Analysis of the Principle and Two Applications for Monte-Carlo Simulations

Minqi Cheng [1] and Jiasheng Guo [2, *]

[1] Kunshan KangChiao school, Kunshan, China

[2] Chongqing No.8 Secondary School, Chongqing, China

* Corresponding Author Email: ckzh85@durham.ac.uk

Abstract. As a matter of fact, stochastic process and sampling algorithms are widely used in the state-of-art numerical simulations. In order to evaluate the random effect, the means of Monte-Carlo simulations are widely adopted and used to obtain a convergence or trending results. With this in mind, this essay mainly talks about the two applications of Monte Carlo simulation and the impact of it toward the society and human race. To be specific, firstly, the origin of Monte-Carlo simulation was revealed and its history of development was elaborated. After that, the basic concept of Monte-Carlo analysis was formulated as well as the sampling process of it is done briefly. All those foreshadows were aimed at assisting the readers to obtain a basic idea of this simulating method and be able to comprehend the relatively sophisticated applications, including financial and computer science knowledge. Overall, these results shed light on guiding further exploration of Monte-Carlo simulations.

Keywords: Monte-Carlo analysis; financial pricing; mathematical simulation.

1. Introduction

The history of Monte-Carlo simulation can be traced from the nuclear program of the United States called Manhattan Project in the 1940s, derived from the famous Monte Carlo casino in Monaco [1]. This method is a statistical sampling technique that has played a crucial role in a variety of scientific research since the end of WWII and has hitherto been a significant method on tackling sophisticated problems that require enormous amount of manpower. The flourishment of electric computers in the post-war America has elicited Monto Carlo analysis to develop, soon possess a vital portion in the research that were in urgent need of a method to assist them to deal with tremendous data. Inspired by two scientists Stan Ulam and John von Neumann, this method focuses on the situation when researchers need to calculate the odds of a variety of outcomes in a stochastic, or random system [2]. To be more specific, the Monte Carlo analysis produce probable outcomes within random inputs. Under this circumstance, this statistical method occupies a critical position in the current studies of science, especially the ones that need intensive simulations and experiments [3]. Furthermore, the fact that Monte Carlo analysis tests a number of variables and take the average of these outcomes to be the result makes much more accurate than the other sampling techniques and is able to surmount the disadvantages of those methods, such as the inefficiency of simulations in some instances.

As the unprecedented development of computer science brought human into the Information Age in the last century, the significance of Monte Carlo simulation goes to a higher degree. In particular, it has been widely adopted by scientists and researchers in the field of science and management. The study of chemical physics is an important branch of science which have great impact on the development of modern material science and study of small particles through the usage of repetitive Monte Carlo simulations. During the process of crystallization and aggregation of alkanes, the Monte Carlo simulation help to determine the thermal state of the material and identify the internal construction of the small aggregates [4]. On the other hand, the Monte Carlo simulation has also been applied in risk managements. Nowadays, many entrepreneur and project managers pay more and more of their attention on the underlying risks of initiating a company or program, therefore they adapt Monte Carlo simulation as a tool of evading potential issues [5]. To be more specific, this approach of risk management reminds the entrepreneurs to avoid the consequence that they ran out of their budget, or the capital chain of the company was broke through simulating the presumptive cost and time of the program. Meanwhile, the Monte Carlo simulation has been applicated on engineering as a strategy of optimization of supercomputers and solve the problems of the algorithms. In another situation, it has been further adapted to geophysical engineering to analyze the slope stability of a variety of factors [6].

The analysis of Monte Carlo method focuses on the essential principle and some basic applications of Monte Carlo analysis, including the implementation of it as a tool in simulation in finance, and social network. In the following paragraphs, the background information of this method would be provided in order to obtain a better comprehension for readers. Afterwards, the descriptive details of the exact operation of Monte Carlo analysis would be separated into two parts and elaborated using vivid examples. In finance, Monte Carlo simulation creates a range of potential prices of one’s estate in the market, indicating the appropriate value of assets; Ultimately, the effect of Monte Carlo simulation on social network studies would be estimated.

2. Basic Descriptions

The basic concept of Monte Carlo simulation could be originated from 18th century, when Georges Louis LeClerc, a renowned French scientist has adopted the method of random sampling in a number of his research. For instance, he verified that as the needles with same lengths fall from a particular point, the possibility for them to interact with the lines that are arranged with the same interval would be $2/\pi$. This has been known as the first example of Monte Carlo simulation for some researchers [7]. The essence of Monte Carlo simulation could be described as a numerical procedure of estimating the expected samples from a particular range of random values. These values are distributed in a given probability density, meaning that there is a function existing for this distribution. As a consequence, the process of doing this simulation could be separated into three major parts: model and convert a specific system into probability density function(pdf), take samples from this function and then calculate the probability of the occurrence of an event in all the samples. The significance of modeling is to figure out what would be the ideal output, meaning that there should be a precise goal of simulating while building a model of Monte Carlo analysis. Furthermore, the method of how to manipulate the inputs in order to attain the accurate possibility of an output would be exploited [8].

Next, after converted the input into pdf function, the method of taking samples from it would be various. In most cases, this operation could be done through different kinds of programming languages such as Python or JavaScript. In addition, the pdf must be a non-negative real valued function, and the result of its integration must be 1. Meanwhile, the integration of the pdf is called cumulative distribution function, which is the method of sampling in a pdf. A cdf for the uniform distribution could be represented as the following:

$$P(x) = \int_R^x p(d)dx = \frac{x-R}{S-R}, R \le x \le S \tag{1}$$

It tells us the probability that a number sampled randomly from the PDF will be ≤ x [9]. In order to attain the final possibility from the model, the only necessary thing is to combine all the results calculated from the above integration and we can find what the result is.

3. The Application of Monte Carlo in Finance and Economy

It is typically simpler to view of commodities and strategies as being essentially random processes due to our poor understanding of the financial system. It could be challenging to gauge the level of risk this ambiguity poses. As an illustration, think about the situation with options, a type of derivative security. The complex link between option payoffs and the prices of other assets is referred to as a "derivative". In essence, it is an agreement that gives one party the option of purchasing or trading a particular item at a defined price, but not the duty to do so.In some cases, it is possible to quickly and easily determine an option's value using formulas. The stochastic technique is frequently used in finance to represent how uncertainty affects financial items like stocks, portfolios, or options. The application of Monte Carlo methods, which entail numerical simulations, is required in this situation. Monte Carlo simulation has become a critical tool in the pricing of derivatives security and risk management [10]. Analyzing risk and justifying exotic derivatives, modeling financial markets with stochastic differential functions and optimizing portfolio allocations were the common problems that this simulation could deal with [11].

Today's financial market participants make extensive use of computing resources. Some of these resources are used in the pricing and risk management of financial assets and their derivatives. The traditional stocks, bonds, and commodities that provide the basis for more complex arrangements like financial derivatives are considered financial assets. Derivatives in financial markets are arrangements whose future payout is based on the price movement or future value of a few fundamental reference resources, which are probabilistic in character. An important issue is the pricing problem, which refers to the process of modifying the appropriate price of the stock based on available information obtained from the markets [12].

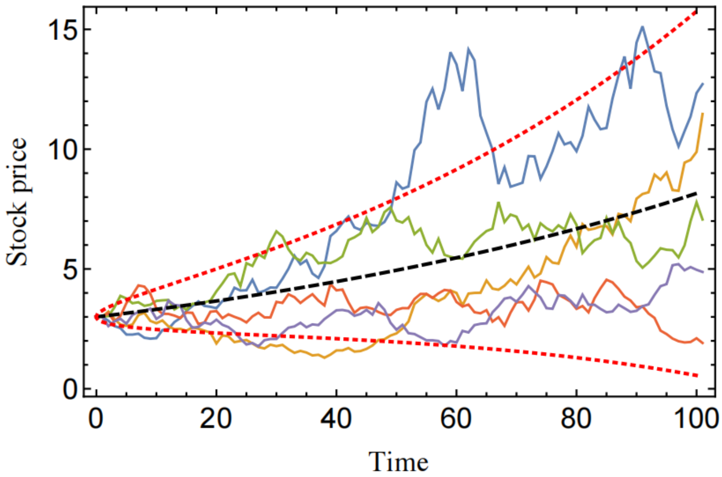

Fig. 1 The price of a stock in the Black-Scholes-Merton Model, a mathematical model for pricing options that behaves randomly as a geometric Brownian motion, changing each time step according to the log-normal distribution.

The MC approach for pricing financial derivatives assesses the projected return of derivative contracts and based the fair price of derivatives on this by modeling the price change path of the underlying assets. With regard to this, the Monte Carlo approach creates a range of potential future prices based on the stochastic process of the underlying asset, and then uses the yield function of the derivative contracts to determine the contract value under each price path. Statistical evaluation of each pathway's worth is the last step. The payment of these contracts is based on the asset's potential stochastic price trend in the future. Brokers need to be able to determine a reasonable price for the derivatives based on market conditions (seen from Fig. 1) [13]. The anticipated return and its spread can typically be accurately estimated using Monte Carlo techniques. However, this takes a lot of runs. Furthermore, this type of financial market modeling shows less prediction accuracy for short times as a result of the assumption of constant drift and volatility parameters [14]. Nevertheless, the accuracy may be increased by modeling these characteristics further as probabilistic combinations of time and other economic indicators.

4. The Application of Monte Carlo in Random Graphs and Social Network

To explain relational information among interacting pieces, network models are commonly utilized. The focus of recent research in social network studies has been heavily on random graph models. It is easy to build the exponentially expanding unpredictable graph paradigm for the network's basic structure by establishing assumptions about the relationships between potential links between nodes within the network, which are seen as random variables. In such frameworks, the boundaries signify the existence of a particular link among participants, while the nodes usually symbolize different social players. The probability of a relation between actors depends on the positions of individuals in an unobserved "social space [15]. Exponential random graph models have the following form:

$$\Pr(Y = y) = \left(\frac{1}{\kappa}\right) \exp\left(\Sigma_A \eta_A g_A(y)\right) \quad (2)$$

where the summation is over all configurations A; ηA is the parameter corresponding to the configuration A; gA(y) = 1 if the configuration is observed in the network y and is 0 otherwise; κ is a normalizing quantity which ensures that Eq. (1) is a proper probability distribution [16].

Using maximum likelihood and Bayesian frameworks, the mathematicians Besag and Clifford describe have generated judgments for the social space. They also provide Markov chain Monte Carlo methods for locating latent variables and figuring out their impacts. Despite that the number of matrices generated, it has been a valid testing method and has shown great power in our small simulation study (with a 3×8 matrix) depends on a given matrix and some test statistic [16]. For the most critical core-Markov random graph models, standard maximum likelihood estimation is not tractable for any but very small networks, because of the difficulties in calculating the normalizing constant in Eq. (2) [17]. This suggests that typical statistical techniques cannot be used to analyze these models. These problems have lately been resolved by improved MC maximum likelihood methods. We draw attention to how simple it is to implement these models' simulation. Without going into details, it can be said that a number of methods, such as the Metropolis algorithm, could replicate the graph's distribution for a specified set of parameter values. Simulation is at the heart of Monte Carlo maximum likelihood estimation. By contrasting the seen graphs with an array of arbitrary graphs produced by a randomized model employing the estimated parameter values, approximate variable estimations can be improved. The framework is presumably degenerate if the parameter estimations never agree. Convergent estimates result in graph ranges where the measured graph is average for all the model effects, according to simulations based on the estimates. The ability to derive trustworthy standard errors for the estimates is one benefit over maximum pseudo-likelihood estimates [18]. The Monte Carlo estimation approaches fixed the number of edges; hence the aforementioned method is reliant on the number of edges used to estimate the parameters. Such models don't have any density parameters. The chance of degeneracy problems will be decreased while only marginally affecting other parameter estimations if the number of edges is fixed. Experience indicates that filtering on edges may not be required, if not for smaller networks, and estimate approaches may converge for the modified criteria with density parameters provided.

Several structural network variables, including degree distribution, aggregation coefficient, influence dispersion, etc., may be estimated using the Monte Carlo method in social network analysis. By basing the assessment on important indications, the Monte Carlo technique may also be used to determine a node's importance in social networks, for instance. Monte Carlo's social network analysis does have some serious drawbacks, though. The complexity of social networks necessitates extensive random sampling and simulation, which raises the computing volume in Monte Carlo. Second, the Monte Carlo technique may encounter memory and computing problems when working with large-scale social networks. To solve these problems, researchers have proposed a few upgraded Monte Carlo techniques, including the layered sampling method and the MCMC method. These methods can decrease the amount and memory use while improving the algorithm's efficacy. Researchers have also sought to combine the Monte Carlo technique with additional algorithms, such as network embedding algorithms and community discovery algorithms, in order to produce more accurate and effective analysis and mining. In conclusion, Monte Carlo has many practical uses in social network research, although certain elements still need to be improved. The use of the Monte Carlo method to social network analysis can be expanded in the future study, leading to the development of more accurate and efficient technologies and algorithms.

5. Limitations and Prospects

The Monte Carlo approach offers a more straightforward way to tackle multi-dimensional or challenging issues, and it is extensively employed in many sectors due to the rapid advancement of computers and the growth of science and technology. The drawbacks of the Monte Carlo method are rapidly becoming more apparent as consumers have increased expectations for computing effectiveness and data quality.

First of all, while many Monte Carlo algorithms are naturally parallelizable, some methods find it difficult to adapt to this new paradigm of computing. There hasn't been much progress lately in terms of effectively integrating Monte Carlo methods into parallel processing systems. Furthermore, given the growing significance of parallel processing, it could be necessary to reevaluate the effectiveness of state-of-the-art algorithms that are difficult to parallelize. When dealing with interactive or real-time computing problems, Monte Carlo techniques typically call for a high number of samples in order to provide reliable results. In order to collect much more data for significantly bigger lattice sizes, simulations and subsequent analysis would need orders of magnitude more processing power. As a result, the Monte Carlo approach might not be suitable for this kind of issue.

Second, since unusual situations do not typically occur in routine simulation sessions, simulating them might be difficult. Using widely recognized variance reduction techniques, such as relevance testing or division, to address this issue can significantly improve the accuracy of estimating the likelihood of a rare occurrence. However, the outcomes might be dramatically distorted if the assumptions are incorrect. Particularly when estimating data towards the tail of the distribution or dealing with exceptional occurrences, the accuracy of the results might be greatly impacted. For instance, if we use the Monte Carlo algorithm to estimate the critical temperature Kc, the predicted number does not agree with the result from the tensor renormalization group within the respective error bars [19].

Finally, numerous Monte Carlo algorithms, like the cross-entropy technique and evolutionary algorithms, are reflexive in that they alter their behavior based on their own random output. These algorithms are effective at handling a wide variety of challenging optimization and estimation issues, but it is sometimes difficult or impossible to investigate the theoretical characteristics of these estimators using available mathematical techniques. Despite significant improvement, there are still a lot of unresolved issues. To minimize variation and increase computing efficiency, certain approaches, for instance, can make use of a variety of variance attenuation strategies. These enhanced algorithms do; however, each have their own benefits and range of use. Therefore, algorithm developers should focus on creating better Monte Carlo algorithms in the future that are more compatible and capable of performing more accurate sampling.

6. Conclusion

A highly helpful mathematical method for investigating uncertain circumstances and offering probability analysis of various outcomes is Monte Carlo simulation. Due to its ease of use and broad application, the Monte Carlo algorithms remains one of the most beneficial techniques to scientific computing. Utilizing MC analysis is founded on a simple, understandable core idea. The adoption of MC simulation has increased across several industries thanks to a multitude of technologies. The prospect of Monte Carlo simulation is promising, it would play an important role in the process of resolving sophisticated estimations and optimizations of unsolved issues in the field of natural science, social science and adaptations in real lives. It is suggested that readers consult the list of references or get in touch with the author if they are curious to learn more about this subject.

Authors Contribution

All the authors contributed equally, and their names were listed in alphabetical order.

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