

Research on Damage Identification of Truss Structures based on Bayesian Updating and Rejection Sampling Method

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Abstract. The research on the early damage detection method of truss structure has important practical value for taking remedial measures in time and reducing the hidden danger of safety. In this study, a specific structural truss beam was designed and the seismic action was simulated. Subsequently, specific nodes are analyzed to assess the resulting structural damage. Bayesian updating method solves the uncertainty problem effectively by introducing the prior distribution and updating the posterior distribution with new observation data. This allows the model to infer global patterns from local information and represent the predicted outcomes in probabilistic form. Compared with the traditional parameter estimation methods, Bayesian updating can make full use of the existing observational data and obtain more reliable and accurate model prediction results in the case of limited data. The Bayesian method of updating the truss structure is flexible and adaptable, allowing updates and adjustments to be made at any given time based on new observational data. As a result, it has significant advantages in analyzing and predicting real-time data and is suitable for scenarios where models need to be dynamically adjusted.

Keywords: Bayesian update, rejection sampling, OpenSees, truss structure, damage identification.

1. Introduction

Truss structures are highly susceptible to uncertainties in node stiffness, external loads, and structural geometric dimensions, especially during operational phases and natural disasters such as typhoons and earthquakes [1]. The consequences of accidents in large-scale projects can be significant, causing substantial loss of life and property. Thus, evaluating the safety performance of truss structures, particularly those supporting and endoskeleton the structure, is crucial [2]. This paper presents an assessment of damage identification in common truss structures to determine the extent of damage and enable relevant personnel to undertake timely safety monitoring and maintenance. This approach can help prevent disasters and save costs [3].

Bayesian updating is a statistical inference method that is widely used for damage identification of truss structures based on their vibration responses [4]. The Bayesian method determines a prior distribution based on historical data and past experience. When new detection data becomes available, the initial distribution is updated by incorporating the new data, effectively reducing the uncertainty of the prior distribution [3]. This study focuses on damage identification under static response using the Bayesian updating method on hyper-static trusses. The simulation of a basic truss structure after an earthquake was accomplished using OpenSees software, as shown in Fig. 1. The test data of node displacement was used to identify the damage location and degree of the truss structure. However, generating samples directly based on complex probability density functions remains challenging. Simple sampling is performed on the proposed distribution, followed by acceptance-rejection operations to achieve sampling on complex distributions [5]. Furthermore, the Bayesian method allows for the quantification of uncertainty related to various parameters such as physical uncertainty of variables, statistical uncertainty of model parameters, and model uncertainty of applied mathematical models for a comprehensive assessment of damage identification [5]. Finally, the experimental data is compared with the actual situation, and optimization methods for damage identification are discussed to improve the effectiveness and accuracy of this approach.

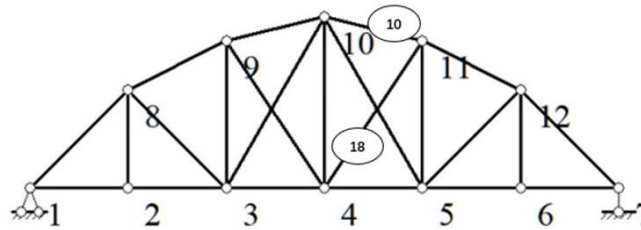


Figure 1. Damage detection model of truss structure

2. Research Methods

2.1. Damage Identification Methods

There are two main categories of damage identification techniques in civil engineering. Local identification techniques utilize visualization or localization methods to accurately identify and analyze the location of structural damage. These techniques have been successfully applied to inspect cracks, welding defects, corrosion wear, relaxation, and instability in specific components. In practice, multiple techniques are often used together to evaluate the overall structural state.

On the other hand, global damage identification techniques focus on dynamically detecting structural damage. The main objective is to determine the current state of the structure based on its dynamic response. This involves comparing the measured modal characteristics of the damaged structure with those of a healthy structure to assess the extent of damage [6].

When studying damage in a given truss structure, the initial step is to conduct a stress analysis of the truss. For instance, if the truss is a non-structural member, the stress analysis is performed by equating the sum of the overall bending moment and force to zero. After determining the specific force magnitude at each node of the truss, further research is conducted on the more susceptible bars.

In the case of a hyper-static truss structure, a qualitative analysis of the susceptible bars is carried out after removing redundant bars and invalid nodes. Subsequently, the obtained results are incorporated into a structural mechanics solver for more detailed investigation.

2.2. Bayesian Updating

In the field of civil engineering, Bayesian updating is commonly employed to update the probability distribution of the structural damage state based on observed data. Bayesian methods rely on three fundamental types of information for statistical inference: population information, sample information, and prior information [7].

Population information includes data from the population distribution and its corresponding family of distributions. Sample information refers to information derived from a sample drawn from the population. Prior information encompasses existing knowledge or experience regarding the statistical inference problem before sampling takes place.

For truss structure damage identification, Bayesian updating can be applied through the following steps. Firstly, it is necessary to establish a damage model for the truss structure, incorporating parameters such as damage location, damage type, and damage extent. Secondly, in the absence of observed data, a prior probability distribution of the damage state can be established based on existing knowledge or experience. This prior distribution may take the form of a uniform distribution, normal distribution, or other appropriate distributions.

Thirdly, a series of observed data can be acquired by monitoring the vibrations of the truss structure, including modal parameters and frequency response. Finally, employing Bayes' theorem, the posterior probability distribution of the damage state can be calculated by combining the observed data with the prior probability distribution. This posterior distribution represents the probabilistic estimation of various damage states given the observed data. It is worth noting that the selection of the prior probability distribution is a subjective matter and can be a source of controversy [8].

2.3. Rejection Sampling

Rejection sampling is a widely employed technique in Bayesian updating and Monte Carlo-based truss structure damage identification. Its primary objective is to randomly sample from the target distribution, such as the posterior distribution, while only accepting samples that adhere to specified criteria and rejecting others. This ensures that the accepted samples accurately represent the target distribution [9].

To identify potential damage locations and extents within the truss structure, observation of its dynamic responses is crucial. By establishing an initial prior distribution based on the observed data, the posterior distribution is obtained by employing the Bayesian formula. In order to generate samples from the posterior distribution, Monte Carlo methods are utilized to sample from the prior distribution. Nevertheless, due to sampling challenges, numerous samples may be inadmissible. Consequently, the rejection sampling method is employed to filter out samples that deviate from the target distribution, accepting only those that align with it. This method effectively reduces the sample size and enhances sampling efficiency for damage identification purposes.

Considering the inherent characteristics of rejection sampling in generating random numbers corresponding to any given probability distribution function, the rejection sampling method will be implemented. It is worth noting that utilizing a function that closely resembles the sampling function in shape significantly improves sampling efficiency [9].

2.4. OpenSees Software

OpenSees is a software tool utilized in seismic response simulations for building structures and geotechnical structures [10]. It facilitates numerical simulations of structures under static loads and incorporates Bayesian theory and structural dynamic test data to update the structural reliability. This approach acknowledges uncertainties in both the excitation experienced by structures and the structure model, including its parameters. By employing Bayesian probability methods, it employs dynamic test data obtained during the service life of the structure to identify the structural parameters. The reliability assessment of a specific truss structure under random dynamic loads is conducted under three conditions: considering only the randomness of the loads, considering the randomness of the loads along with the prior distribution of the structure model parameters, and considering the randomness of the loads coupled with the updated distribution of the structure model parameters. A comparison is made between the natural frequencies and mode shapes of the actual structure and the finite element model after updating, followed by an analysis of the results derived from the updated reliability calculation.

3. Loading Scheme Results and Discussion

3.1. Experiment 1

The experiment began by varying the magnitude of the downward loading at node 11 of the truss model (Fig. 1) and the elastic modulus of the material, and conducting 10,000 simulation experiments. The results of the former (Fig. 2) indicate that regardless of the changes in the applied load magnitude, the number of experimental failures remains stable between 9600 and 9650. The results of the latter (Fig. 3) show that regardless of the changes in the elastic modulus of the material, the experiments yield the same conclusion. In fact, despite significant variations in the conditions, the changes caused by the data are not substantial.

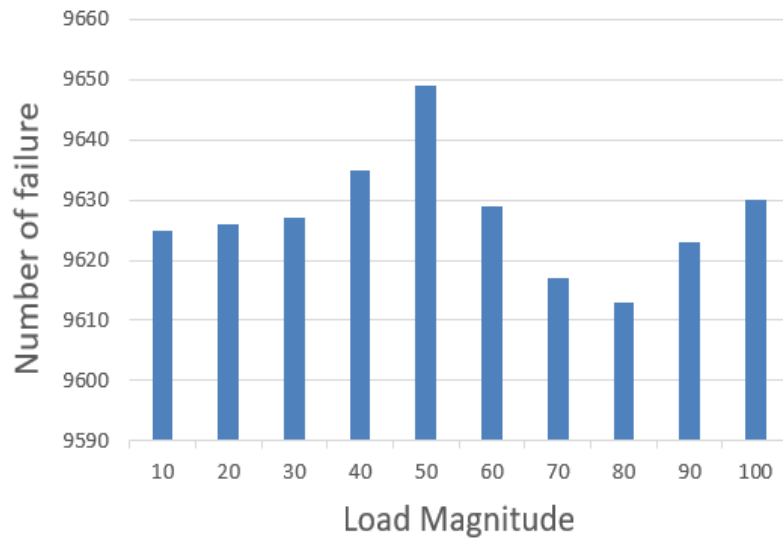


Figure 2. The experimental results under changing the load of No.11 node

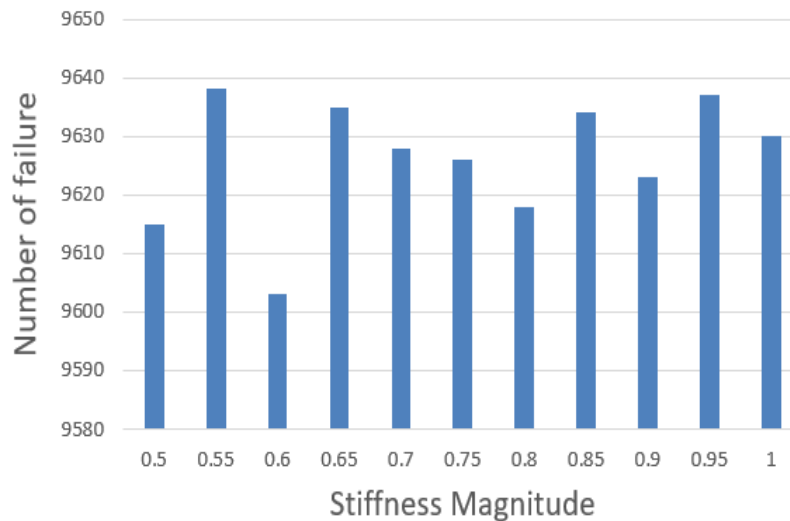


Figure 3. Experimental results under changing material stiffness

3.2. Changing the Loading Point

3.2.1. First loading

The experiment selected nodes 6 and 11 as the loading points, with a loading magnitude of -100 kN. The measurement points were located at nodes 4, 5, 6, 10, 11, and 12. The stiffness sampling data intervals for the 10-bar and 18-bar are shown in Fig. 4, respectively. The experimental results do not fit well with a normal distribution. Under the rejected sampling conditions, the distribution of the stiffness sampling data for the 10-bar and 18-bar is shown in Fig. 5, respectively. Compared to the situation before rejecting the sampling, there is a significant improvement in the fitting degree.

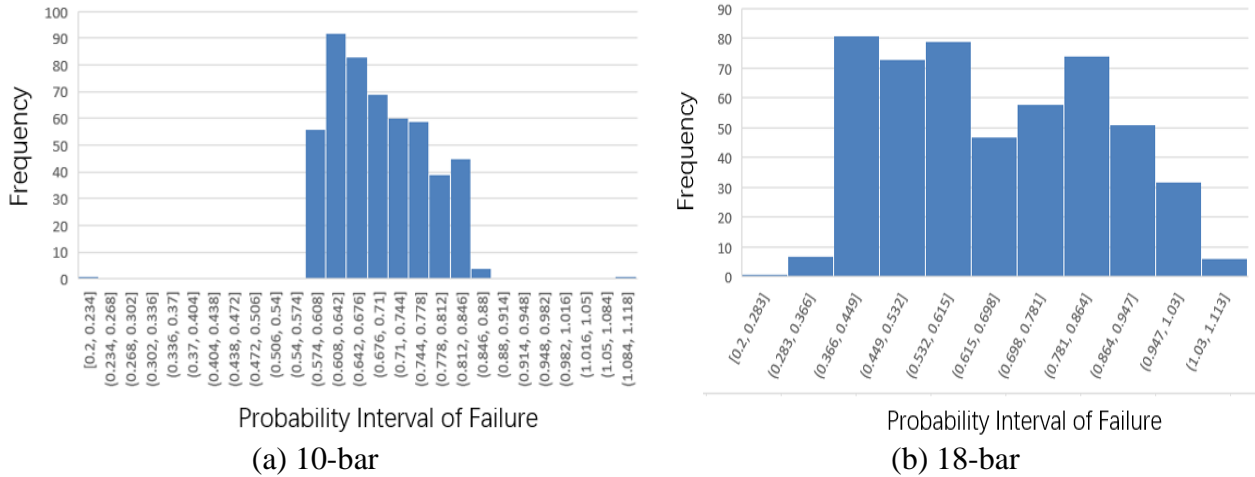


Figure 4. The results of experiment with loading points of 6 and 11

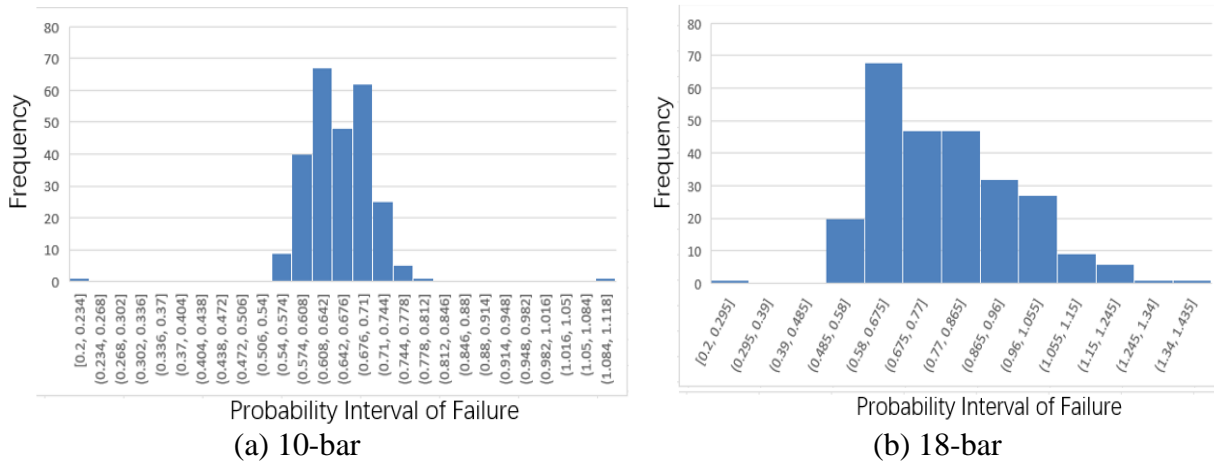
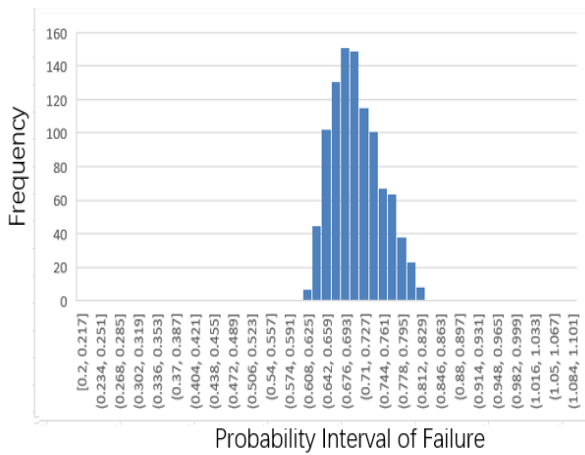


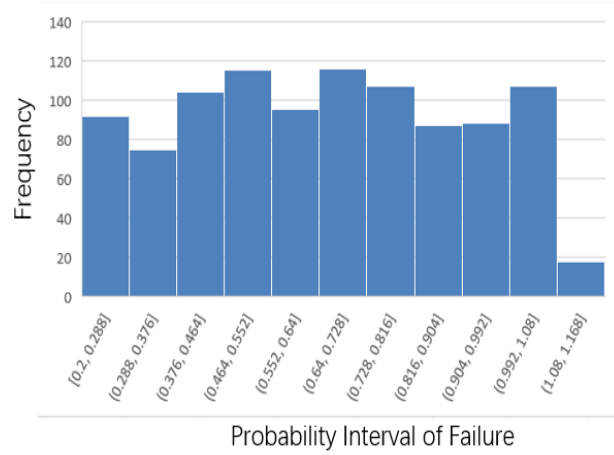
Figure 5. Experimental results of rejection sampling under loading points 6 and 11

3.2.2. Second loading

The loading points were selected as nodes 4, 6, and 11, with a loading magnitude of -100 kN. The measurement points were located at nodes 4, 5, 6, 10, 11, and 12. The stiffness sampling data for the 10-bar and 18-bar are shown in Fig. 6, respectively. The experimental results are almost identical to the first loading point selection, and they do not fit well with a normal distribution. In fact, the fitting degree is even worse for the 18-bar compared to the results of the first loading experiment. Under the rejected sampling conditions, the distribution of the stiffness sampling data for the 10-bar and 18-bar is shown in Fig. 7, respectively. There is a significant improvement in the fitting degree compared to the situations before rejecting the sampling and the rejection of sampling in the first loading experiment.

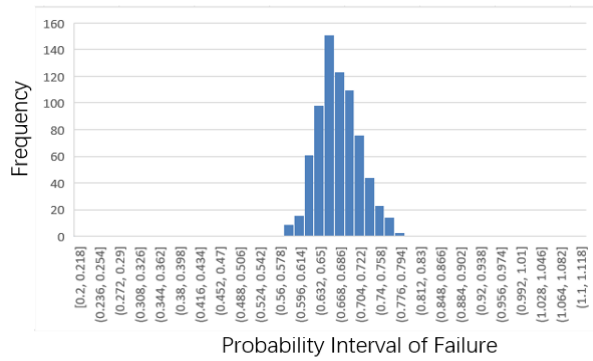


(a) 10-bar

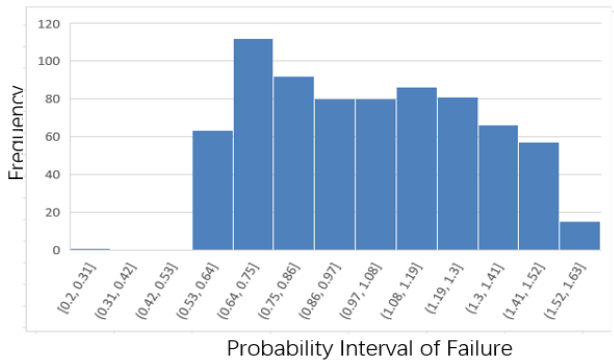


(b) 18-bar

Figure 6. The results of experiment with loading points of 4, 6 and 11



(a) 10-bar



(b) 18-bar

Figure 7. Experimental results of rejection sampling with loading points of 4, 6 and 11

3.2.3. Third loading

The loading points were selected as nodes 4, 5, 6, and 11, with a loading magnitude of -100 kN. The measurement points were located at nodes 4, 5, 6, 10, 11, and 12. The stiffness sampling data for the 10-bar and 18-bar are shown in Fig.8, respectively. Similarly, the experimental results are almost identical to the previous two loading point selections, and they do not fit well with a normal distribution. The fitting degree is also worse for the 18th bar compared to the results of the previous two loading experiments. Under the rejected sampling conditions, the distribution of the stiffness sampling data for the 10-bar and 18-bar is shown in Fig. 9, respectively. There is a significant improvement in the fitting degree compared to the situations before rejecting the sampling and the rejection of sampling in the previous two loading experiments.

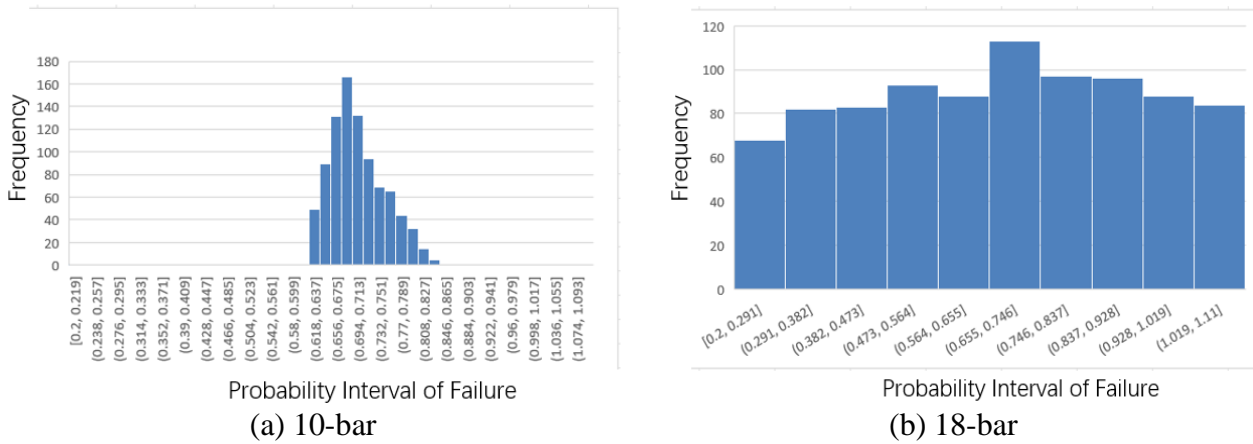


Figure 8. Experimental results with loading points of 4, 5, 6 and 11

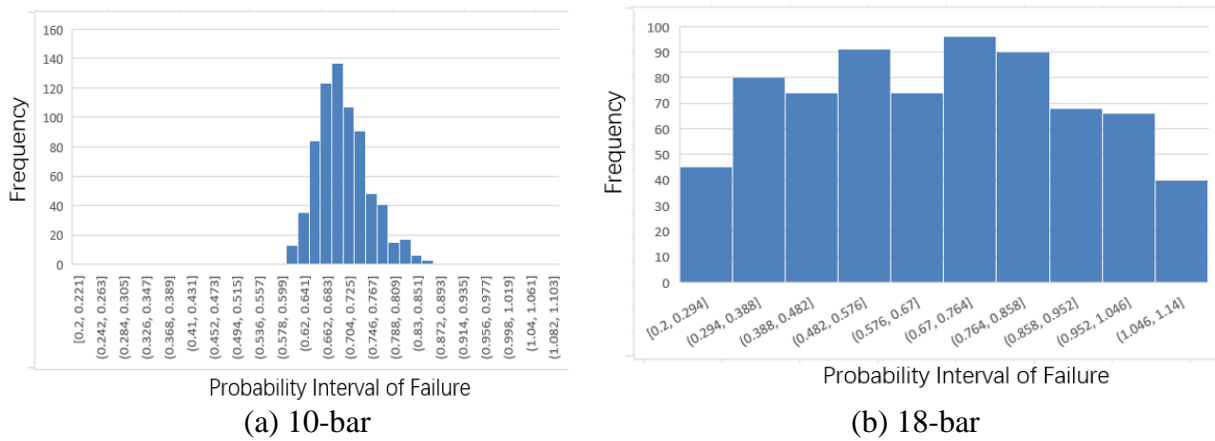


Figure 9. Experimental results with loading points of 4, 5, 6 and 11 after refusing sampling.

3.2.4. Forth loading

The loading points were selected as nodes 4, 5, 6, and 11, with a loading magnitude of -100 kN. The measurement points were located at nodes 4, 5, 6, 10, 11, and 12. Under the actual data conditions, without rejecting the sampling, the distribution graphs of the stiffness sampling data for the 10-bar and 18-bar are shown in Fig. 10, respectively. Similarly, the experimental results are almost identical to the previous two loading point selections, and they do not fit well with a normal distribution. The fitting degree is also worse for the 18-bar compared to the results of the previous three loading experiments. Under the rejected sampling conditions, the distribution of the stiffness sampling data for the 10-bar and 18-bar is shown in Fig. 11, respectively. There is a significant improvement in the fitting degree compared to the situations before rejecting the sampling and the rejection of sampling in the previous three loading experiments.

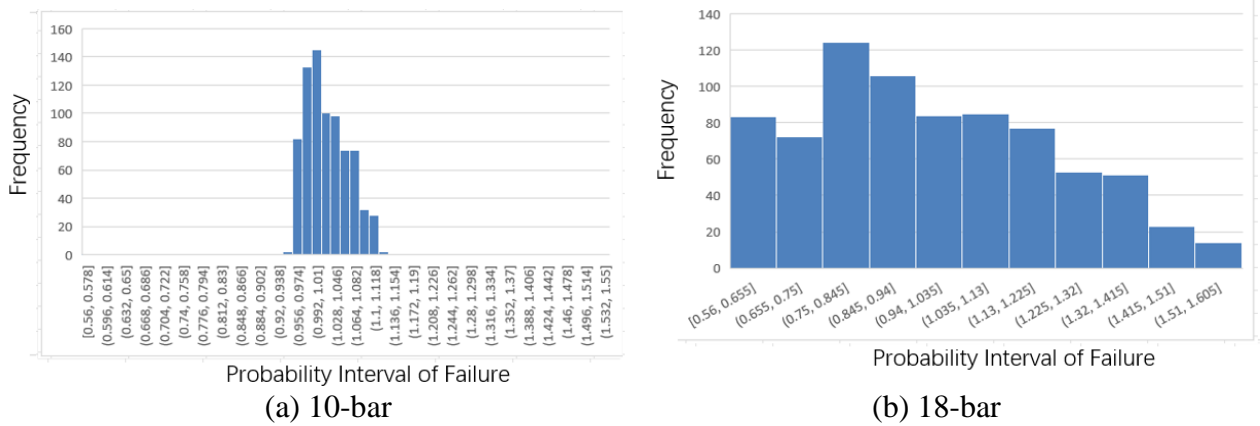


Figure 10. Experimental results with loading points of 4, 5, 6 and 11 after refusing sampling under non-real conditions

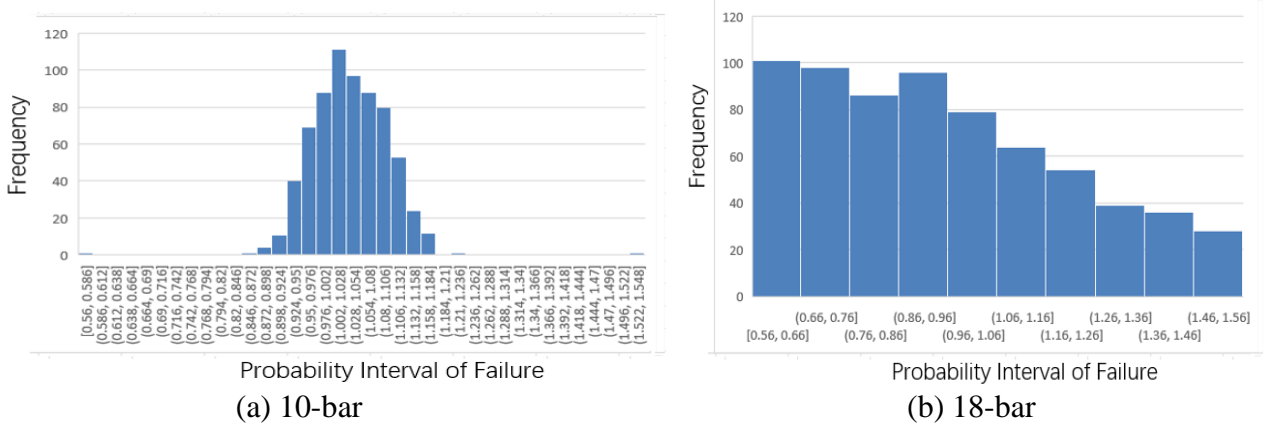


Figure 11. Experimental results with loading points of 4, 5, 6 and 11 after rejecting sampling in real situation

To sum up, the results show that the number of loading points will directly affect the concentration of data. Secondly, rejection sampling will make the data more accurate, and the data distribution of real data and model data is basically the same.

3.3. Relative Error and Absolute Error

The relative error refers to the relative value compared to the true condition. Taking into account the influence of various errors, the acceptance range is set at 5% of the true value, which means that all successful experimental results must be based on the deformation of the bars not exceeding 5% of the initial dimensions. The absolute error refers to the difference in the true condition. The acceptance range is set to 0.5mm, which means that all successful test results must be based on the deformation of the rod not exceeding 0.5mm. Under the actual data conditions, the loading points were selected as nodes 4, 5, 6, and 11, with a loading magnitude of -100 kN. The measurement points were located at nodes 4, 5, 6, 10, 11, and 12. In the experiment under the assumption of absolute error, Fig. 12 shows the distribution of the 10-bar and 18-bar, respectively. In the experiment under the assumption of relative error, Fig. 13 shows the distribution of the 10-bar and 18-bar, respectively. It can be observed that compared to the relative error, the accuracy of the data under the absolute error assumption is higher. The number of successful cases in 10,000 selections is only single-digit, but the effective sample size is too small to exhibit a normal distribution.

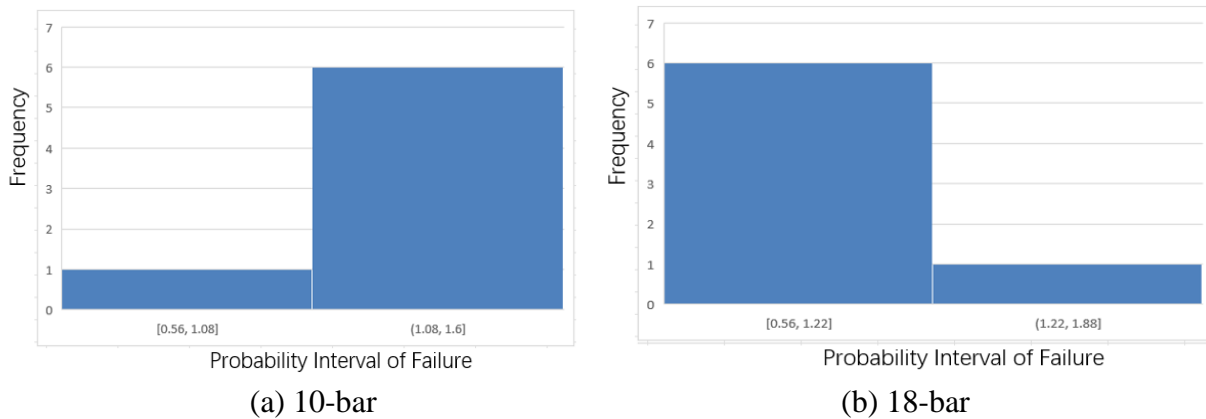


Figure 12. Experimental results of rejection sampling with loading points of 4,5,6 and 11 under absolute error

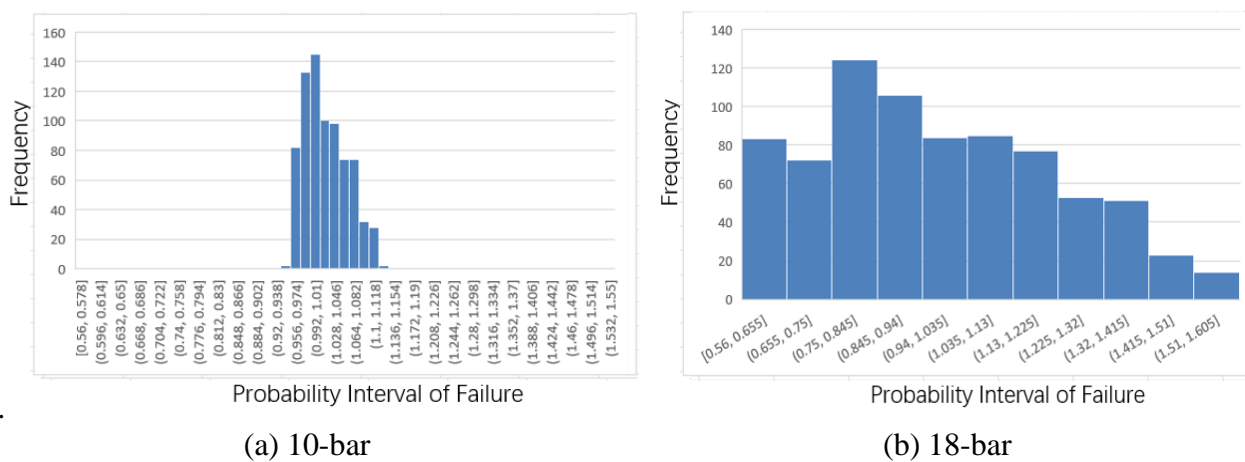


Figure 13. The experimental results of rejection sampling under the loading points of 4, 5, 6 and 11 under the relative error

4. Conclusion

The experiment showed that varying the load magnitude and material elastic modulus had little influence on the data: regardless of how the applied load magnitude or material elastic modulus was changed, the number of failed experiments remained between 9600 and 9650. This indicates that the variations in conditions did not have a significant impact on the data. Reject sampling improved the fit of the stiffness sampling data: prior to reject sampling, the distribution of the stiffness sampling data did not match a normal distribution. The number of loading points and reject sampling affected the concentration and accuracy of the data: increasing the number of loading points could enhance the concentration of the data. Additionally, the application of reject sampling techniques improved the accuracy of the data. The distribution of the real data was essentially the same as the model data, exhibiting higher accuracy when conducting experiments under the assumption of absolute error. When conducting experiments under the assumption of relative error, these data showed lower accuracy compared to the absolute error condition.

Although the Bayesian updating method for truss structures has promising prospects in theory, it still faces challenges in practical applications. Bayesian methods involve a lot of uncertainty, such as selecting appropriate prior distributions, handling high-dimensional problems, and computational complexity, which require further research and solutions. However, with the continuous development of statistics and computational technology, it is believed that these challenges will be gradually

overcome, and the application prospects of Bayesian updating methods for truss structures will become more extensive in various fields.

Authors Contribution

All the authors contributed equally and their names were listed in alphabetical order.

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