

Analysis of Principle and Applications of Fourier Transform for Acoustics

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Abstract. As a matter of fact, since the invention of Fourier transform has been invented, various fields have used the measure to extract signals and eliminate the noise. In recent years, with the rapid development of computer fields, the fast Fourier transform (FFT) is pretty simple to implement. On this basis, numerical solutions and dealing approaches are proposed in lots of fields including optics and acoustics. With this in mind, this paper will detailly analyze and discuss the implementation of Fourier analysis into acoustic. To be specific, this study will demonstrate the principle for FFT analysis as well as the situations of using FFT. At the same time, the applications in the acoustic field will be systematically estimated. According to the analysis, the current limitations will be proposed, and the future prospects will be presented. Overall, these results shed light on guiding further exploration of FFT analysis in different subjects and fields.

Keywords: Fourier analysis; FFT; acoustic.

1. Introduction

A function meeting particular criteria can be represented using the Fourier transformation by combining sine and/or cosine trigonometric functions linearly, or by using their integrals [1-3]. The most typical acoustic property for converting audio signals from time domain to frequency domain is the Fourier transform [4]. The conversion of a signal using an FFT takes place from time coordinates in a time space to FFT-based coordinates in a frequency space [5].

The continuous Fourier transform formula is shown above, where $f(t)$ is the expression of the temporal signal and $F(\omega)$ is the transformed result. Of course, in reality, the expression of the signal is not known, but information is obtained through discrete sampling. So, one needs to use discrete Fourier transform. Due to the requirement of FFT that the signal is periodic continuation, and arbitrarily truncated signals are difficult to meet this characteristic, directly performing FFT transformation can lead to frequency leakage and introduce abnormal frequencies. By using a window function to suppress the start/end of the signal, it approaches zero, making the boundaries of each cycle of the signal smooth enough to reduce the frequency of leakage [6]. The window function needs to make the main lobe width as narrow as possible to achieve high frequency resolution; Simultaneously, the sidelobe attenuation should be maximized to reduce spectrum leakage at other frequencies

2. Basic Descriptions

Fourier analysis was initially used as a tool for thermal process analysis, but the methodology had typical reduction and analytical features. Optional The function can be expressed as a linear combination of sinusoids by a constant decomposition, the sinusoidal function is relatively simple, and is already well studied in physics, which is very similar to the theory of atomic theory in chemistry. Surprisingly, modern mathematics finds that Fourier transform has a very good nature And they marveled at the miraculous miracle. The Fourier transform is a linear operator that can still be a Tori operator if the proper norm is applied. The Fourier transform's inverse transformation is simple to find and has a shape that is quite close to the positive conversion. A differential calculus eigenfunction known as the sine wave basis function has the ability to convert the solutions of linear differential equations into regular algebraic equations. Physical systems with linear time invariance have an

invariant frequency, and by combining responses to various frequency sinusoidal signals, one can determine how the system reacts to complex excitation [7].

According to the well-known convolution theorem, the Fourier transform can simplify convolution calculations by converting complex convolution processes into simple product operations. The fast Fourier transform algorithm (FFT), often known as a digital computer, can be used to generate a discrete Fourier transform. The rise component of all sinusoids gradually contracts as the superposition grows, and the descending portion of all sinusoidal waves is offset to the horizontal point. This is how rectangles are created.

This allows sine waves to be superimposed with waveforms other than rectangles. Without a modifier, the term "Fourier transform" usually refers to "Continuous Fourier transformation." The complex exponential function, which is essential to the Continuous Fourier transform, is represented by the square product function. The inverse transformation in the aforementioned equation is actually a representation of the Continuous Fourier transform, which shows the function in the time domain as an integral of the function in the frequency domain. On the other hand, forward conversion is a key component of both time- and frequency-domain functions. Functions are generally referred to as basis functions and as Fourier transform image functions. Forming Fourier transform pairs are basis functions and image functions [8].

If this is an odd (or even) function, the cosine (or sine) component is zero, and the conversion at this time can be referred to as cosine transform (or sine transformation). The main entry of Fourier series: the continuum form of Fourier series and Fourier transform is actually an extension of Fourier series, and the integral is actually an addition type addition operator. In the case of a periodic function, the Fourier series representation is the period of the function, the Fourier series of the function, where the sum can be expressed as the amplitude of the real frequency component, and the Fourier expansion coefficient, which is equivalent to the case of the real valued function. Separate time Discrete time primary entry for the Fourier transform Transform of Fourier The sequence of Z in the specified domain is transformed using the discrete time Fourier transform (DTFT). The DTFT is defined if set to the sequence. The comparable inverse transform is a DTFT, which is typically employed for spectral analysis of discrete time signals and is periodic in the frequency domain and discrete in the time domain.

In this case, discrete Fourier transform (DFT) is performed. If one considers the first frequency component with the lowest frequency as '1', one has the most basic unit for constructing the frequency domain. For our most common rational number axis, the number "1" is the basic unit of the rational number axis. The basic unit of time domain is "1 second". If one sets an angular frequency to Sine wave $\cos(\omega t)$ As the foundation, the basic unit of frequency domain is With '1', it is necessary to have '0' to form the world. $\cos(0t)$ is a sine wave with an infinite period, which is a straight line. The direct current component is often known as the 0 frequency in the frequency domain. Fourier series superposition simply modifies the overall waveform in relation to the number axis upward or downward; the wave's shape remains unaffected.

3. Applications in OCT

With 68 soil samples from the long-term positioning experimental area of Fengqiu Ecological Experimental Station of the Chinese Academy of Sciences as materials, the available phosphorus in soil was measured by Fourier transform infrared photoacoustic spectroscopy. With Olsen-P as the dependent variable, partial least squares method and artificial neural network model were constructed by Fourier transform infrared photoacoustic spectroscopy, and the model was used for prediction. The results showed that the correlation coefficient (R^2) of the partial least squares model was 0.96, the corrected standard deviation was 1.79mg/kg, and the validation standard deviation was 5.25mg/kg; The calibration coefficient of the artificial neural network model is 0.84, the calibration standard deviation is 2.40mg/kg, and the validation standard deviation is 5.43mg/kg. Both models can be used for predicting soil available phosphorus, and the partial least squares model is superior to the artificial

neural network model. The characteristic of this method is that it does not require sample pretreatment and the determination does not cause damage to the sample, providing a new means for rapid determination of soil available phosphorus.

The OCT can capture three-dimensional spatial information (x, y, and z) quickly, precisely, and sensitively inside the sample. According to the principles of optical low coherent interference, the OCT achieves sensitive resolution imaging of spatial information in the depth (z) direction, as shown in Fig. 1. The reference arm is a linear, transportable mirror that may have its optical path size altered. A single layer of reflective surface makes up the sample arm. The interference signal detected by the detector is the optical path difference if the light source employed in the interferometer is monochromatic (having a long coherent length, such as a laser) [9].

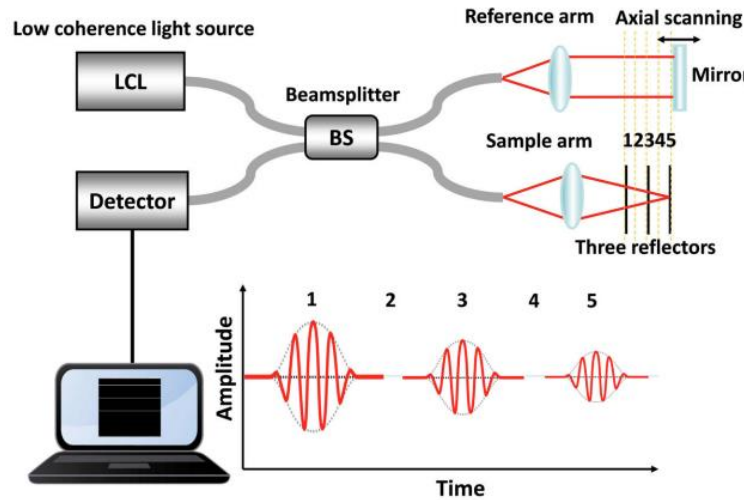


Fig. 1 A sketch for OCT system.

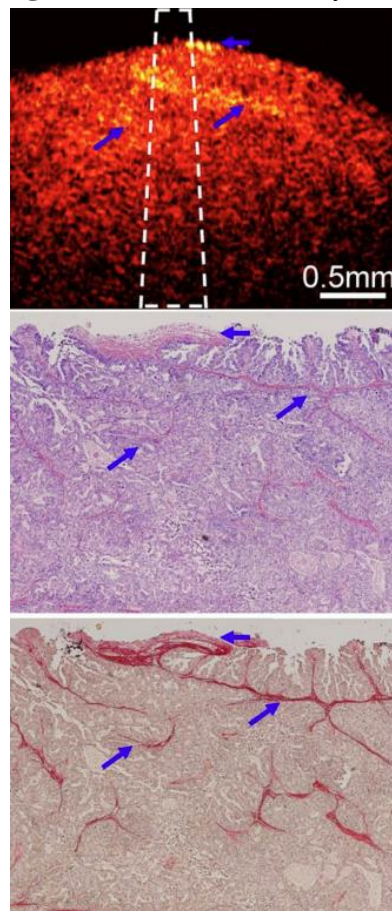


Fig. 2 A typical results of OCT.

However, if the interferometer's light source has a large spectrum (wave number k with a constant width), each spectral component creates a straightforward oscillation of the cosine function (interference fringe), as seen in FIGS. 1c and 1D, but the oscillation frequency is different. As a result, it was found that the interference fringes of various spectral components form a superposition of the growth only when the optical path difference is close to zero, and the interference signal rapidly attenuates with an increase in the optical path difference and forms an interference envelope as shown in Fig. 1. It is commonly utilized for spatial positioning and ranging and is known as optically low coherent interference. To achieve OCT three-dimensional imaging, a two-dimensional spatial scanning mechanism (often implemented by X and Y optical scanning mirrors) can be introduced in the X and Y planes perpendicular to the depth direction. OCT fundus imaging, which typically chooses the 800 nm band with weak absorption because of the effect of water absorption in ocular tissue, chooses the 1300 nm band with weak scattering because of the issue of absorption and scattering in fundus choroidal imaging. One can use a trade wavelength scheme with a 1000 nm wavelength. A typical analysis results.

In addition, for specific applications such as optical material detection, the OCT can use visible light wavelengths and significantly improve axial resolution. Accordingly, the operating wavelength of the system can be determined based on different application objects. Central wavelength λ After determining zero, a wide spectrum light source can be selected to enhance the axial resolution of the Oct. Theoretically the axis resolution of OCT δ Bandwidth Z $\Delta\lambda$ In an actual application, the axial resolution of the OCT is ultimately a passive optical device such as a broadband light source, detector, coupler, circulator, and optical dispersion effect. However, with the advance of technology, the axial resolution of OCT is improving [10].

Eliminating high-frequency noise is a crucial step in the analysis and extraction of spectral features for reliable detection of distant sensing objects using infrared spectroscopy. In the area of signal time-domain analysis, a new infrared spectral filtering technique is suggested employing spectral continuum processing and fast Fourier transform. This technique uses the Fast Fourier Transform to transfer the infrared spectrum to the frequency domain after first performing spectral continuity removal on it.

A fast fourier inverse transform is used to translate the frequency domain signal into the time domain after a low-pass filter is used to remove high-frequency noise. The signal is then subjected to spectral continuity recovery in order to produce the filtered infrared spectrum signal. Comparative investigations have demonstrated that the continuous fast Fourier filtering approach solves the Gibbs problem in conventional fast Fourier infrared spectral filtering by having better and faster filtering effects than traditional time-domain filtering methods. This technique satisfies the demands of infrared spectral ground object recognition for high-quality spectra and is simple to use, quick to execute, and has effective filtering effects.

One knows that the phase of a pulse wave function consists of two parts: one is the spatial phase factor. Another factor is the temporal phase factor. For spatial interference, it reflects the phase difference between two waves with the same frequency at different spatial coordinates. By introducing spatial carrier frequency, this spatial phase difference can be separated. For spectral interference, it reflects the phase difference between two pulses with the same spectral component at time coordinates. By introducing time delay, this temporal phase difference can be separated. When two pulses enter the spectrometer, The spectral coordinate system is converted into a spatial coordinate system, and spectral interference is expressed in the form of spatial interference fringes. Therefore, the processing of spectral interference fringes can be solved by using the processing method of spatial interference fringes [11].

One of the OCT's major performance characteristics is imaging speed, and the technology is primarily segmented across multiple generations according on this parameter. The second generation OCT and Fourier domain OCT, which can reach hundreds of Hz and up to 8 kHz, are commonly referred to as the first generation OCT, the real time domain OCT, and its agscan line scanning speed. The imaging speed is several tens of kHz, and is the mainstream of current commercial Oct.

The invention of a line array CMOS camera with a high-speed scanning light source has allowed for the attainment of imaging speeds of several hundred kHz and even several MHz, even if the third generation OCT is still based on the Fourier domain spectral detection technique. As previously mentioned, in order to collect complete depth information in the depth direction, the time domain OCT must scan the reference arm once, and this mechanical scanning mechanism considerably reduces the speed of the Oct. Utilizing the spectral resolution detection approach, a parallel acquisition of depth data is accomplished in the Fourier domain OCT [12].

When using spectral region OCT, the scanning speed of the linear array camera and the light source both affect how quickly images are captured. The development of optoelectronic industry technology affects the wire array camera and sweep sources. If parallel detection in the X and Y directions, or surface scanning (volume scanning), is achieved using the OCT signal detection method, it is possible to achieve the super high speed imaging of the OCT. Since its inception, optical coherence tomography (OCT) has rapidly developed and gained wide-ranging uses. Regarding the imaging depth, the imaging speed, and the imaging accuracy, the resolution and sensitivity were significantly enhanced. The use of optical coherent tomography techniques is expanded by improving picture depth, and real-time 3D imaging is made possible by improving imaging speed. The basis and possibility for major early diagnosis of diseases like cancer are provided by improved imaging resolution, which has increased the resolution of imaging technologies to the level of cell and molecular biology

4. Limitations and Prospects

Fourier analysis lacks the analysis of the time domain, for example, a signal to detect how large a certain frequency is, Fourier is suitable, but a signal to analyze where a certain frequency is maximum, Fourier is not good. So, there's a kind of short-time Fourier analysis, which is to cut a long signal into a number of short signals and analyze the short signal to get their respective spectrum, and then one gets a time spectrogram with information in the time domain and frequency domain. It does solve some problems, but one of the problems of short-time Fourier analysis is that the short signal is cut short, and can't get the desired frequency information accuracy, and the long cut can't get the desired time domain information accuracy. The other thing is that the short-time Fourier is a little bit uglier mathematically, a little bit more engineering. Then comes the wavelet analysis, wavelet analysis has continuous wavelet analysis, continuous wavelet analysis discretization and discrete wavelet analysis, the first continuous wavelet analysis, wavelet analysis is based on: no longer use the sine of infinite length of the signal as the basis, but use a time domain information and frequency domain information of the waveform as the basis of transformation.

The idea of wavelets is that these "small" wavelets as a basis can have both time domain information and frequency information, and the mathematical theory is complete, such as when there is an inverse transform, when it is redundant, what can be a wavelet and what can't be a wavelet, what wavelets are good, and because of the uncertainty principle, you can't know the exact time domain information and frequency domain information at the same time so how do you go Looking for the highest resolution and so on, and then discrete wavelets are basically multiresolution analysis, which is essentially a multirate filter bank, where the high frequency information is accurate, the frequency information is inaccurate, and the low frequency spectrum information is accurate and the time domain information is inaccurate, unlike short time Fourier analysis where the frequency interval and the time domain interval are fixed. Such "multi-scale" is more suitable for some scenarios. For example, if an animal makes a roar, the high frequency is usually the head of the sound so I need to know when the animal is roaring, the lower frequency might be the sustain part of the sound so I need to know the fundamental frequency to figure out what kind of animal it is.

5. Conclusion

To sum up, this study investigated the FFT applications in acoustic. To be specific, the formulae are given and analyzed initially. Subsequently, the principle for the FFT is demonstrated and discussed. Subsequently, the specific applications have been discussed using OCT case. Afterwards, the limitations of current FFT analysis is given and further prospects are implemented accordingly. Overall, these results offer a guideline for FFT analysis in acoustic.

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