

Analysis of the Principle and Applications of Random Walk: Finance, Diffusion Equations and Game Design

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Abstract. As a matter of fact, as a common simulation types, random walk is widely applied in various fields in recent years incorporating with the Monte-Carlo simulations. With this in mind, this study summarizes and provides an analysis of the principle and applications of random walks in three diverse fields, i.e., finance, game strategy, and game design. To be specific, it begins by introducing the historical background and significance of random walk research as well as the basic mathematical descriptions. Recent advancements and applications of random walks are discussed. In reality, the analysis includes an explanation of the basic concepts and mathematical representations of random walks, followed by three specific applications with real-world scenarios and results. At the same time, the study also discusses the limitations of current random walk applications and offers insights into future research prospects. Overall, these results shed light on guiding further exploration of random walk applications.

Keywords: Random walk; finance; game strategy; hypergraphs.

1. Introduction

Random walk theory has been a fundamental concept in various fields, including finance, physics, and computer science. This introduction provides a fundamental overview of the historical evolution and significance of random walk research. Random walks have a rich history. It started from the mid 18s, and its models are still used by many researchers now. Questioned by Karl Pearson, Random Walks then have a wide influence on different areas of study. Actually, the problem of Random Walks was addressed before Pearson by Lord Rayleigh. As Random Walks models develop, it applies to many fields [1].

To understand random walk applications better, it is crucial to establish a solid foundation in the fundamental concepts of random walks. A model of successive random steps on a path, is recognized as Random Walk. This can be in multiple dimensions, such as the basic simulations of one-dimensional random walk, two-dimensional random walk, and three-dimensional random walk. For example, when calculating probabilities for statistical use of different fields, researchers find phenomenons that can be modeled by random walk models. a 3D random walk model can simulate the diffusion between liquids and motions of molecules. By calculating the probability of a certain path appearing, researchers can summarize the statistic for further physical research.

In recent years, the application of random walking has surged in various fields, such as finance, game strategy, and hypergraph. In finance, it can be used to explore the occurrence of record-breaking events in random walks with asymmetric jump distributions [2], which discusses an application to financial study. The random walk model is used to predict stock prices and scholars state that the research has practical applications, such as comparing the analytical results to the record statistics of daily stock prices from the Standard & Poor's 500 index. This application helps understand how the presence of an overall trend in stock prices affects the number of upper records [3]. In game decision making, random walk helps decision makers to make faster and better decisions. In many games, character movement is based on random walk algorithms [3]. This is especially common in simulations, sandbox games, and games with open-world environments. Researchers analyzes four studies exploring strategic thinking in a number-guessing game and highlights that nonrandom strategic choices can be advantageous, but their effectiveness depends on the behavior of the opponent [4, 5]. Others introduce a research study that applies a random walk framework to analyze hide-and-

seek games on complex networks. a way to use random walk theory for game strategy: by evaluations of search efficiencies [6]. Random Walk is used to develop a spectral theory for hypergraphs. This can be in various ways to analyze, model, and extract information from complex data structures. A hypergraph extends the concept of a traditional graph by allowing hyperedges that connect more than two nodes. Random walks are used to develop a spectral theory for hypergraphs with edge-dependent vertex weights: hypergraphs where every vertex v has a weight $\gamma_e(v)$. This paper is for reviewing and analyzing the applications of random walk with a main focus on the fields of finance, game strategy, and hypergraph.

2. Basic Descriptions

Random walk theory is a mathematical concept used to describe a random or probabilistic process in which an entity (usually a point in space or a value on a numerical line) moves in discrete steps based on a set of random results or events. It is a fundamental model for understanding stochastic processes and has wide applications in various fields such as physics, finance, biology, and computer science. Random walks are often defined in discrete time, meaning that at each time step, the walker makes a move based on random outcomes. The most basic random walk is one-dimensional random walk, where the walker moves along a line. This can be simplified as a randomly-moved point on a numberline [7].

Let X_t represent the position of the walker at time t . The walker takes each step of a certain size, which can be constant or follow a certain probability distribution. The walker's step direction (left or right) is determined by a random process. Let S_i represent the i -th step, where i is a positive integer indicating the step number. The walker's position at each step can be mathematically represented as:

$$X_t = X_0 + \sum_i S_i \quad (1)$$

where X_t is the position at time t , X_0 is the starting position (initial condition), S_i is the step size at the i -th step. If the step sizes have associated probabilities, one can denote $P(H)$ as the probability of a positive step and $P(T)$ as the probability of a negative step. Then, the expected step size is:

$$E[S_i] = P(H) \times (\text{positive step size}) + P(T) \times (\text{negative step size}) \quad (2)$$

The random walk theory explores the behavior and characteristics of X_t as t increases, such as expected position, variance, and probability distribution of reaching a specific position after a certain number of steps. It is a fundamental concept used in various applications to simulate and analyze phenomena affected by random or uncertain events.

3. Applications in Finance

Random walk theory has important applications in the financial field. It is often used to simulate stock prices and financial market behavior. The Efficient Market Hypothesis (EMH) is one of the key concepts related to finance and random walks, which suggests that stock prices follow random walks and therefore cannot consistently outperform the market by predicting stock price trends. An example of pricing is shown in Fig. 1. Research in this field has explored the recording statistics of biased random walks and their applications in financial data analysis. By studying how stock prices evolve due to movements similar to random walks, researchers have gained a deeper understanding of the impact of market dynamics and trends on stock price records. Random walk and market efficiency test: evidence from capital markets in the United States, China and Europe in the context of the global COVID-19 pandemic.

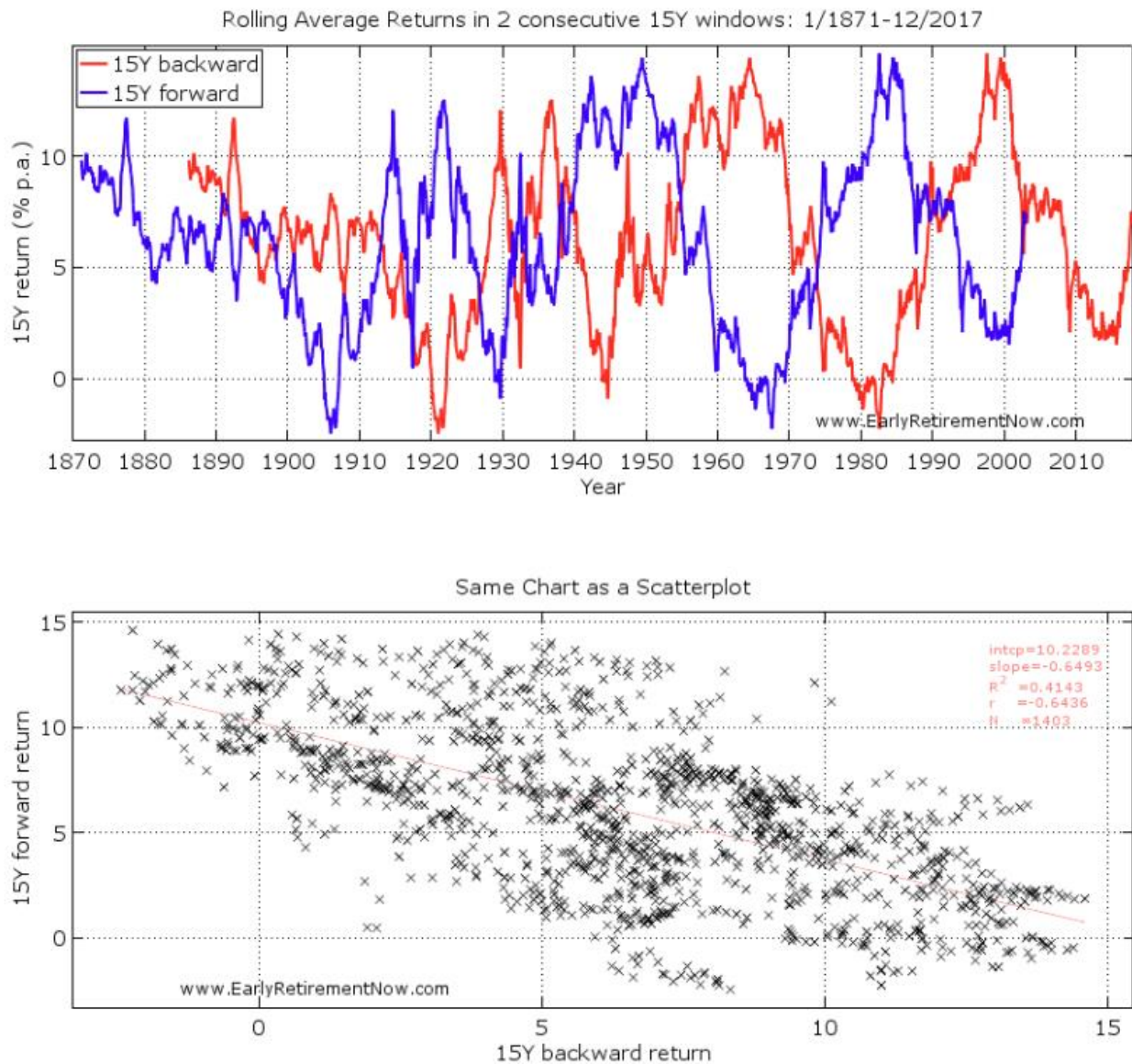


Fig. 1 Price prediction based on random walk.

An analysis was conducted on the stock markets of China and India, and it was found that these two countries did not exhibit a weak state of market efficiency. Random walks are used for testing [3]. It reveals that returns show long-term correlation, supporting the view that problematic stock markets are inefficient and have not reached the mature stage of market development. In the French capital market, from day 2 to day 10, the random walk hypothesis was rejected, and from day 11 to day 16, one found that the market tended towards equilibrium. Evaluate the structural breakthrough of the analyzed market, which saw a significant breakthrough in March 2020, with the exception of the Spanish and Chinese markets.

Preisou study investigates the application of random walk theory to efficient markets and its effectiveness in predicting stock returns in the Nigerian Stock Exchange (NSE), and explores the Efficient Market Hypothesis (EMH) [7]. The results indicate that although past returns are a good guide for predicting the future, there is no doubt that there is insufficient data and information in predicting NSE stock returns. The investors should comprehensively consider past and current stock performance when predicting future returns, and suggest that investors diversify their investment portfolios and increase the market sensitivity of the US Securities and Exchange Commission and NSE to market activities. The conclusion of this study is that it advocates that future research needs to focus on other variables of the economy, directly focusing on developing countries and other growing markets. The research results confirm that all variables are statistically significant.

Other scholars tested the Efficient Market Hypothesis (EMH) to examine the information efficiency of cryptocurrency markets [8]. It uses three random walk tests to verify weak EMH and used event study methods to test semi strong EMH. The analysis results indicate that 6.04% comply with the weak form EMH and 2.695% comply with the semi strong market hypothesis; also, among cryptocurrency exchanges established before November 2017, large exchanges are more likely to meet weak and semi strong EMHs.

4. Applications in Game Strategy

Random walk theory has also been applied to game strategies, especially in analyzing the decision-making process in games. In various types of games, number guessing game the movement of characters is usually dominated by random walk algorithms. For example, in simulation games and open world games, characters may follow random paths, resulting in various unpredictable behaviors.

Previous study discusses four studies that use random walk theory exploring strategic thinking in a number-guessing game and highlights that nonrandom strategic choices can be advantageous, but their effectiveness depends on the behavior of the opponent. The findings suggest that players may prefer strategic thinking when they perceive it as providing an edge, and that successful strategies may sometimes involve breaking conventional rules [4]. In hide and seek games, as discussed in the study [6], random walking provides a framework for evaluating search strategies. These studies help improve the design of games and decision algorithms, and enhance the player experience. The study proposes two approximation methods for predicting efficient search strategies. One method is based on an equilibrium analysis of the random walk strategy, providing approximate but reasonable estimates of optimal search strategies. The second method utilizes an effective drift concept in random walks on networks and is expected to be efficient in networks with specific characteristics.

Others explore the model construction and dynamic interaction process of both parties in game sports. The study described the positional interaction between players from both sides during the relative stage of a single net/wall game. The results indicate that relative phase displays certain aspects of tennis tactical interaction [9]. At a more abstract level, the interaction between two teams in handball was studied by studying the relationship between two scoring processes. Each process can be seen as a random walk. The moving average of the probability of scoring shows something similar to instantaneous intensity. The mobile correlation (length=20 ball controls) describes the instantaneous relationship between the strength of two teams. There is a significant positive correlation as well as a significant negative correlation between each stage. This highlights the importance of viewing the interactive relationships in these competitions from a dynamic perspective [10].

5. Applications in Hypergraphs

Random walks have played an important role in developing the spectral theory of hypergraphs, extending their applications to complex data structures. Hypergraph is used to represent the relationship between hyperedges connecting multiple nodes. In addition, the research applied random walk to hypergraphs with edge related vertex weights [5, 10]. A sketch is shown in Fig. 2. This study enhances our understanding of complex data structures and their spectral characteristics, and has potential applications in fields such as data mining, social network analysis, and knowledge representation.

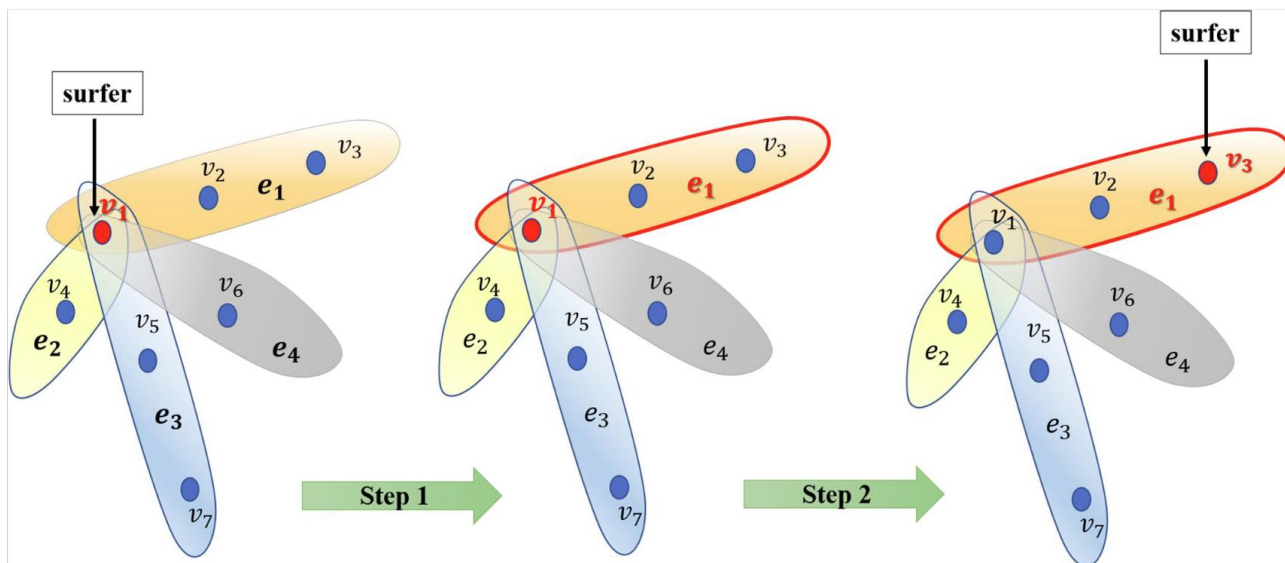


Fig. 2 A sketch of applications of random walk in hypergraphs.

6. Limitations & Future Outlooks

Although random walk theory is widely used, its limitations such as simplicity assumption and data requirements should be noticed. Random walk models typically assume simple step sizes and probabilities, which may not fully reflect the complexity of real-world systems. Random walk models may not take into account specific factors that affect decision-making in games, finance, or hypergraphs, resulting in over simplification. In addition, random walk applications may require a large amount of data, while real-world data may not always be readily available or may not meet idealized assumptions. Random Walk models still have potential in the future. Continuous research on financial applications of random walks may provide more accurate predictive models for stock price trends. The combination of machine learning technology and random walk principle can provide better insights into market behavior. Game developers can further explore random walk algorithms for character behavior to create more dynamic and adaptable non player characters. Reinforcement learning techniques can be combined with random walks to improve NPC decision-making. Using random walks in hypergraphs may lead to more complex methods for knowledge extraction, community detection, and recommendation systems.

7. Conclusion

In summary, this article explores the principles of random walks and their various applications in the fields of finance, game strategy, and hypergraphs. The history of random walk theory can be traced back to the 18th century, providing a powerful framework for understanding random processes and probabilistic behavior. In finance, random walk is a component of the efficient market hypothesis and has been applied to predict stock prices, evaluate market efficiency, and analyze financial data. These applications provide valuable insights into the dynamic nature of financial markets. In the field of game strategy, random walk algorithms can assist in the movement of characters in various types of games, enhancing gameplay and decision-making processes. This has had an impact on strategic thinking and player experience, breaking traditional rules and promoting innovative strategies. In addition, random walks have been extended to the field of hypergraphs, enabling the development of spectral theory for complex data structures. This opens the door to applications in fields such as data mining, social network analysis, and knowledge representation. Although the random walk model has been widely used, it also has limitations, including oversimplification and data requirements. However, the future prospects are promising, with opportunities to improve predictive models, integrate machine learning, and enhance NPC behavior in games. Developing techniques with

random walk theory may open up new insights and applications in these different fields. In short, random walks are still a valuable tool for understanding and simulating complex probability phenomena.

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