Analysis of the Principle and Applications of Fractal

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Abstract. As a matter of fact, fractal is one of the mystery in math applications which attracts a large number of scholars even in recent years. With this in mind, a brief introductions for definition of fractals and a presentation of a brief history of fractal theory will be presented in this study. To be specific, this study will provide explanations on basic principles of fractal geometry like self-similarity and scaling. At the same time, this study will also give the idea of fractal dimensions and examples of fractal shapes. In detail, the essay will include two specific fractal applications in mechanism and biomedicine. According to the analysis, challenges and limitations of fractal theory will be mentioned potential applications in various fields will be discussed. Eventually, this study will conclude summary of key points and importance of fractals in all fields. Overall, these results shed light on guiding further exploration of fractal investigation.

Keywords: Fractal geometry; Hausdorff dimension and measure; mechanical transmission; biomedical image.

1. Introduction

Fractal theory was invented by Benoit Mandelbrot in 1973. Fractals are often described as "never-ending patterns" because they can be infinitely magnified and still retain their intricate details. They are formed by a simple process of repeating in a loop. In 1904, Koch curve, the first artificial curve that has structure that is similar in parts and wholes was described by Helge von Koch, and it is called self-similar structure [1-3]. The curve is formed by creating an equilateral triangle, removing the one thirds in the middle, building another equilateral triangle in the empty place where the sides were removed, then repeating the process.Fractal is a set F with following properties. It has a fine structure, that is detailed on arbitrarily small scales. It is irregular that annoy be described by traditional geometry. It is self-similar. The fractal dimension of it is always larger than its topology dimension. The definition of F is simple or recursive [4].

Fractal theory added fractions as dimensions rather than only integers for the aspect of measuring. Fractal dimension is a measure of the complexity or irregularity of a fractal shape or pattern. It is a way of quantifying the amount of detail or structure in a fractal [5]. The dimension is a non-integer value. For example, a line is contained in a fractal dimension of 1, because it is one-dimensional; and a square be described by a fractal dimension of 2, since it is two-dimensional. However, a fractal shape like the Koch snowflake has a fractal dimension between 1 and 2, because it has a finite area but an infinite perimeter. People take the parameter fractal dimension for describing objects with high complexity that cannot be depicted as integer dimension like 1,2. For example, shorelines and Koch curve.They are too complex so that they will be infinite in the scale of 1 dimension-a point, but in the scale of 2 dimension they will be zero [6]. So, the dimension used to describe such an object will be a number between 1 and 2, which is a fraction.

In order to measure the non- integer dimension, people introduced Hausdorff measure in 1919. Hausdorff measure is a mathematical concept that quantifies the size or content of sets in metric spaces. It is named by the German mathematician Felix Hausdorff. The Hausdorff measure It assigns a non-negative real number to a set, representing its size or "Hausdorff content" in different dimensions. The measure considers the minimum amount of covering required to completely cover the set, with smaller covers indicating a higher dimensionality or complexity of the set. The Hausdorff measure is widely used in various branches of mathematics, such as fractal geometry and mathematical analysis, to study the propertie and structure of sets in metric spaces.

2. Analysis of Principles and applications

2.1. Principles

The self-similarity of a fractal is a principle that the partial magnification of the fractal is similar to the overall appearance. There are three categories of it, exact self-similarity, approximate self-similarity, statistical self-similarity. Exact self-similarity is that the whole object is made up with infinite number of a similar object which is different in ratio with itself. This type of similarity is strict and precise such as Koch curve and Candor set. Approximate self-similarity is a more common type of similarity. When observing an object at different scales, the structure observed is approximate similar. It is existed in a limited extent. The self-similarity will not exist if it is beyond the limitation. Taking vegetation roots as an example, the structure observed when zoom in would approximate similar with the overall one. Statistical self-similar object does not have a similar appearance, but it is similar in statistical meaning. The fractal dimension can remain constant when zoom in.

Self-similarity is common in nature, such as shorelines and plant vain. Most of them are approximate self-similarity or statistical self-similarity. Scale invariance is a principle of fractal. Zooming in or out will not change the pattern, complexity, and irregularity of the partial fraction of a fractal. It means the pattern details will not be observed changes under different scales. The object with scale invariance must satisfy the properties of self-similarity. Self-similarity only exists within a range of intervals with scale invariance. If the object does not have self-similarity exceeding the interval, fractal will not exist.

Self-affinity is a continuation of self-similarity. It is self-similarity if the variation ratio between segments to overall in all directions is same. It is self-affinity if the variation ratio between segments to overall in all directions is uncertain same. Self-similarity is an exception of self-affinity.

Fractal geometry is a challenge to traditional geometry mainly with Euclidean geometry. There are two major differences between fractal geometry and regular geometry. First, regular geometry has a characteristic scale. For example, this study will observe a circle with the scale that is larger than that of its diameter. However, with a scale that is smaller than its diameter, one could only see a part of it, that will be an arc. If one takes a smaller scale to examine it, it will be a small section of a line. Contradictory, fractals do not have such a characteristic scale. It contains factors in all scales. Complex details will be observed within any scale. Second, unlike only integer dimension could be used to describe Euclidean geometry, fractal geometry could use fraction dimension to describe it. That is fractal dimension. Fractal objects, like Koch curve, are irregular and complex that one cannot be depicted with Euclidean geometry language [7].

2.2. Applications

Fractal theory and fractal geometry have been developed to apply in many fields, like mechanism, medicine. Fractal theory can apply on mechanical transmission. Based on the above characteristics, fractal theory is able to express complex systems more approaching to their true characteristics and states and can better satisfy the diversity and complexity of objective things. The contact surface and accuracy on manufacturing are the major factors affecting the mechanical transmission traits. The early assumption of an object was rigidity, and then elastic deformation was considered. However, some micro differences were overlooked as these studies are based on traditional dimensions, as shown in Fig. 1 [8].

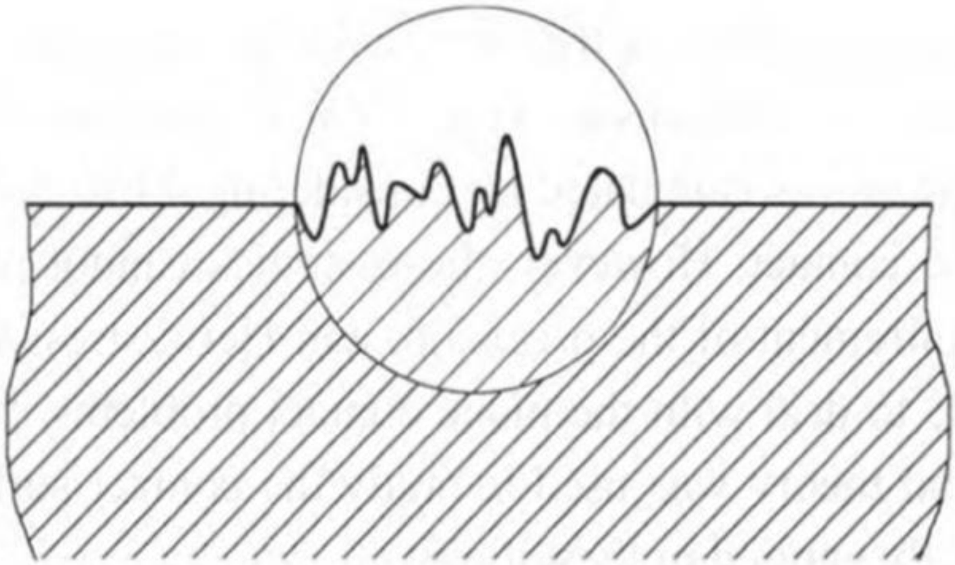

Fig. 1 Micro topography of a surface

Applying fractal theory can serve to break the limitations and deeper the study. In 1995, SRINIVASAN and WOOD used the fractional Brownian motion model to form incorrect data as an ability of fractal dimension and superimposed it on the ideal contour of the sliding bearing. The feasibility of applying fractal methods to solve tolerance specification problems in engineering design was elaborated. A specific method, which is based on Fourier transform in two-dimension for calculating fractal dimension on the surface and analyzing the anisotropy of three-dimensional surfaces of machine has been proposed by Jiang Zhuangde. He also obtained fractal parameters on the surface by averaging the power spectrum with the same nominal amplitude in each direction. Machine surface has statistic self-similarity and Fractal theory can be used to describe it. And proposed by Liu, fractal theory would develop to 3-dimensional. It can be more accurately to describe morphology [9].

A multiple linear regression model with the independent variable deviation and fractal dimensions, in which case the gear quality was a non-independent variable. A field experiment showed that the proposed method is consistent with identification efficiency. Other scholars proposed a modification to the fractal rough surface contact model, and that is extending the improved rough contact model developed to a complete contact model for fractal surfaces [7]. The relationship between fractal dimension and surface roughness is an exponent one. The milled surface has fractal properties. The fractal dimension of it remains unchanged while the change of milling speed occurred. The transverse fractal crack of the axle under pure winding is considered irregular on the surface and can be characterized by non-uniformity. The result was obtained, and it showed that the obeisance will increase as same with the increase of the dimension in the fractal surface. Peng and Guo used the binary Weierstrass Mandelbrot fractal function to create a coarse surface morphology model and used the fractal micro contact model for establishing an adhesion model for plastic deformation micro convex bodies [10]. The elastic-plastic adhesion model can be solved with putting the Maugis-Dugdale model and its expansion with the Morrow method together. Others approved that the influence of an adhesion is decided critically by the fractal dimension. Qian established a model for calculation in contact characteristics of the heavy clutches by applying the fractal theory, and got the effects that the fractal dimension, contact face, contour profile size exerted on the articulus friction force, the temperature of friction could change role of the clutches [11].

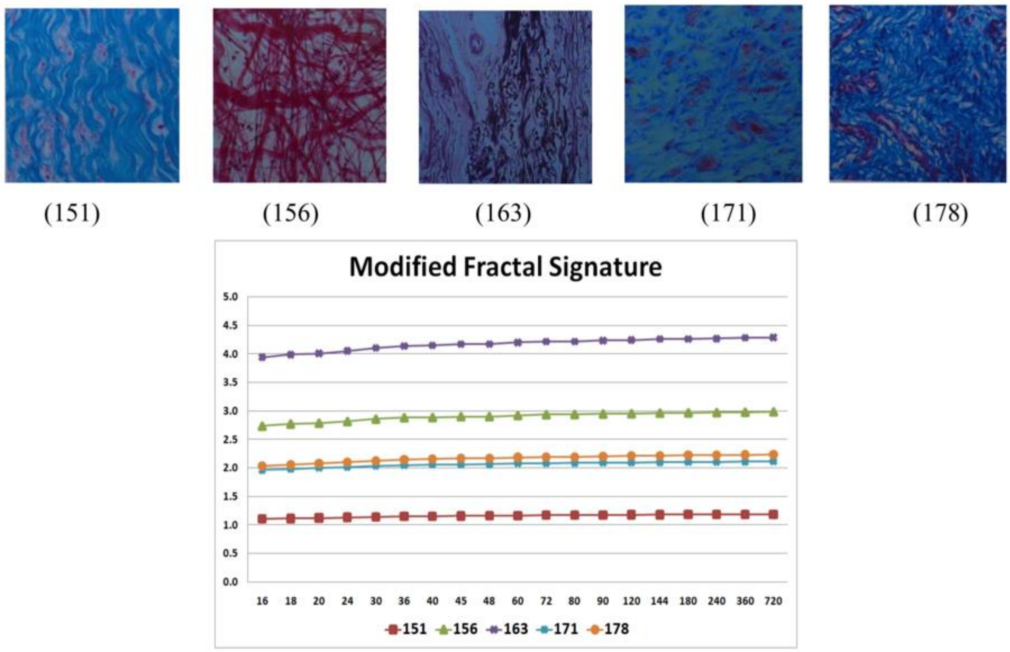

Fig. 2 An example of the result with fractal signature method

Nowadays, it is widely believed that physical systems exhibiting chaotic behavior are universally apparent in nature. An approximate system dynamic can be obtained with using statistical description because it is accessible to follow their trajectories within a short time period. One can take the phase diagrams of complex dynamical systems into considerations and analyze them as images. The stable constant set of a system usually has complex geometric structures, which will be fractals or multi-fractals (union of several fractal sets, each with their own fractal dimension, seen from Fig. 2). The multi-fractal form, as a method of describing such sets, relies on the fact that highly non-uniform probability distributions caused by the non-uniformity of the system typically have high scaling traits, such as self-similarity. As a result, one can combine fractal characteristics with non-uniformity measures, which will present us statistical characteristics with the generalized dimensions. In 1997, an improved fractal signature method based on calculating Minkovsky dimension was proposed, which is consistent with the box dimension of non-empty bounded sets in R. This method is applicable to different subject areas and has been applied to the classification of biomedical product images. There are three definitions.

Definition 1. R^n contains a nonempty bounded set F, $\Omega = \{\omega_i = 1,2,3,...\}$—a covering F, $N_\delta(F)$ -- the set number in Ω in which case their diameters are non-greater than δ. Let $dim_H F$ and $dim_T F$ symbolize Hausdorff and topological dimensions F respectively. F will be fractal if $dim_H F > dim_T F$. Box-counting (capacity) dimension is defined by $dim_B F = lim_{\delta\to 0} \frac{log_2 N_\delta(F)}{-log_2 \delta}$.

Definition 2 δ-parallel body F_δ can be defined as, $F_\delta = \{x \in R^n: |x-y| \le \delta, y \in F\}$.

Definition 3. Let F be a solid and limited set in R^n, $F_\delta - -\delta$ parallel body F, $ol^n(F_\delta)$,i.e., n-dimensional volume of F_δ. For a constant s, if δ→0, the maximum of $Vol^n(F_\delta)/_{\delta^{n-s}}$ is positive and bounded, one concludes that F maintains s in Minkovsky dimension, and it is represented by $dim_M F$.

In practical use, the image F is deformed into a few non-overlapping sub-images and is calculated for every sub-image using formulas. Then all dimensions will be summarized and give the fractal dimension that can be obtained. And it is convenient to study the contour of the image, where in all single cells the corresponding the fractal dimension value is computed.

3. Limitations and Prospects

The practical use for Hausdorff measure is limited. Calculating the Hausdorff dimension of a fractal could be complicated and hard. It is relative easier to calculate the Box dimension of one fractal with estimating and observing. It is calculated by dividing the logarithm of the number of boxes needed to cover the fractal object at a given scale by the logarithm of the scale factor. In many conditions, the box dimension of a fractal is equal to its Hausdorff dimension. But there is one drawback for it. It may confuse the dimensions, since a small point set could have a non-zero dimension with it. Thus, the application for box dimension is limited strictly. There is no definition of dimension that is applied for all fractals. Sure, here's a draft of an essay on the limitations and potential of fractals. One of the main limitations of fractals is their lack of universality. While some fractals, such as the Mandelbrot set, are highly complex and exhibit intricate patterns, others are relatively simple and repetitive. Moreover, not all natural phenomena can be accurately modeled using fractals. For example, the shapes of clouds and mountains are not strictly self-similar, and therefore cannot be represented by fractals alone. Another limitation of fractals is their sensitivity to initial conditions. Fractals are often generated using iterative algorithms that depend on the precise values of their input data. Tiny changes in these arguments can result in significant changes in the final fractal shape. This sensitivity to initial conditions can make it difficult to use fractals for predictive modeling or control.

Despite these limitations, fractals have enormous potential for future research and applications. One area of active research is the study of fractional calculus, which extends the traditional calculus to non-integer orders. Fractional calculus has been used to model complex systems that exhibit memory and long-range correlations, such as financial markets and biological networks. Fractals provide a natural framework for fractional calculus and can help to better understand the behavior of these systems. Another area of potential for fractals is in the development of new materials and structures. Fractal patterns have been found in many natural materials, such as bones, lungs, and leaves. These patterns provide optimal designs for strength, flexibility, and efficiency. By mimicking these patterns, engineers can create new materials and structures that are stronger, lighter, and more durable than traditional designs.

4. Conclusion

Fractal is a mathematical concept that describes complex patterns that repeat themselves at different scales. Fractals are self-similar, and they are created through a process called iteration, where a simple geometric shape is repeated over and over again. Fractals have many applications in science, art, and technology. Like in analyzing the mechanical transmission and in biomedicine to analyze complex biological structures.Overall, fractals are fascinating and important mathematical objects that have captured the imagination of scientists, artists, and mathematicians alike.Fractals have both limitations and potential for future research and applications. While they may not be universally applicable or predictable, they offer a powerful tool for modeling complex systems and designing new materials and structures. As research in fractals continues to advance, one can expect to see even more exciting developments in this field.

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