (1) from formula (2):

$$\phi - P = \lambda \cdot N + (I_\phi - I_p) + (\varepsilon_\phi - \varepsilon_p) + (d_{mul}^\phi - d_{mul}^P) \quad (3)$$

Because the whole week ambiguity will not change between adjacent epochs, Difference between epochs for Formula (3):

$$\varepsilon = \frac{(\phi_{i+1} - \phi_i) - (P_{i+1} - P_i)}{\lambda} \quad (4)$$

Theoretically,  $\varepsilon$  should fluctuate within a certain range. If there is a cycle jump in the actual situation, the above formula will exceed the specified range. Therefore, whether cycle slip occurs can be determined according to whether it is greater than the threshold.

## 2.2 Doppler integral method

The relationship between Doppler observation value and carrier phase is:

$$D = \frac{d\phi}{dt} \quad (5)$$

In the formula,  $D$  represents the change rate of instantaneous carrier phase;  $\phi$  Is the carrier phase observation value;  $t$  is the observation time. Then there are:

$$\phi_{i+1} = \phi_i + \int_{t_i}^{t_{i+1}} d(t)dt \quad (6)$$

In the formula,  $\phi_i$  is the carrier phase observation value;  $d(t)$  is Doppler observation value;  $t$  is the time of the observation epoch. The integral value of Doppler observation value is recorded as  $\Delta dop_{i+1}$  that if cycle slip occurs between adjacent epochs, it will be compared with  $\phi_{i+1}$  and then with  $\phi_i$ . The error only includes the influence of observation noise and should be within a certain range:

$$(\phi_{i+1} - \phi_i) - \Delta dop_{i+1} < \varepsilon \quad (7)$$

If  $n$  cycle jumps occur, the size of the cycle jump is:

$$n = INT\left[\frac{(\phi_{i+1} - \phi_i) - \Delta dop_{i+1}}{\lambda}\right] \quad (8)$$

In the formula,  $INT[*]$  is the rounding function.

## 3. Inter satellite differential reconstruction Doppler cycle slip detection method

A stationary signal transmitting tower broadcasts a signal with frequency  $f$ , and the receiver operates at speed  $v$ , then the signal frequency  $f_r$  received by the receiver is no longer the transmission frequency  $f$  of the signal, but  $f + f_d$ . We call this phenomenon that the signal receiving frequency changes with the relative motion between the signal transmitting source and the receiver as Doppler effect, and  $f_d$  as Doppler frequency shift. Thus, the Doppler shift  $f_d$  is equal to the difference between the signal reception frequency  $f_r$  and the transmission frequency

$$f_d = f_r - f \quad (9)$$

Based on the basic theory of electromagnetic wave propagation, we can strictly deduce the following calculation formula of Doppler frequency shift:

$$f_d = \frac{v}{\lambda} \cdot \cos \beta \quad (10)$$

It can be seen that the Doppler effect reflects the change of the connecting distance between the signal transmitting source and the receiver, which is directly proportional to the projection of the operating speed of the receiver in the direction of signal incidence.

Formula (10) is extended to dynamic emission sources:

$$f_d = \frac{(v - v^s) \cdot I^s}{\lambda} = -\frac{(v^s - v) \cdot I^s}{\lambda} = \frac{-\dot{r}}{\lambda} \cdot f_0 \quad (11)$$

In the formula,  $v$  is the receiver speed,  $v^s$  is satellite speed,  $I_s$  is the directional cosine vector of the satellite relative to the receiver, it can be expressed as:

$$I^s = \frac{1}{\sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}} \cdot \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} \quad (12)$$

Doppler value is closely related to Doppler integral. Doppler value represents the change rate of satellite speed relative to receiver in a certain time, while Doppler integral represents the displacement change of satellite and receiver in a certain time. When obtaining the precise ephemeris of the satellite, only a rough positioning solution combined with the receiver speed can reconstruct the Doppler value. In this way, the accuracy of the calculated Doppler value is higher and more stable.

The integration of Doppler observation value reflects the change of carrier phase, which contains certain error, and it is greatly affected by the sampling rate, about 2cm / s. when reaching the 30s sampling rate, the Doppler integration contains about 4 weeks of sampling error. As shown in the figure below, at the sampling rate of 1 second, the waveform of Doppler observation value integral and carrier phase change value is similar, while the reconstructed Doppler integral is closer to a straight line, indicating that the value of reconstructed Doppler integral is more stable, because it really reflects the relative distance between the satellite and the receiver. When the receiver is static, the Doppler value is mainly affected by the speed of the satellite. Take the x component as an example ,decompose formula (11):

$$f_d^x = (v_x^s - v_x^R) \frac{x_s - x_R}{\lambda * \sqrt{(x_s - x_R)^2 + (y_s - y_R)^2 + (z_s - z_R)^2}} \quad (13)$$

When the satellite coordinates are accurate, the directional cosine of the satellite and the receiver can be regarded as a nonlinear equation of the direction. Linearize it at the  $x_0$  ,Only the first-order term is retained to obtain:

$$\frac{x_s - x_R}{\sqrt{(x_s - x_R)^2 + (y_s - y_R)^2 + (z_s - z_R)^2}} = \frac{x_0 - x_R}{S} + \left( \frac{(x_0 - x_R)^2}{S^3} - \frac{1}{S} \right) * (x - x_0) \quad (14)$$

In the above formula,  $\left( \frac{(x_0 - x_R)^2}{S^3} - \frac{1}{S} \right) * (x - x_0)$  is the deviation term, and its coefficient is an infinitesimal. In other words, the influence of the change of receiver coordinate value on the observed value of reconstructed Doppler can be ignored. The reconstructed Doppler integral reflects the real distance between epochs, and the real distance is affected by the relative velocity of satellite and receiver.

In the t observation epoch, the measured value of Doppler frequency shift can be written as:

$$f_d = \dot{\phi}_r^s(t) - \dot{\delta}_r + \dot{\delta}^s - \dot{\varepsilon} \quad (15)$$

In the formula,  $f_d$  is the reconstructed Doppler value,  $\dot{\phi}_r^s(t)$  the carrier phase change rate,  $\dot{\delta}_r$  is the receiver clock error change rate and  $\dot{\delta}^s$  is the satellite clock error change rate.  $\varepsilon$  is the influence of other errors that can be ignored in a short time interval.

Doppler observation reflects the change rate of carrier phase including various errors, while reconstructed Doppler reflects the change rate of distance between satellite and receiver including various errors. The corresponding reconstructed Doppler integral is:

$$Dop = \Delta\phi - \Delta dt_r + \Delta dt^s + \Delta\varepsilon \quad (16)$$

In the formula, in the reconstructed Doppler integral,  $\Delta\phi$  is the change value of carrier phase,  $\Delta dt_r$  is the receiver clock drift,  $\Delta dt^s$  is the satellite clock drift,  $\Delta\varepsilon$  and the remaining errors.

In Fig. 1, the waveform of the change of carrier phase is completely different from the reconstructed Doppler integral, because the receiver clock error between epochs is not equal, and the receiver clock error is the parameter to be estimated for positioning, that is, the receiver clock error of this epochs is unknown, although some scholars have proposed many polynomial fitting modeling methods of receiver clock error to decompose the receiver into three unknowns like satellite clock error, That is, it includes the clock drift and frequency drift of the receiver, but the receiver clock drift is unstable and often presents irregular changes. Therefore, the calculated receiver clock drift still has a large

error. The experimental result in Fig. 2 is the satellite clock drift calculated in combination with the final ephemeris. Compared with Fig. 1 and Fig. 2, it can be seen that when the reconstructed Doppler integral value plus the receiver clock error is roughly the same as the waveform of the carrier phase. It can also be seen from Figure 1 that the receiver clock error includes about 2 cycles of error for the detection cycle slip of reconstructed Doppler integral value, and it is extremely unstable.

The satellite clock drift can be obtained from the broadcast ephemeris. In a short time, the atmospheric error can be almost ignored. If the receiver clock error can be eliminated, the reconstructed Doppler integral value can accurately reflect the change of carrier phase. In positioning, the difference method is often used to eliminate the error. The receiver clock error is a common error for satellites. If the difference is made between satellites, a combined value without receiver clock error can be obtained. If the reference satellite has no cycle slip, the reference satellite can be used to detect the cycle slip of other satellites. So we can get the formula:

$$\varepsilon = (Dop_j - Dop_i) - [(\phi_{j+1} - \phi_j) - (\phi_{i+1} - \phi_i)] \quad (17)$$

The change of  $\varepsilon$  should be within a certain range, and the threshold can be set to judge whether cycle slip occurs. The trapezoidal integration method is used for the calculation of Doppler integration method and reconstructed Doppler integration method. Although the satellite speed changes very stably, the satellite speed is not really uniform acceleration, so the trapezoidal integration method still contains integration error, which is related to the sampling rate. Both Doppler observation value integration method and reconstructed Doppler method are affected by the sampling rate. The integral error of reconstructed Doppler caused by sampling rate is small. Fig. 3 and Fig. 4 show the integration errors of three satellites at the sampling rate of 1s and 30s respectively. The experimental results show that when the sampling rate is 1s, the integration error is completely negligible, while when the sampling rate reaches 30s, PRN15 and PRN12 contain integration errors of about half a cycle, PRN12 changes stably, PRN15 changes less than half a cycle from the first epoch to the 100th epoch, and the change trend is stable.

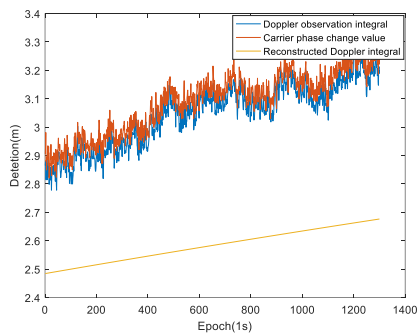


Fig. 1 Distance change.

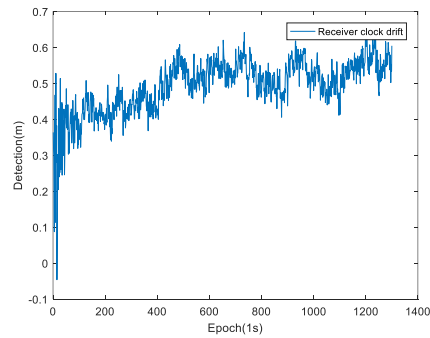


Fig. 2 Receiver clock drift.

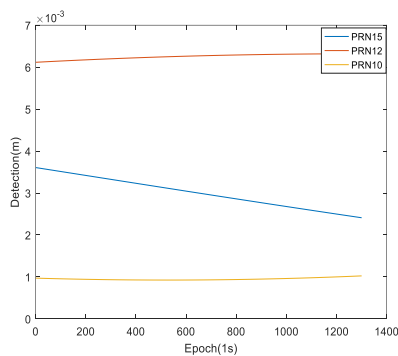


Fig. 3 Integral error.

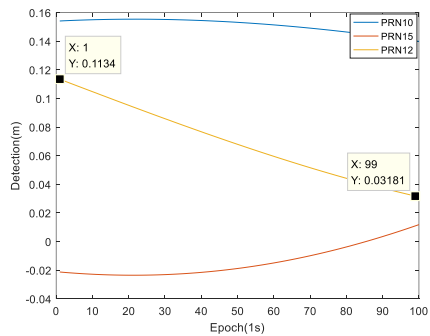


Fig. 4 Integral error.

## 4. Experimental analysis

In order to verify the effectiveness of the method, this paper uses two schemes to compare with two common single frequency cycle hop methods and inter satellite differential reconstructed Doppler methods.

### 4.1 Scheme 1 different sampling rates

From experiment 1 to experiment 3, the observation data of 1s sampling rate on September 18, 2019 collected by the laboratory and hexinxingtong receiver are used. Four cycle jumps are added manually in the 300th epoch, two cycle jumps are added manually in the 600th epoch, and one cycle jump is added manually in the 900th epoch. It can be seen from Fig. 5 to Fig. 7 that the three methods can detect a small cycle jump, while the detection error of inter satellite difference method is the smallest and that of Doppler integration method is the largest. It can be seen from Fig. 5 that the influence of receiver noise is eliminated when inter satellite difference is used, The atmospheric errors such as tropospheric error and ionosphere error can be ignored in a short time, and the satellite clock can also be ignored in a short time. Therefore, the reconstructed Doppler method of inter satellite difference only contains small integration error.

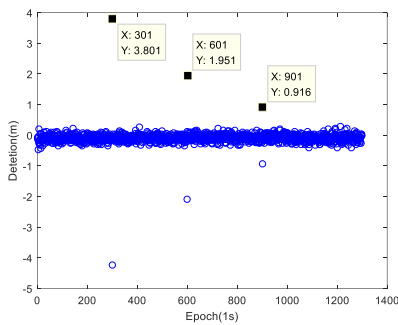


Fig.5 Doppler integration method.

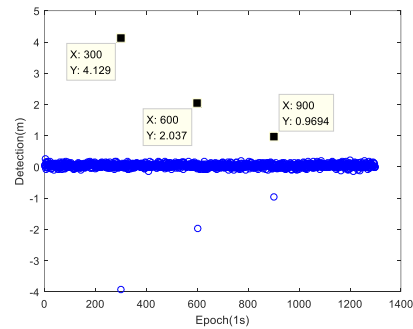


Fig.6 Phase pseudorange combination method.

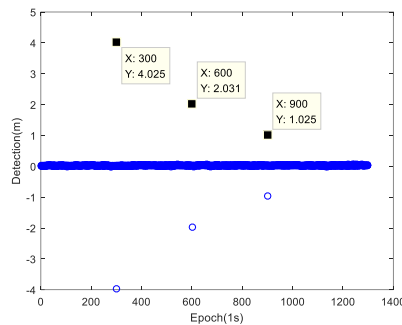


Fig.7 Inter satellite differential Doppler reconstruction method.

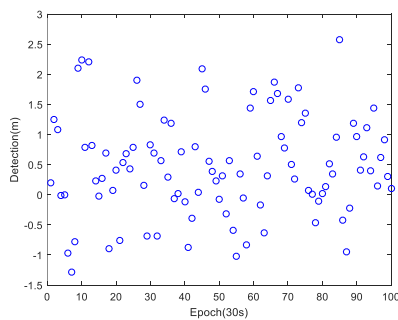


Fig.8 Doppler integration method.

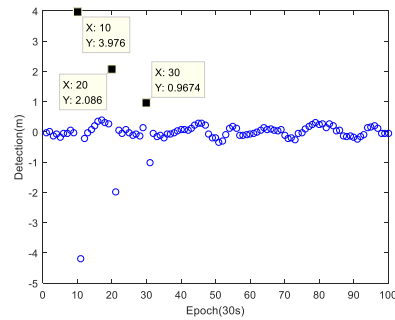


Fig.9 Phase pseudorange combination method.

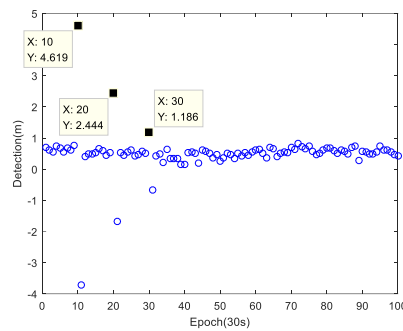


Fig.10 Inter satellite differential Doppler reconstruction method.

Experiment 4 to experiment 6 are the observation data of 30s sampling rate with hexingxintong, as shown in Fig.8 to Fig.10. 4-week cycle jump is added manually in the 10th epoch, 2-week cycle jump is added manually in the 20th epoch, and 1-week cycle jump is added manually in the 30th epoch. The experimental results show that the Doppler integral method is greatly affected by the sampling rate and contains 4 weeks of error at the 30 second sampling rate, so it can not be detected accurately. Both the phase pseudo range combination method and the inter satellite differential reconstruction Doppler method can detect one week of small cycle slip. The experimental comparison results are shown in table.1.

Table 1. Scheme 1.

sampling rate	Epoch	cycle jump	Doppler integral method	Phase pseudorange combination method	Inter satellite differential Doppler reconstruction method
1s	300	4	√	√	√
	600	2	√	√	√
	900	1	√	√	√
30s	10	4	×	√	√
	20	2	×	√	√
	30	1	×	√	√

#### 4.2 Scheme 2 different receivers

Because the phase pseudo range combination method is greatly affected by the pseudo range residual, its cycle slip detection ability will be weakened when the observation noise output by the receiver is large. Experiments 7 to 9 are the 30s data of IGS observation station, as shown in Fig.11 to Fig.13. Add 4-week cycle jump in the 100th epoch, 2-week cycle jump in the 200th epoch and 1-week cycle jump in the 300th epoch. The experimental results show that the observation noise of the phase pseudo range combination method is too large, resulting in high observation error. In this experiment, 4-week and 2-week cycle slips can be detected, but there are many misjudgment points due to too large noise, as shown in the red dot in Fig. 12. The reconstructed Doppler method of inter satellite difference can still detect a small cycle slip in one week, but due to its integration error, there are blind spots as shown in the red dot in Fig. 13. The experimental comparison results are shown in table.2.

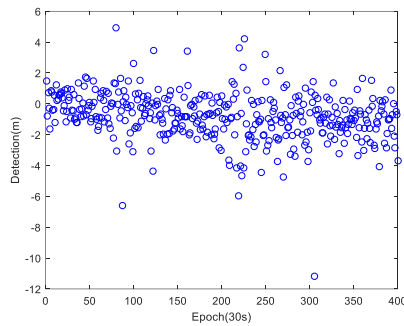


Fig.11 Doppler integration method.

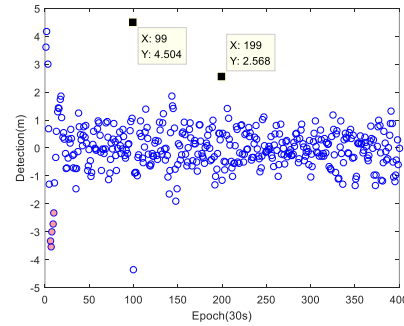


Fig.12 Phase pseudorange combination method..

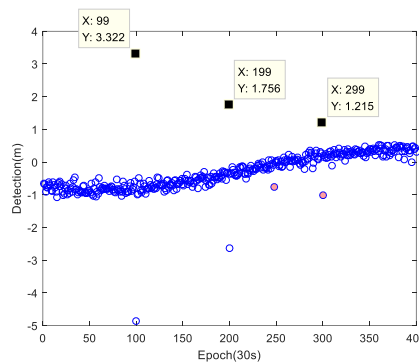


Fig.13 Inter satellite differential Doppler reconstruction method.

Table 2. Scheme 2.

Receiver	Epoch	cycle jump	Doppler integral method	Phase pseudorange combination method	Inter satellite differential Doppler reconstruction method
hexingxingtong	10	4	√	√	√
	20	2	√	√	√
	30	1	√	√	√
IGS	100	4	×	√	√
	200	2	×	√	√
	300	1	×	×	√

## 5. Conclusion

Aiming at the problem that the traditional phase pseudo range method is greatly affected by pseudo range noise and the Doppler integration method is greatly affected by sampling rate in single frequency cycle slip detection, a reconstructed Doppler method based on inter satellite difference is proposed in this paper. Its core idea is to use the satellite velocity and clock error information in the precise ephemeris to calculate the accurate Doppler integration value with only a rough positioning solution, It can accurately reflect the change of station satellite distance between epochs, and then use the method of inter satellite difference to eliminate the error caused by the clock drift of the receiver. If the other errors can be ignored, it can accurately reflect the change of carrier phase. In the experiment, the small cycle slip of one cycle can be detected no matter whether the sampling rate of 1s or 30s is adopted or the data of different receivers are adopted.

## Acknowledgments

This work was financially supported by ZhiQiang Zhang fund. First of all, I need to thank my mentor Cai Chenglin for his guidance in writing my thesis, Secondly, I would like to thank my classmates for their help, Finally, I would like to thank the conference for its recognition of my paper.

## References

- [1] Liu Ning, Zhang Qin, Zhang Shuangcheng, Wu Xiaoli. Algorithm for Real-Time Cycle Slip Detection and Repair for Low Elevation GPS Undifferenced Data in Different Environments[J]. Remote Sensing, 2021, 13(11).
- [2] Yu Xianwen, Xia Siqi. A highly adaptable method for GNSS cycle slip detection and repair based on Kalman filter[J]. Survey Review, 2021, 53(377).
- [3] Li Dehai, Dang Yamin, Yuan Yunbin, Mi Jinzhong. Improved Cycle Slip Repair with GPS Triple-Frequency Measurements by Minifying the Influences of Ionospheric Variation and Pseudorange Errors[J]. Remote Sensing, 2021, 13(4).
- [4] Yang Zhouming, Liu Xin, Guo Jinyun, Xia Yaowei, Chang Xiaotao, Renaudin Valerie. An Enhanced Method for Detecting and Repairing the Cycle Slips of Dual-Frequency Onboard GPS Receivers of LEO Satellites[J]. Journal of Sensors, 2020, 2020.
- [5] Xu Xiaofei, Nie Zhixi, Wang Zhenjie, Zhang Yuanfan. A Modified TurboEdit Cycle-Slip Detection and Correction Method for Dual-Frequency Smartphone GNSS Observation[J]. Sensors (Basel, Switzerland), 2020, 20(20).
- [6] Jiaojiao Zhao, Manuel Hernández-Pajares, Zishen Li, Liang Wang, Hong Yuan. High-rate Doppler-aided cycle slip detection and repair method for low-cost single-frequency receivers[J]. GPS Solutions: The Journal of Global Navigation Satellite Systems, 2020, 24(3).
- [7] Xianwen Yu, Siqi Xia. A highly adaptable method for GNSS cycle slip detection and repair based on Kalman filter[J]. Survey Review, 2020.
- [8] Engineering; Researchers from National University of Defense Technology Discuss Findings in Engineering (Real-time Quadruple-frequency Cycle Slip Detection and Repair Algorithm Based On the Four Chosen Linear Combinations)[J]. Journal of Mathematics, 2020.
- [9] Sensor Research; Beihang University Details Findings in Sensor Research (A Cycle Slip Detection Framework for Reliable Single Frequency RTK Positioning)[J]. Journal of Technology, 2020.
- [10] Xinyang Zhao, Zun Niu, Gaoxu Li, Qiangqiang Shuai, Bocheng Zhu. A New Cycle Slip Detection and Repair Method Using a Single Receiver's Single Station B1 and L1 Frequencies in Ground-Based Positioning Systems[J]. Sensors, 2020, 20(2).
- [11] Nikolai S. Kosarev, Konstantin M. Antonovich, Leonid A. Lipatnikov. The method of cycle-slip detection and repair GNSS measurements by using receiver with high stability frequency oscillator[J]. Contributions to Geophysics and Geodesy, 2019, 49(3).