# Pricing European Call Options with Visualization Based on the Binomial Model, Monte-Carlo Simulation, and Classical Black-Scholes Model

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**Abstract.** After the emergence of financial derivatives, pricing has been focused by a considerable number of mathematicians. With stochastic analysis and some classic probability theories, many new pricing formulas appeared in the quantitative finance field. The improvement of pricing methods of different financial securities has essentially made prices more precise and more strict, thus greatly promoting the development of modern financial markets. In this research paper, the author reviewed three significant option pricing models in mathematical finance, which are the binomial model, classical Black-Scholes model, and Monte-Carlo simulation, and applied these three models to give the price of European options. Based on the results of these three models, the author conducted some visualization work and statistical work to compare the differences of the models. Furthermore, in order to determine the most suitable model in this case, the machine learning technique is applied to classify the price data by analyzing the confusion matrices in each model.

**Keywords:** European call options; option pricing; binomial model; Black-Scholes model, Monte-Carlo simulation.

## 1. Introduction

The Black-Scholes formula had a significant impact on the financial industry. [1]. The creation of this formula promoted the development of the modern financial derivatives market. Many of the strategies and pricing models utilized in finance today have their roots in the theories and approaches put out by these two economists [1]. Before discussing option pricing, it is necessary to consider some significant concepts about options.

Financial derivatives are products whose prices and payoffs depend on the stochastic development of the financial variables that they are linked to [2]. The emergence of options and other financial derivatives is an important milestone in the development of modern finance. The right to swap stock at a predetermined strike at any time on or before a predetermined date is provided by the financial derivative known as an option [3]. The specified price is defined as the striking price [3]. The given date is the expiration date [3]. There are various standard types of options: Call options give on the right, instead of the obligation, to purchase a certain quantity of a financial good at a specific cost which is also called exercise price or strike price or strike [4]. A call option under the price $S_t$ at time is a derivative taking the form of a contract, the holder and the writer. This is specified by a maturity time $T$ and a strike $K$. The holder pays the option premium also called the option price, to the writer at inception of the contract and then has the right to purchase the product at time $T$ at strike $K$. The option will be exercised if $S_T > K$, because then the holder is buying the stock for less than its worth; in principle (s)he could immediately sell it, and make a profit of $S_T - K$. Often, options are ‘cash settled’: on exercise the writer pays the holder $S_T - K$ in cash. On the other hand, if $S_T < K$, the holder do not choose to exercise and the option expires worthless. The payment from the call option can be expressed as $C(T) = \max\{S_T - K, 0\} = (S_T - K)^+$, where $(X)^+$ denotes the greater of $X$ and 0 [5]. The capacity, instead of the responsibility, to sell a financial good at a strike is provided by put options. [4]. A put option with the price of $S_t$ at time are the two parties to the derivative transaction.This is specified by a maturity time $T$ and a strike $K$. The holder enters into a contract with the writer, pays a charge, which is the option premium, and is then given the right to sell the product at time $T$ for strike $K$. If the holder decides to execute this transaction, the writer, having accepted the premium, is obligated to buy one unit of the stock in return for a payment $K$. The option will be exercised if $S_T < K$, because then the holder is selling the stock for more than its worth; in principle (s)he could immediately buy it, and make a profit of $K - S_T$. Often, options are 'cash settled': on exercise the writer pays the holder $K - S_T$ in cash. On the other hand, if $S_T > K$, the option expires worthless since the holder won't exercise it. The payoff in this case is expressed as $P(T) = \max\{K - S_T, 0\} = (K - S_T)^+$, where $(X)^+$ demonstrates the greater of $X$ and 0 [5]. Furthermore, European options grant the right to execute the transaction on the expiration date, often known as the maturity date [4].

In order to discuss some significant factors in option trading, several financial concepts should be reviewed. A forward contract is one of the most straightforward derivatives. In the financial market, forward contracts are usually transacted [6]. In a forward contract, the party who makes the aggrement to purchase the asset is taking a long position, and the other is defined as taking a short position [7]. An evaluation of an option's theoretical value is made based on all the information that is currently accessible. The option's fair value can be obtained through option pricing models. The invention of the pricing theory for options represents a significant turning point in the history of contemporary finance. The growth of modern financial markets has been tremendously aided by the advancement of pricing methodologies, which has effectively made prices more exact and stringent.

This paper focuses on three significant option pricing models which are the binomial model, classical Black-Scholes model, and Monte-Carlo simulation, to price European call options under the no-arbitrage market, and compare the results of these models.

## 2. Methodology

### 2.1. Data Source

The dataset used in this paper comes from the yahoo finance, which is the NASDAQ European call option index [8]. The dataset clearly shows the mathematical factors needed for pricing in those three models, including volatility, interest rate, and strike price.

### 2.2. Description of the Conditions

Before discussing different option pricing models, understanding the idea of risk-neutral probability, which is frequently employed in option pricing and may appear in many option pricing models, is crucial. Theoretical probabilities of future events that have been risk-adjusted are known as risk-neutral probabilities [9].

This idea is predicated on two basic ideas: In the first, it is shown that an asset's current value is its projected payment. The second premise demonstrates that there are no market prospects for arbitrage [10].

### 2.3. Methodology Description

A binomial option pricing model is the simplest way to value the options. The completely efficient market assumption needs to be satisfied. The author believes that the price of the underlying asset will either increase or decrease throughout the term according to the binomial model. It is feasible to determine an option's payoff given the information of the asset and strike. The payoffs might be discounted to get the option's current value.

Another often employed option pricing model is the classical Black-Scholes model. The model was created for European call options. This model also needs to satisfy several assumptions. The following are the presumptions on the distribution of stock prices. The stock's continuously compounded returns are uniformly distributed and time-independent. Future payouts and the volatility of constantly compounded returns are both predictable and consistent [11]. This formula was created to price options on no equities without payment dividends.

Another advanced technique for valuing options is the Monte-Carlo simulation. To determine the discounted anticipated option payoffs using this technique, the author simulates potential future stock values [12]. In this paper, the author discusses simulation in continuous time, which is different from the binomial model.

## 3. Results and Discussion

### 3.1. Model Results

The result of Black-Scholes model can be clearly visualized by Greeks. Greeks represent sensitivities of a derivative value to changes in the underlying parameters used to determine its price [13]. More specifically, if the author denotes by $V$ be the price of a financial product, which needs the information of the stock price $S$, a risk-free rate $r$, a volatility $\sigma$, and with maturity $T$, then the Greeks are defined as follows.

Delta ($\Delta$): It gauges how quickly the option price fluctuates. $\Delta = \frac{\partial V}{\partial S}$. Gamma ($\Gamma$): It gauges how quickly changes in reaction to the stock price changes. $\Gamma = \frac{\partial^2 V}{\partial S^2} = \frac{\partial \Delta}{\partial S}$ . Vega (ν): It gauges how sensitive a system is to variations in volatility $\sigma$. $\nu = \frac{\partial V}{\partial \sigma}$. Rho ($\rho$): It gauges responsiveness to variations in the risk-free interest rate. $\rho = \frac{\partial V}{\partial r}$ . Theta ($\Theta$): It gauges how sensitive the derivative's value is to the passage of time. $\Theta = -\frac{\partial V}{\partial T}$ . Elasticity or Lambda (λ): It is the leverage that compares the option value percentage change to that of the stock price. $\lambda = \frac{\partial V}{\partial S}\frac{S}{V} = \Delta \times \frac{S}{V}$. Psi ($\psi$): It is option value change divided by that of the dividend yield ($q$). $\psi = \frac{\partial V}{\partial q}$ . In particular, it is feasible to have the following explicit expressions: $\Delta = N(d_1)$ , $\Gamma = \frac{N\prime(d\_1)}{S\sigma\sqrt{T}}$ , $\nu = S\sqrt{T}\,N'(d_1)$, $\rho = KT\,e^{-rT}N(d_2)$, $\Theta = -\frac{S(0)N\prime(d\_1)\sigma}{2\sqrt{T}} - rKe^{-rT}N(d_2)$ [14]. One of the results of pricing is as follows.

**Table 1**. Black-Scholes model

| Name | Premium | Delta | Gamma | Vega | Rho | Theta | Psi | Elasticity |
|---|---|---|---|---|---|---|---|---|
| bscall | 7.382 | 0.803 | 0.031 | 0.098 | 0.164 | -0.012 | -0.201 | 5.442 |

Visualizing the Greeks of the previous example can be conducted to show the graphs of the previous nine factors.

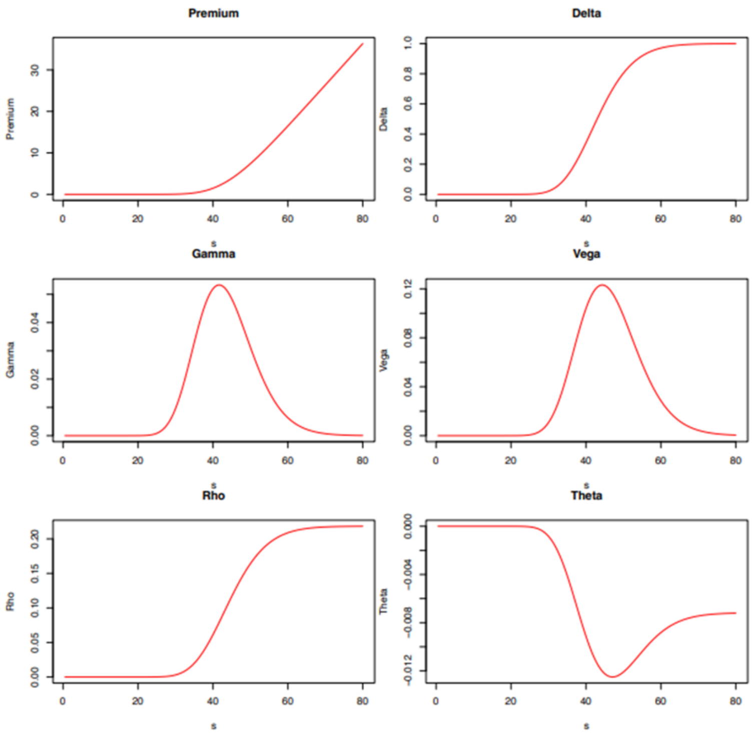


**Fig. 1** Visualization of the Greeks

Fig. 1 demonstrates that this model with the application to pricing the European call option of the dataset shows stability. From the Gamma graph and Vega graph, the curves tend to be the same, and the same situation is for the Delta graph and Rho graph.

For the binomial model, the author obtained two prices of each option in one stage. Fig. 2 is obtained based on one option with two stages in the dataset. More details will be discussed in the next part in this paper.

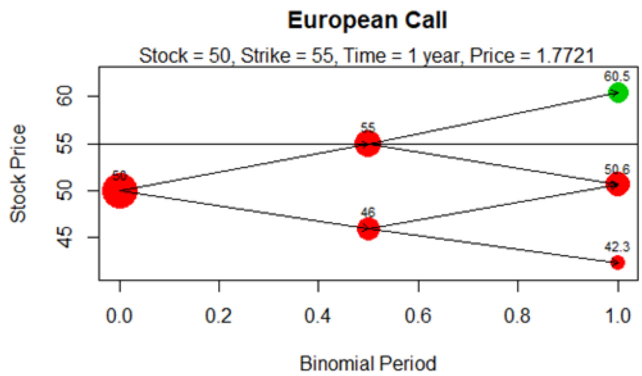


**Fig. 2** Binomial model in two stages

There are an endless number of time points between any two points in time for the Monte-Carlo technique in continuous time [12]. As a result, each variable has a specific value at each instant. The author employed the geometric Brownian motion of the stock price in this situation. Random walk refers to the idea that because price changes are independent of one another, future stock values cannot be forecast using previous trends. Under contrast to the simulation under the binomial model, the author calculated the stock price at maturity using a continuous time simulation. Finding the option's payment at maturity and discounting it to the present value are the next steps, which are identical to what the author accomplished for simulation in the binomial model.

### 3.2. Discussion

In order to evaluate the effectiveness of these three pricing models, some machine learning models are helpful in classifying the price data by comparing with the actual data. One machine learning technique called adaptive boosting model (AdaBoost) was implemented to classify the price data. AdaBoost all belong to ensemble machine learning system, only AdaBoost is able to adaptively reweight the training set [15], which may potentially give a satisfying prediction result. In the binomial model, considering that at the first stage two prices will be obtained from one option, based on the K-means method, the price data can be divided into three classes.

A table called a confusion matrix is used to represent and assess the efficacy of a classification model [16]. For a perfect confusion matrix, the author would expect to find only values on the leading diagonal because values off-diagonal indicate misclassification [17].

Accuracy is a common statistic used for classification model assessment [18]. It is an illustration of a categorization model's total predictive power. A confusion matrix, which is the total of the primary diagonal elements divided by the sum of all the elements, can often be used to compute accuracy with relative ease [17].

Recall is the proportion of values correctly predicted as belonging to a default class to the actual number of values in that class. Precision, also known as the positive prediction value (PPV), is the percentage of values in a default class that were correctly classified to the total number of values [18]. Usually, memory and precision have an antagonistic connection [19]. It is therefore possible to increase one at the expense of the other.

The F1-score combines recall and accuracy. It is a precision and recall harmonic mean. The F1-score, which runs from 0 to 1, indicates the robustness of a classifier by indicating both the number of cases that a classifier successfully classifies and the number of missing instances. Usually, the higher the F1 Score, the better the model becomes [20].

Based on the analysis of these parameters, the confusion matrices of the three models are as follows.

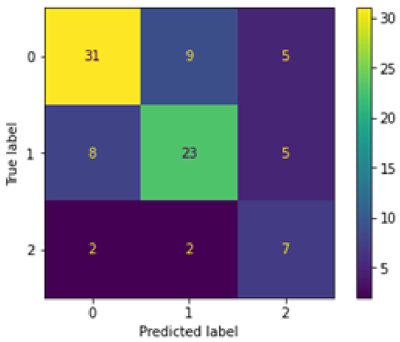


**Fig. 3** Confusion matrix for the Black-Scholes model

In Fig. 3, although most of values lay along the primary diagonal, the incorrect classifications cannot be ignored. It is obvious that high percentages of the incorrect classifications are still noted in all of 3 statuses. The instability characteristic of the Black-Scholes model in this case well represented in this confusion matrix.

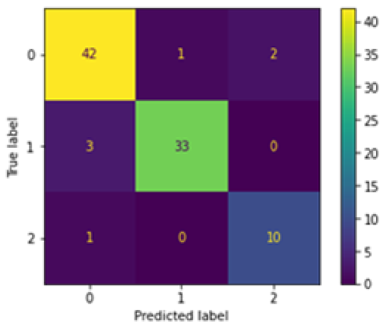


**Fig. 4** Confusion matrix for the binomial model

It can be seen from the Fig. 4 that almost all values lay along the diagonal, and 0 occurs twice in the off-diagonal positions. Notably, the only 1 incorrect classification as perfect pricing. Meanwhile, it can be seen that the number of incorrect classifications of each status (off-diagonal value) is no more than 3, which means that the borders between three cases are obvious. The good classification performance might be due to the fact that at each iteration AdaBoost strives to maintain the covariance between classifiers low and decouple the following classifier from the previous one [15]. Therefore, it is not surprising that this model was able to give a higher accuracy than before.

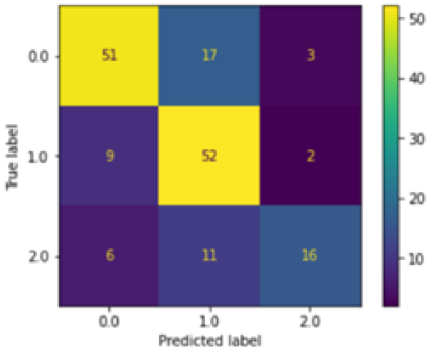


**Fig. 5** Confusion matrix for the Monte-Carlo simulation

It is obvious in Fig. 5 that the majority of values lay along the diagonal. Nonetheless, it was evident that the majority of misclassifications occurred between labels-0 (perfect pricing) and label-1 (slight mistaken pricing), which may be derived from the vague decision borders between the two classes. This potentially make classifiers hard to recognize it, resulting in occasional misclassifications. Moreover, label-2 (severe mistaken pricing) had the smallest correct predicted value on the diagonal for the Monte-Carlo simulation and had a relatively higher percentage of inaccurate classifications compared with the other two classes, likely because the model had not been adequately trained for this set of data, considering that the sample size of this label was the lowest in selected dataset. The f1-score of the perfect pricing class was 74%. The training data of perfect pricing and slight mistaken pricing status performed well.

## 4. Conclusion

Using real data from the NASDAQ European call option index, and under the no-arbitrage assumption, three models, the binomial model, Monte-Carlo simulation, and classical Black-Scholes model were applied to price the European call options. The results have been visualized in this paper. In order to discuss the best model for pricing the European call options in this case, the author conducted k-means and elbow method to define three pricing statuses and applied adaptive boosting machine learning model to compare the accuracy and efficiency of the three models. According to the confusion matrices and analysis of the relevant data, the binomial model tends to be the best choice in this option pricing case due to the outcomes of the adaptive boosting machine learning model, as it demonstrates the prices in two cases. For the classical Black-Scholes model, the machine learning outcomes show that the instability due to the market with more than two assets. The non-arbitrage assumption in the market might not be applied. In reality, the market might exist the opportunities of arbitrage. Monte-Carlo Method tends to be the second-choice model in this case. In the future, more precise pricing method might be applied to this case, and the discussion of the best pricing model in this case might be updated.

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