# Analysis of the Applications for Monte-Carlo Simulations in Real Estate Modeling, Radiation Therapy and Brachytherapy

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**Abstract.** Monte Carlo simulation is a frequently utilized modeling technique present in PMBOK, primarily in the quantitative risk analysis process of the risk management knowledge area. It serves as one of the fundamental tools for conducting quantitative risk analysis in a project. Monte Carlo simulation can estimate project schedule or cost, as well as assist in creating a schedule plan. This study will introduce the Monte Carlo simulation regarding its historical background, derivation process, algorithms, and significant role in the fields of economics and physics. The text uses clear, concise language with passive tone and avoids biased or ornamental language. Precise technical terms are utilized, with explanations for abbreviations. The structure creates a logical flow of information with causal connections between statements, adheres to conventional academic sections, and maintains regular formatting. Additionally, the text is free from grammatical errors and follows consistent citation and footnote style. Based on the analysis, the Monte Carlo simulation method efficiently resolves cash flow randomness and uncertainty related to project investment. Financial analysts and project decision makers can be relieved from taxing mathematical calculations, and the computer can conduct numerous numerical simulation experiments within a relatively short span of time, enhancing decision-making efficiency.

**Keywords:** Monte-Carlo simulation; real estate modelling; radiation therapy; brachytherapy.

## 1. Introduction

Monte Carlo simulation is a method that simulates the probability of various outcomes of a process [1]. To mitigate this unpredictability, Monte Carlo simulation is helpful in selecting appropriate methods. The experimental process is considered unpredictable due to the use of random variables, such as uncertain variables or risk. It requires generating random samples of input values to create statistical distributions of potential outcomes [1]. Monte Carlo simulation models have the ability to predict the likelihood of different outcomes when dealing with probabilistic components. To determine a range of potential outcomes, Monte Carlo simulation randomly assigns values to uncertain variables [1] and then calculates the average results. This model is useful for predicting and forecasting how risk and uncertainty may impact a system [2]. The 1777 method of calculating pi by French scientist Georges Buffon[3], in which he threw a needle onto a surface of parallel lines, is regarded as one of the earliest forms of Monte Carlo methods. Place a series of needles with a length of S onto parallel lines. The distance between the lines, d, must exceed the length of the needles, i.e. greater than S. The primary question is to determine the probability of a needle intersecting a line. It is assumed that the array is of sufficient size, and that the needles are distributed uniformly in their direction across the interval $[0,\pi)$. We must assume that the needles are uniformly distributed within the region, which implies that the axes of these needles are random values within the region [3].

When a pin intersects a line, this is equivalent to a rectangle with a width of $l \sin \theta$ intersecting the same line. It is important to consider the probability of this occurrence for a given $\theta$:

$$P(\text{intersection} \mid \theta) = \frac{l \sin \theta}{d} \tag{1}$$

$\theta$ is unified on $[0, \pi)$ so that

$$P(\text{intersection}) = \frac{1}{\pi d}\int_0^{\pi} \sin \theta \, d\theta = \frac{2l}{\pi d} \tag{2}$$

Thus, if n pins are placed, then if the experiment is performed several times, the average number of intersections with a line is:

$$\pi = \frac{2nl}{Md} \tag{3}$$

Thus, if "n" pins are placed and the experiment is conducted multiple times, the mean number of line intersections can be calculated as $\pi \approx 355/133$ [3]. The initial Monte Carlo techniques showed promise but were impractical due to their high experimental costs. Consequently, physicists at Los Alamos in the 1940s utilized Monte Carlo methods to address problems. Advancements in Monte Carlo methods occurred in the 1980s [4], ultimately leading to their widespread use in the statistical community for evaluating high-dimensional integrals. More and more scientists will use Monte Carlo methods. This heightened interest also spurred researchers from other fields to develop various methodological and theoretical advancements beyond the typical scope of Monte Carlo integrals [4]. The origin of Monte Carlo simulation is linked to Monte Carlo, a city in Monaco that was once a renowned gambling location in Europe [5]. This simulation technique rests on probability calculations, which are also fundamental to the world of gambling. As a result, the method is called after a renowned gambling hub [6].

## 2. Basic Description

One of the most direct examples, in the early days of Monte Carlo's invention, was the dice [7]. With 36 distinct combinations possible, it is feasible to manually determine the probability of any particular outcome. Nevertheless, to obtain more precise predictions, Monte Carlo simulation can be employed to simulate upwards of 20,000 dice rolls (or more) [7]. To apply the Monte Carlo method to real-world problems, adhere to these key steps:

Develop a simple and feasible probabilistic statistical model that utilizes the problem's attributes to derive precise probability distributions or mathematical expectations.

Provide a variety of sampling techniques for the different distributions of random variables used in the model [8].

Conduct statistical analysis on the simulation results to create statistical approximations of the problem solution and its accuracy level [8].

To estimate the probable price fluctuation of a stock or asset, one can create a Monte Carlo simulation utilizing Microsoft Excel or analogous software [8]. The said fluctuation involves two components: drift and random inputs. Data such as fluctuations in asset values, standard deviations, means, and other variables can be modeled using Monte Carlo simulations to calculate the probability of specific outcomes. From the definition of probability, the likelihood of an event can be estimated by its frequency of occurrence in numerous trials, and when the sample size is sufficiently large, this frequency can be regarded as the event's probability. Thus, numerous random samples can be drawn from the variables impacting its reliability. These values may then be collectively substituted into the functional equation to determine whether the structure fails. Ultimately, its failure probability can be derived. The Monte Carlo method is founded on this approach for analysis.

## 3. Monte Carlo Simulation in Commercial Real Estate Modeling

In the past, assessing real estate market risks has posed challenges due to limited data availability, non-normally distributed returns, and inadequate methodologies. Evaluating project investments requires critical indicators such as Net Present Value (NPV) and Internal Rate of Return (IRR), which demonstrate the project's return on investment. Special attention should be given to technical vocabulary for precise and clear communication. The report must consistently utilize appropriate terminology. Simulation results play a significant role in informing investment decisions and in assessing project feasibility [9]. Project investment appraisal provides a dependable measure for assessing return on investment. The two key criteria for this appraisal are net present value (NPV) and internal rate of return (IRR) [9]. To assess potential investment risks, sensitivity factor simulations are performed alongside Monte Carlo simulations. The resulting data is used to inform investment decisions and evaluate whether the project is feasible.

When modeling, the risk variable's probability density function is included in the risk assessment model. The most common distribution forms for probability density functions are the uniform, triangular, and normal distributions [10]. The probability density functions for these different distributions are as follows:

Uniformly distributed probability density function:

$$f(x|a,b) = \begin{cases} \frac{1}{b-a}, a < x \ll b \\ 0, others \end{cases} \tag{4}$$

Triangularly distributed probability density function:

$$f(x|a,b.m) = \begin{cases} \frac{2(x-a)}{(b-a)(m-a)}, a \ll x \leq b \\ \frac{2(b-x)}{(b-a)(m-a)}, m < x \ll b \end{cases} \tag{5}$$

Normal distribution probability density function:

$$f(x|\mu,\sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{6}$$

In the uniform and triangular probability density functions, α denotes the minimum value, while b denotes the maximum value, with m indicating the most likely value. Meanwhile, in the normal probability density function, μ represents the mean of the numerical distribution, and σ2 indicates its variance [10]. Two crucial factors to evaluate a project's investment are the net present value (NPV) and the internal rate of return (IRR), as they provide an accurate reflection of a project's profitability [10]. The paper presents a project investment risk assessment model, which comprises NPV analysis, combining Value at Risk (VaR) and Conditional Value at Risk (CVaR), and IRR analysis, based on standard deviation rate. The following is its model [11]:

$$\begin{cases} NPV = \sum_{t=0}^{n} (C_I - C_O)_t (1+i)^{-t} \\ P(\delta_{NPV} \ll R_{VaR\alpha}) = \alpha \\ R_{VaR\alpha} \ll E(NPV) - NPV^* \\ R_{CVaR\alpha} = \frac{1}{\alpha}\int_0^{\alpha} NPV_r dr \end{cases} \tag{7}$$

With constrains:

$$\begin{cases} NPV(i_{IRR}) = \sum_{t=1}^{n} (C_I - C_O)_t (1+i_{IRR})^{-t} = 0 \\ \omega_{i_{IRR}} = \frac{\sigma_{\delta_{IRR}}}{E(\delta_{IRR})} \end{cases} \tag{8}$$

In these equations, n represents the project duration, (CI-CO)t represents the overall cash flow in the year in which the project is carried out, t, δNPV represents the simulated NPV distribution, RVaRα stands for the risk of loss value at a confidence level of 1-α, and NPV* denotes the NPV value at a specific confidence level[11]. E(NPV) represents the anticipated value of the simulated NPV, while RCVaRα denotes the likelihood of loss at a 1-α confidence level. The internal rate of return is represented by iIRR, and the standard deviation of the simulated IRR distribution is symbolized by σδIRR. Lastly, the expected value of the simulated IRR distribution is portrayed as E(δIRR), and ωiIRR denotes the standard deviation of the IRR distribution [11].

## 4. Monte Carlo Simulation in Radiation Therapy

The Monte Carlo Method can be applied in a variety of fields because its purpose is to generate a collection of results using random factors and analyze those findings similarly to statistical samples.

It includes physics [12]. In physics, Monte Carlo simulation is frequently used to model particle motion and system behavior [13]. For instance, a crucial technique for studying the physics of nuclear medicine, radiology, and radiation therapy is the Monte Carlo (MC) simulation. A precise evaluation of the absorption of dose distribution in the entire body as well as specific organs and tissues is required for treatment planning in radiation therapy (RT) [13]. Analyzing surgical risk during treatments requires determining the patient's radiation exposure [13]. A typical result is illustrated in Fig. 1.

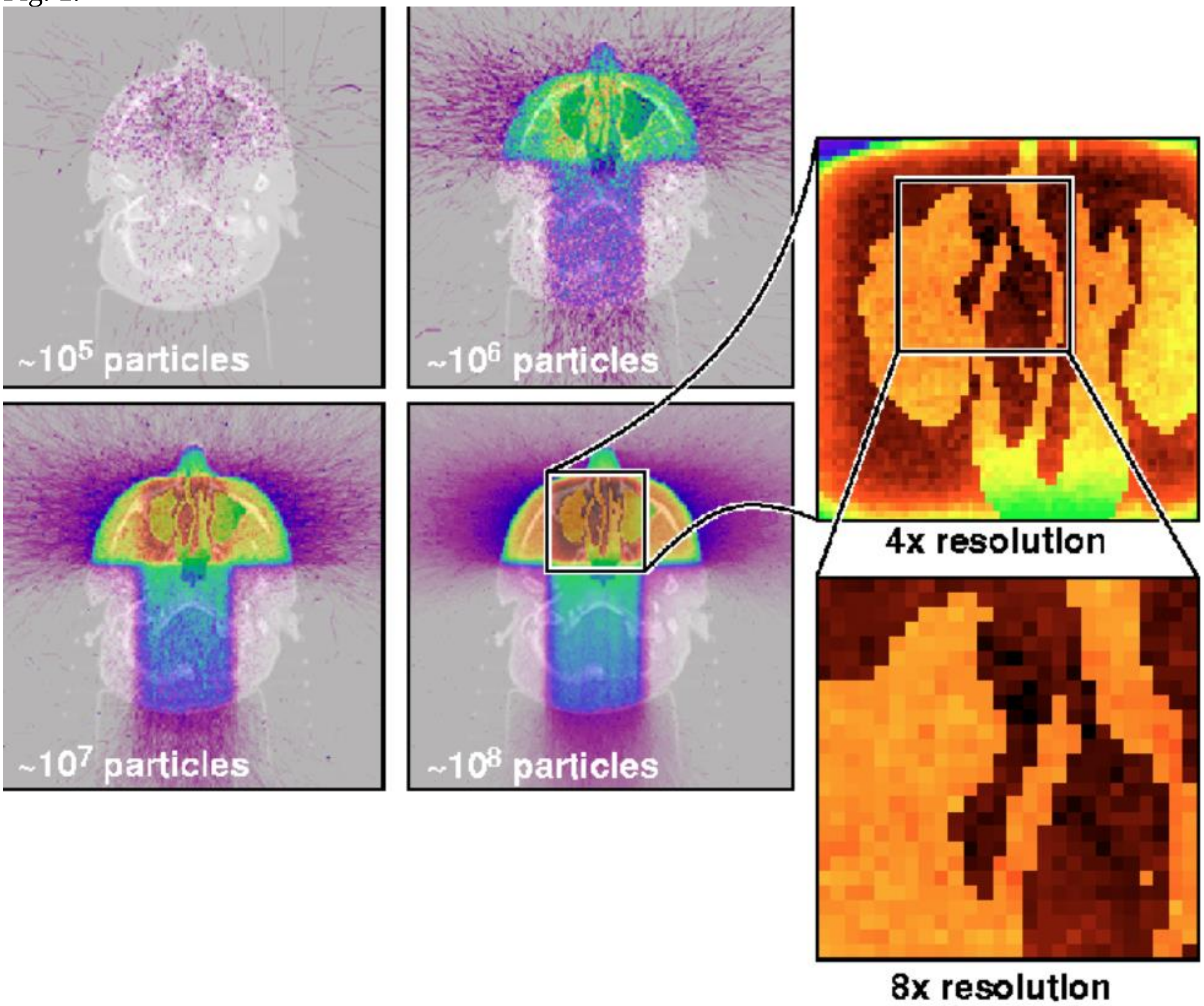


**Fig. 1** Monte Carlo simulation in radiation therapy.

The primary variable under consideration in this sentence is dose D, or the amount of energy deposited per unit mass of medium. Its unit is units of gray (1 Gy = 1 J/kg). In Monte-Carlo simulations, the energy in a volume Edep is typically expressed using eV (1 eV $= 1.60217646 \times 10^{-19}$J). By taking into account the volume of interest and density, we may convert it to Gy [2]:

$$D[Gy] = (E_{dep}[eV] \times 1.60217646 \times 10^{-19}[J/eV])/(\rho[kg/cm^3]/V[cm^3]) \quad (9)$$

Here, D is the first stage in evaluating radiation's biological impacts, including both stochastic and deterministic effects. The following equation describes the statistical uncertainty at pixel k, where N is the total number of primary events, and dk, i is the amount of energy deposited in pixel k at primary event i:

$$D_k = \Sigma_i d_{k,i} \quad (10)$$

$$S_k = \sqrt{\frac{1}{N-1}\left[\frac{\Sigma_i d_{k,i}^2}{N} - \left(\frac{\Sigma_i d_{k,i}}{N}\right)^2\right]} \tag{11}$$

$$\epsilon_k = 100 \times S_k / D_k \tag{12}$$

## 5. Monte Carlo Simulation in Brachytherapy

In recent years, the characterization of brachytherapy devices has benefited greatly from the use of MC modeling. To determine dosimetric parameters including air intensity, radial absorbed dose function, anisotropy function, and absorbed dose rate constant in liquid water, the researchers conducted MC simulations. Then, by contrasting these simulations with actual measurements, they were found to be accurate [13]. A Typical result is shown in Fig. 2.

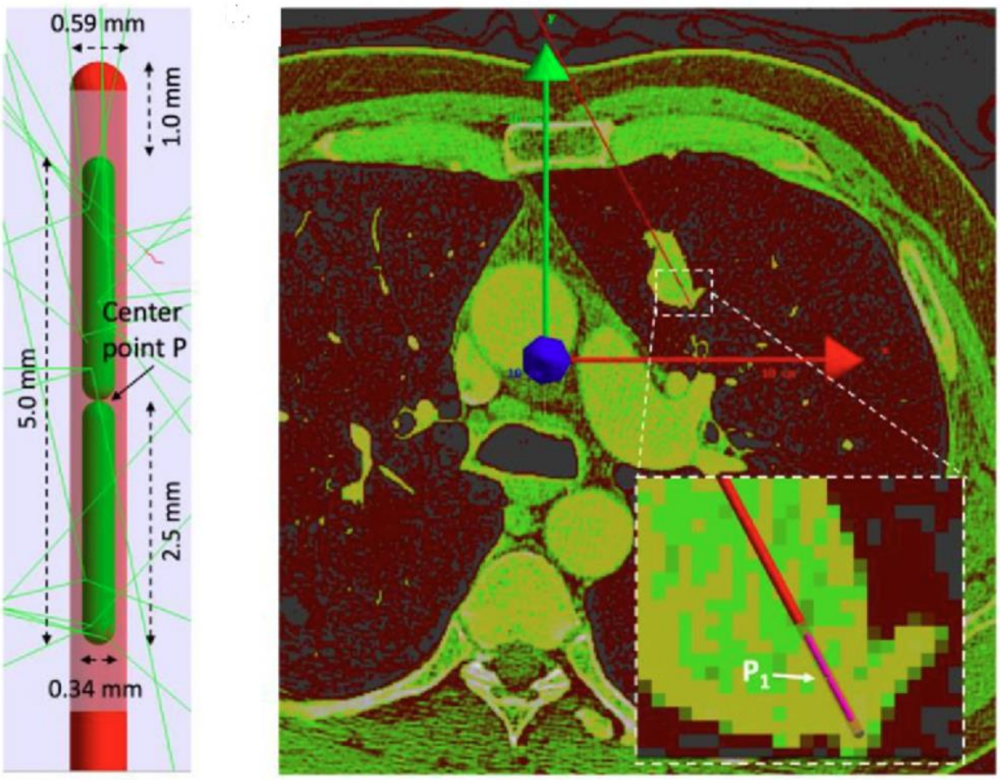


**Fig. 2** Monte Carlo simulation in brachytherapy.

## 6. Limitations and Prospects

The fact that Monte-Carlo Simulation only offers statistical estimates of results rather than precise numbers is one of its drawbacks [14]. This makes it challenging to present results in a way that is readable and easy to grasp. Another significant drawback is that it requires expensive, specialised software to complete due to its complexity [14]. As a result, it is challenging for anyone without a strong intellectual foundation to apply it directly. Equipment purchases for Monte-Carlo Simulation research may be also a problem for researchers with insufficient research funding. It is almost impossible to avoid the issue that process complexity may result in errors that produce incorrect outcomes that could be deceptive [14].

## 7. Conclusion

Entering various variables enables the probabilistic Monte Carlo simulation method to quantify the probability of events. It is a useful tool that can be used in a number of fields, such as biology, computer science, and finance. In order to estimate the probability of an event, Monte Carlo simulation works by periodically creating random samples from a distribution of probability. For example, to calculate the likelihood of flipping a coin and obtaining heads, the simulator generates 100 random values between 0 and 1. If 50 of those values are less than or equal to 0.5, it can be estimated that the probability of receiving heads is 50%. The powerful tool of Monte Carlo simulation can be used to analyse complicated systems, anticipate outcomes, and estimate the likelihood of uncommon events, among many other challenges. It can, however, be computationally expensive, particularly for issues involving numerous random samples since they need tools and equipment that are now cheap. Despite its drawbacks, a lot of researchers and professionals find Monte Carlo simulation to be a useful tool. It is an effective way to understand intricate systems and arrive at wise judgement.

## Author Contribution

All the authors contributed equally and their names were listed in alphabetical order.

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