Principle and Application of Complex Function in Geography, Optics, Communication Engineering

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Abstract. As a matter of fact, complex function has been widely used in various fields. This study explores the applications of complex functions in the fields of geography, optics, and communication engineering. By analyzing practical problems in different domains, scholars have discovered the unique value and prospects of complex functions in these areas. By integrating complex function theory with geographic data analysis, one has achieved efficient processing and analysis of geographic data and especially in calculating the minimum height. In the field of optics, one has utilized complex function methods to study the imaging and transmission characteristics of optical systems and image fusion and have achieved satisfactory results. Additionally, this study has introduced complex functions into the field of communication engineering, studying transforming the signal in the time domain into the frequency domain. The findings of this research enrich the theoretical content of scientific fields such as geography, optics, and communication engineering, providing strong support and references for related work.

Keywords: Complex function; geography; optics; communication engineering.

1. Introduction

In 16th century, when Cardano, an Italian Mathematician was solving the cubic equation, he was confused about after taking square root of a negative number, whether the result was a number. Since Cardano did not know the answer, he called it imaginary. The word, ‘imaginary number’, was first mentioned by Descartes in the 17th century. In the following two centuries, mathematicians tried to explain complex variable function geometrically and correspond to plane vectors in order to solve problems existing in real life. It was two amateur mathematicians, Wessel (Norwegian) and Robert Argand (French) who first made simple geometrical explanation of complex number [1]. In the 18th century, the theory of complex function was established, and comprehensively developed in the 19th century [2]. Similar to how calculus' direct extension dominated mathematics in the 18th century, the branch of complex functions dominated mathematics in the 19th century. This was due to the thorough development of the theory of complex functions. The most comprehensive area of mathematics at that time was complex function theory, which some mathematicians referred to as the mathematical delight of the century and one of the most harmonious theories in all of abstract science. In the year 1831, Carl Friedrich Gauss mentioned the ‘complex plane’, so people were able to combine complex functions with vectors. Then, in the 20th century, the theory of complex variable functions made further progress, and Verstras's students, the Swedish mathematician Levler, the French mathematicians Poincaré and Adama, etc., all did a lot of research work, opened up a broader field of research in the theory of complex variable functions, and contributed to the development of this field.

The theories of complex function nowadays have become an important part of mathematics. As there are plenty of perfect theories and exquisite skills in the complex number system, these theories have promoted the development of many subjects. When coming to some practical problems, such theories are very useful and sometimes simpler than other approaches. For instance, when solving differential functions, it is easier to calculate using complex function theories compared with method of undetermined coefficients.The theories of complex variable function have had a significant impact on the growth of several fields of mathematics, including differential equations, integral equations, probability theory, and number theory. Moreover, these theories have been widely used in other subjects. In the study of vibration and wave in mechanics, alternating current in electricity,

electromagnetic wave, interference diffraction in optics, wave function in quantum mechanics, diffusion in physics, etc., complex functions can be used [1].

The complex functions play an important role in mathematics and other fields nowadays. Since complex function theories are able to deal with various abstract practical problems. And sometimes easier to calculate and analyze than normal ways. More and more researchers attach more importance on this field. Therefore, it is necessary to clarify in several fields that complex functions can be utilized. Therefore, in this paper, applications of complex function in three different fields will be concluded. Before concluding the applications, there will be a basic description of complex functions, which mainly explains key theories and basic definitions in complex function theories. Then, it will come to the three applications of complex function. Firstly, applications in Geography will be studied. In Geography, the applications in analyzing terrains will be studied, especially in calculating the minimum height of trapezoidal filter in earth dams [3]. Secondly, applications of complex functions in Optics will be studied. In this part, complex function will be used as a template for image fusion[4]. Thirdly, the applications of those theories in communication engineering will be discussed. In this field, Fourier series and Laplace transform will be applied. After concluding all the three applications, the limitations and future outlooks will be illustrated. Eventually, there will be a summary of the whole paper to make a conclusion of contents.

In conclusion, this paper aims at concluding applications of complex function in Geography, Optics and Communication Engineering, and illustrating the traits of complex functions in those fields. In this paper, Mathematics will be combined with and applied to Communication Engineering and Geography. The author sincerely hope that this paper could provide sufficient help for future researches in all the three fields.

2. Basic Descriptions

Complex numbers can be defined as ordered pairs (x, y) of real numbers that are to be interpreted as points in the complex plane, with rectangular coordinates x and y, just as real numbers x are thought of as points on the real line [5]. One writes x = (x, 0) when real numbers are shown as points (x, 0) on the real axis, making it apparent that the real numbers are a subset of the set of complex numbers. When x= 0, y≠0, pure imaginary numbers, or complex numbers of the type (0, y), are defined as points on the y-axis. The imaginary axis is thus referred to as the y axis. A complex number, (x,y) or x+iy, can be customarily denoted by z in the complex plane. A sketch is shown in Fig. 1.

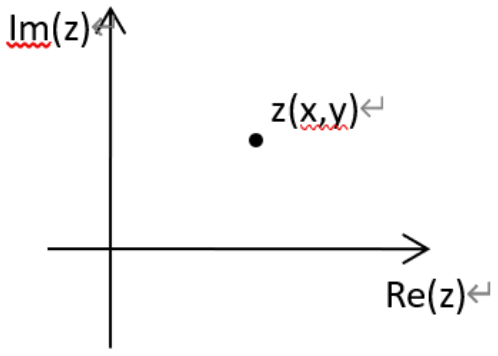


Fig. 1 Sketch of complex number.

Only when both the real part and imaginary part of the complex numbers are the same, the complex numbers are equal to each other. As a result, the phrase "$z_1 = z_2$" denotes that z_1 and z_2 represent the same complex plane point.The sum z_1+z_2 and product z_1z_2 of two complex numbers z_1= (a, b) and z_2 = (m, n) are defined as follows: For the sum rule, it is defined as $z_1 + z_2$ =(a,b)+(m,n)=(a+m,b+n); For the product rule, it is defined as z_1z_2 =(a,b)(m,n)=(am-bn,an+bm). It is pronounced that the sum of two complex numbers is a new point on complex plane whose coordinate is (the sum of two real parts, the sum of two imaginary parts). As $i^2 =$

$-1, z_1 z_2$=(a+ib)(m+in)=am-bn+i(an+bm). When expanding the equation, it is the same way in real number system. The difference z_1-z_2 and the quotient z_1/z_2 are defined as follows: For the difference rule, it is similar to that in real number system as well. The difference of two complex numbers, z_1-z_2, is the combination of the difference between the real parts and the difference of the imaginary parts, which means z_1-z_2=(a-b)+i(m-n). In the division rules, $z_1 \div z_2$ will be written into $\frac{a+ib}{m+in}$. Then multiplying the conjugate of denominator to make the denominator become a real number (m^2+n^2). Afterwards, expanding the numerator to a polynomial, the result is got then:

$$z_1/z_2 = \frac{am+nb+i(bm-an)}{m^2+n^2} \tag{1}$$

According to the division is the inverse operation of multiplication, all roads lead to Rome, it is easy to obtain the law of complex division [6]. If the imaginary part equals 0, these rules are correct as well, which means that real number is the subset of complex numbers. As a result, the real number system is expanded upon by the complex number system.

3. Applications in Geography

The application of complex function in terrain analysis is mainly embodied in the generation of terrain model and the analysis of terrain surface. The generation of terrain model refers to describing the height change of terrain through complex variable function, so as to generate the real terrain model. The analysis of terrain surface is to use the property of complex variable function to analyze the characteristics and change rules of terrain surface. Through mapping relations, the complex function can convert a complex plane into a terrain surface when creating terrain models. For instance, the fluctuation in argument and modulus of a complex function can be used to describe a region's topography. Mountains, rivers, sand dunes, and other types of terrain can all be precisely produced by choosing the right complex function and adjusting the parameter values. When creating a terrain model, complex variable functions have strong adjust-ability and flexibility and can adapt to the needs of various terrains.In recent years, the application of complex function in climate model is a hot research direction in the field of meteorology. The use of complex functions can help meteorologists and climatologists better understand and predict complex climate phenomena and provide effective numerical simulation methods.

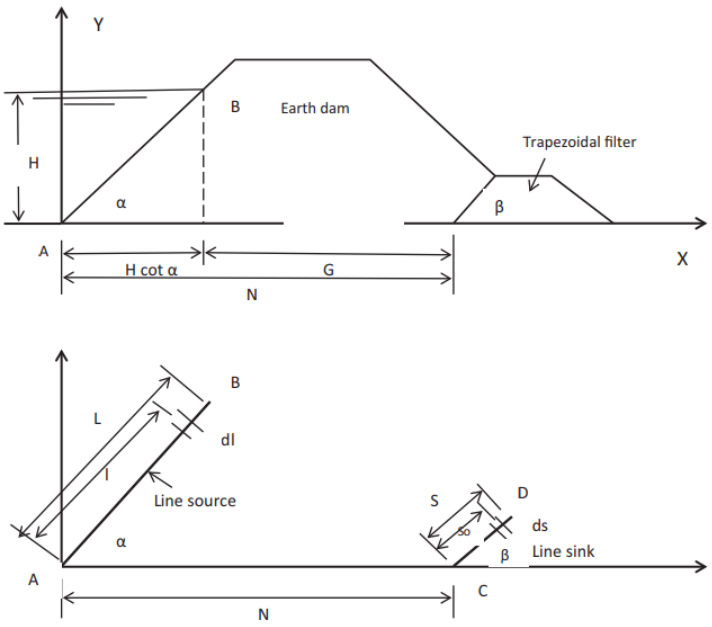


Fig. 2 The dam profile showing the line source and line sink [3].

In this section, the complex function method of calculating minimum height of trapezoidal filter will be discussed in detail. For the dam profile shown in Fig. 2, the upstream face of the dam is viewed as a line source, and the upstream face of the filter is regarded as a line sink, using the complex function theory to determine the system's complex potential, which is used to construct equipotential and stream functions. Then, one considers the complex potential of the line source AB. As the complex potential equals to the sum of the equipotential and `stream function of line source AB. Through calculation, it is easy to get the relative results. Similarly, the complex potential of the line source CD can be got. And then combining the two complex potential, the complex potential of the whole system then can be calculated. The phreatic surface through the earth dam equation should then be written down. The distance N given in Fig. 2 is disregarded in order to achieve accurate findings for the phreatic surface through the earth dam and the filter height.Additionally, a formula for the seepage discharge that passes through an earth dam per unit of dam length has been developed. The minimal height of the trapezoidal filter can then be determined by data analysis.

4. Applications in Optics

The complex function is widely used in the optical wave transmission problem and can be applied to many aspects in the optical field. One of the important applications is the establishment and analysis of light wave transmission models. The complex function can help us to describe the behavior of light wave in the process of transmission, so as to explore the propagation law of light. In the problem of light wave transmission, one often needs to study the transmission characteristics of light waves in different media. The complex function can provide a convenient way to describe the propagation behavior of light waves in complex media. By using the characteristics of complex function, one can find a suitable representation, so as to better understand the transmission process of light waves in the medium. The application of complex function in the problem of light wave transmission mainly involves two aspects: refraction and diffraction. Refraction is the deviation of light rays when they travel through a medium of different refractive indices. The complex function can help us calculate the path and Angle of refracted light, so as to better understand the law of light propagation in different media. Complex function can not only be applied in light transmission problems but also in image fusion problems. In this section, complex function will be used as a template for image fusion. The invention of pixel image acquisition opened wide possibilities for image processing. Traditional fusion methods suffer from detail loss, low resolution, and application scenarios limitation [7]. In this situation, it will be very simple to propose a new fusion technique employing complex function.Within the scope of the proposed fusion technique, one of the two partial images is chosen to serve as the real component and the other as the imaginary part of the complex function, which acts as the complex form of the fused image.Since the amplitude and phase of every complex function can be used to uniquely characterize it, there are two groups of methods (amplitude and phase) available for the fusion of incomplete images.

The t- or φ- algorithms, where $t = tan\,\phi$ is the ratio of the partial pictures and afterwards $\varphi = \tan^{-1} t$ can be used to synthesize the phase images.Because there is no constraint as to which of the two functions must be chosen as the real component and which one must be the imaginary part, the complex function can be built in either of the two methods, $\psi_{pos} = v + iu$ or $\psi_{neg} = u + iv$ .It turns out that the Vis-image u is a positive image, whereas the IR-image v is more likely to be a negative image, explaining why the phase images seem to be positive images.

5. Applications in Communication Engineering

With the advent of the information age, higher requirements have been put forward for the quality and capacity of communication. Transmission technology is closely related to the level of communication engineering, which helps to improve the quality of communication and provides better communication services for the application of communication engineering. Therefore,

strengthen the research on transmission technology, clarify the type of technology in communication engineering, explore the application of this technology, and improve the quality of transmission network, so as to provide technology for the scientific and technological development of the communication industry [8].

Complex functions are a crucial mathematical tool in communication engineering, with several applications in modulation and demodulation, digital signal processing, filter design, and more. Complex functions can be used in communication engineering to more precisely analyse and process signals, enhancing the efficiency and dependability of communication systems. Complex functions have numerous other uses in communication engineering in addition to the filter construction, digital signal processing, modulation, and demodulation listed above. The uses of complex functions in signal detection and estimation, power spectrum analysis, channel simulation, and so on are described here. The application of complex functions mainly focuses on fluid mechanics and electromagnetism. s

Based on the understanding of the wide application range of complex variable functions and further understanding of their application status in communication engineering, the results show that the application of complex functions in communication engineering is mainly reflected in its learning link between signal and system, and through professional analysis of the real frequency domain of continuous time signals and specific research on the real frequency domain of continuous time systems.[9]In this part, different mathematical ideas and methods should be applied and discussed based on proficiency in mathematical logical thinking, including knowledge of Laplace and Fourier transformations, combined with various specific knowledge of linear algebra [10]. For example, Calculating the Fourier transform and power spectrum density function are two key examples of how complex functions are used in power spectrum analysis. The signal may be converted from the time domain to the frequency domain using the Fourier transform, and its frequency energy distribution can be calculated. The Fourier transform of the signal may be used to construct the power spectral density function, which is a function that represents the change in signal power with frequency. The energy distribution of the signal over various frequency bands may be determined by computing the power spectral density function, enabling the signal's frequency domain analysis.

In conclusion, complex variable functions have many uses in communication engineering. From filter design to signal detection and estimation, from power spectrum analysis to channel simulation, complex variable functions are essential. In order to increase the effectiveness of analysis and processing and create more precise and effective user systems, one may fully use the qualities and characteristics of complex variable functions in the study of communication engineering challenges.

6. Limitations and Prospects

Although this study has made certain breakthroughs and progress in the fields of geography, optics, and communication engineering, there are also some shortcomings. Firstly, in the field of geography, one can further explore and optimize methods for complex variable functions in the application of terrain analysis and climate models. Secondly, in the field of optics, our research mainly focuses on the imaging and transmission characteristics of optical systems, and further in-depth research is needed for the application of other optical phenomena. Finally, in the field of communication engineering, one should further expand the application scope of complex functions in transforming and analyzing signals, and increase the exploration of theory and practice.

The future research directions and practical suggestions are as follows: In the field of geography, the application of complex functions in geographic information systems can be further explored, and methods for map data processing and geospatial data analysis can be improved. In the field of optics, the application of complex functions in other optical phenomena can be studied, such as diffraction, interference, and so on. In the field of mathematics, the application of complex functions in differential equations and complex plane geometry can be further promoted, and the application prospects in other mathematical fields can be expanded. In communication engineering, complex

function theories can be applied in other aspects like analyzing signals. In conclusion, complex function theories still have a large amount of untapped potential.

7. Conclusion

In summary, this study delves into the applications of complex functions in fields such as geography, optics, and communication engineering. By combining the theory of complex functions with practical problems, scholars have achieved a series of beneficial results in fields such as geography, optics, and communication engineering. In geography, researchers apply complex functions to terrain analysis and made some calculations. Through the complex variable function method, one can efficiently process and analyze geospatial data in geographic information systems, providing strong support for research and application in the field of geography. In the field of optics, mankind used the complex function method to study the imaging and transmission characteristics of optical systems, image fusion as well. These research results provide a strong theoretical basis for the design and optimization of optical devices, promoting the development and progress of optical technology. In the field of Communication Engineering, reserachers introduce Fourier transform into transforming signal. Through the application of complex variable functions, one has successfully solved some complex transforming signal problems.

References

[1] Li X. Application of Complex variable Function in solving electromagnetic field problem. Dalian Maritime University, 2011.

[2] Yang H. The application and development history of complex variable function. Mathematics World (Late), 2019, 5: 48+47.

[3] Rezk M, Elela M M A. Minimum height of the trapezoidal filter in earth dams using Complex Function Theory. Alexandria Engineering Journal, 2015, 54(4): 1219-1224.

[4] Khaustov Y, Khaustov D, Hryvachevskyi A, et al. Complex function as a template for image fusion. Results in Optics, 2021, 2: 100038.

[5] Brown J W, Churchill R V. Complex variables and applications. McGraw-Hill, 2009.

[6] Yang Z. Developing rational Thinking in Successive Studies: taking Multiplication and division of Complex Numbers as an example. Middle School Mathematics, 2022, 15: 19-21.

[7] Xu B, Li S, Yang S, et al. MSPIF: Multi-stage progressive visible and infrared image fusion with structures preservation. Infrared Physics & Technology, 2023: 104848.

[8] Huang J. Discussion on the application and future development of communication engineering transmission technology. China New Communications,2023, 25(07): 4-6.

[9] Luo J. Application Analysis of complex variable Function in Communication Engineering. Communications World, 2017, 21: 54-55

[10] Chen P. The application of complex function in communication engineering. Information recording materials, 2021, 22 (01): 51-52.