Analysis of Principle and Applications of Series Expansion

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Abstract. As a matter of fact, the series expansion has been widely used in various fields since the proposed. The key reason is that it can be used for simplifying and approximation for complex structure, functions as well as situations. On this basis, it is able to offer an easy routine to calculate some of the values. With this in mind, this study mainly discussed the use of Taylor series expansion in chemistry and daily life, and obtained the freezing point reduction formula, which is not a small challenge for us. However, this expansion of Taylor's series is limited in many other areas of daily life and not all areas and similar problems can be solved. The significance of this paper is to help Taylor series in the freezing point, Taylor series to a certain extent to solve the problem of freezing point in mathematics. Overall, these results shed light on guiding further exploration of series expansion.

Keywords: Taylor series; series expansion; order approximation.

1. Introduction

The study of polar numbers began in the early 1930s, when several great scholars conducted the most basic research on them, classified them and unified them into several theorems. Tippett, Dodd, Fisher, Frechet et al. systematically integrated and rigorously proved the number of poles. He created three theorems and had a profound impact on the history of human mathematics. In the 1980s and 1990s, the polar model was gradually improved, forming a relatively complete polar system. Various theories of polar deepening are still being further explored [1-4].

Although the application of the polar number is relatively rare in daily life, the appearance of the random extreme value phenomenon is usually a large-scale natural disaster that occurs once in a hundred years or some major events that have a major impact on human society and even destroy the biological balance [5, 6]. Today's polar numbers can also be applied to the natural environment, the economy and a range of other places and even applied to the calculation of extreme values to study the distribution intensity of earthquakes [1, 7-9]. For example, through a series of extreme value variation types and selection of different models for practical use. For example, knowing the distribution type and magnitude of earthquakes, we can determine what type of extreme value distribution is related. In addition, there is another application of extreme values such as extreme search control, which can handle some energy regulation through the adjustment of the system [4]. It is generally suitable for reducing energy consumption in various situations [10]. According to the value of different situations to achieve the ideal effect through the study of ESC theory. At the same time, the maximum estimated value in the weather is also within the study range of the extreme value. Examples include modern architecture, hydrology and a range of weather changes such as temperature or rainfall [11, 12]. The assessment of typhoon risk and wind speed is also within the scope of extreme value estimation. Through some key parameters of the physical model. To simulate speed, size, path, maximum size. Extremes also play an important role in our lives.

2. The uniqueness of Taylor's expansion.

If f(x) can be expanded into a power series of x, then this expansion is unique and must be consistent with the McLaughlin series of f(x). In fact , if (x) has a power series expansion in some neighborhood (-R ,R) of a point x,0:

$$f(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^3 \tag{1}$$

Then there must be

$$a_0 = f(0) \tag{2}$$

$$a_1 = f'(0) \tag{3}$$

$$a_2 = f''(0)/2! \tag{4}$$

$$a_n = f^n(0)/n! \tag{5}$$

Taylor 's formula can approximate the function by constructing a polynomial with coefficients of each derivative at a certain point. The application of Taylor expansion to derive the formula of freezing point reduction

3. Applications

Under A certain external pressure p, a small amount of solute B is dissolved in solvent A to form a dilute solution. The temperature at which pure solvent A is precipitated from the solution is the freezing point of the dilute solution (This place, let's call it Tf). It is lower than the freezing point Tf* of pure solvent A under the same external pressure p. The decrease of freezing point is expressed as ΔTf. So let's start with the phase equilibrium equation, and use the Taylor expansion formula to infer between ΔTf and the composition of solution the quantitative relationship [13, 14].

It is known from the phase equilibrium relationship that when the external pressure is p, when the pure solvent A liquid and solid two phases coexist, there are:

$$\mu_{A(1)}(T_f^*, x_{A(1)}) = \mu_{A(s)}^*(T_f^*, x_{A(s)}) \tag{6}$$

Under the same external pressure p, when a small amount of solute B is added to form a thin solution, at this time, the composition of liquid-solid two-phase solvent A changes dxA (l) and dxA (s) respectively, and the freezing point becomes T_f*+dT:

$$\mu_{A(1)}(T_f^* + df, x_{A(1)} + dx_{A(1)}) = \mu_{A(s)}^*(T_f^* + dT, x_{A(s)} + dx_{A(s)}) \tag{7}$$

Applying the Taylor expansion of the multivariate function, expanding the function on both sides of it equal sign at (Tf*, xA (l)), and ignoring the third and subsequent terms in the expansion, yields:

$$\mu_{A(1)}\left(T_f^*, x_{A(1)}\right) + \frac{\partial \mu_{A(1)}\left(T_f^*, x_{A(1)}\right)}{\partial t} dT + \frac{\partial \mu_{A(1)}\left(T_f^*, x_{A(1)}\right)}{\partial x_{A(1)}} dx_{A(1)} = \mu_{A(s)}^*\left(T_f^*, x_{A(s)}\right) + \frac{\partial \mu_{A(s)}^*(T_f^*, x_{A(s)})}{\partial t} dT + \frac{\partial \mu_{A(s)}^*(T_f^*, x_{A(s)})}{\partial x_{A(S)}} dx_{A(1S)} \tag{8}$$

$$-S_{m,A(l)}^* dT + \frac{\partial \mu_{A(1)}\left(T_f^*, x_{A(1)}\right)}{\partial x_{A(1)}} dx_{A(1)} = -S_{m,A(s)}^* dT \tag{9}$$

Let's bring another formula into the above formula, we can get that:

$$-S_{m,A(l)}^* dT + RT dx_{A(1)}/x_{A(1)} = -S_{m,A(s)}^* dT \tag{10}$$

So,

$$-S_{m,A(l)}^* dT + RT dln x_{A(1)} = -S_{m,A(s)}^* dT \tag{11}$$

Thus, one derives:

$$dlnx_{A(1)} = \frac{\Delta_{fus}H^*_{m,A}}{RT^2}dT \tag{12}$$

Integrate both sides of the formula, we can get that:

$$lnx_{A(1)} = -\frac{\Delta_{fus}H^*_{m,A}}{RT}\left(\frac{T_f^* - T_f}{T_f^* T_f}\right) \tag{13}$$

The above formula is then combined with the Taylor expansion, we can get that:

$$\ln(1 - x_B) \sim -x_B \sim -\frac{n_B}{n_A} = -b_B M_A \tag{14}$$

Combine the above two formulas and then simplify them, and one obtains:

$$\Delta T_f = -\frac{R\left(T_f^*\right)^2 M_A}{\Delta_{fus}H^*_{m,A}} b_B \tag{15}$$

From this, we get the freezing point reduction formula. There is also the physical application of the Taylor expansion. Application in the magnetic field problem of Helmholtz coils. A typical sketch is shown in Fig. 1.

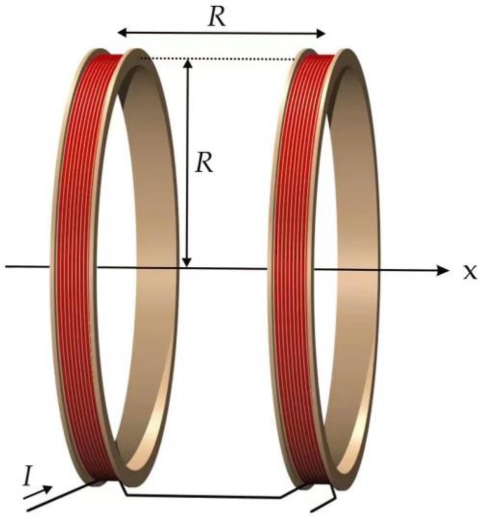

Fig. 1 Helmholtz coils.

Everyone knows that there is a uniform magnetic field on the central axis of the Helmholtz coil, although it is mentioned in many textbooks, it is not explained. Let's start with a brief introduction to the Helmhertz coil magnetic field. The Helmholtz coil is a pair of coaxial circular coils parallel to each other and connected, and the current direction in the two coils of each coil V coin is the same, and the distance d between the coils of the same size is exactly equal to the average radius of the circular coil (seen from Fig. 2).

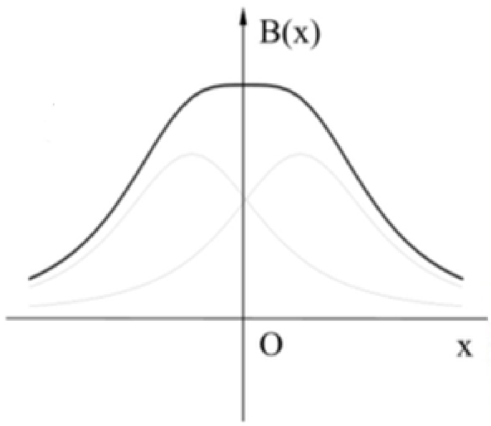

Fig. 2 Magnetic field distribution.

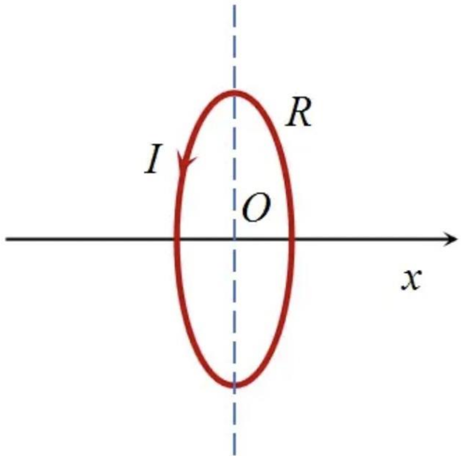

Fig. 3 Coil with current.

Let x be the distance from the center point 0 of a point on the central axis of the Helmholtz coil, then the magnitude of the magnetic induction intensity B at any point on the axis of the Helmholtz coil is

$$B' = \frac{\mu_0 NIR^2}{2}\left\{\left[R^2 + \left(\frac{R}{2} + x\right)^2\right]^{-\frac{3}{2}} + \left[R^2 + \left(\frac{R}{2} - x\right)^2\right]^{-\frac{3}{2}}\right\} \tag{16}$$

The magnitude B of the magnetic induction intensity at center 0 on the axis of the Helmholtz coil for

$$B_0' = \frac{8}{5^{3/2}}\frac{\mu_0 NI}{R} \tag{17}$$

If one wants to explain the magnetic field of the Helmholtz coil, one can get the result with the Taylor expansion. For a coil with current (seen from Fig. 3), the magnetic field generated on its axis can be obtained from Bio-Savard in the form:

$$B = \frac{\mu_0 IR^2}{2(R^2+x^2)^{3/2}} \tag{18}$$

According to the principle of magnetic field superposition, the magnetic field on the axis of the Helmholtz coil can be expressed as the sum of the magnetic fields generated by the left and right coils:

$$B = \frac{\mu_0 IR^2}{2(R^2+(R/2+x)^2)^{3/2}} + \frac{\mu_0 IR^2}{2(R^2+(R/2-x)^2)^{3/2}} \tag{19}$$

Dimensionless refers to the elimination of dimensional influences so that data with different characteristics are comparable. Normalization and standardization are the specific practices to achieve dimensionless:

$$B' = \frac{2BR}{\mu_0 I}, x' = x/R \tag{20}$$

Then the magnetic field can be written as:

$$B' = \frac{1}{(1+(1/2+x\prime)^2)^{3/2}} + \frac{1}{(1+(1/2-x\prime)^2)^{3/2}} \tag{21}$$

Expanding the formula according to Taylor, one can get such a result

$$B = \frac{16}{5\sqrt{5}} - \frac{2304}{625\sqrt{5}}x^4 + o(x^5) \tag{22}$$

It can be seen that after the constant term, it is directly a small quantity of the fourth order, rather than the common small quantity of order 1 and 2, which means that in the vicinity, a uniform magnetic field of considerable length will appear. And that's true on the image.

4. Conclusion

As a matter of fact, many of the articles on Taylor Series have been combined with other fields, such as computer science and the natural environment. In different fields, Taylor series can help technicians in different fields solve different problems through its special properties and framework. For example, Taylor series can be used to complete the calculation of forestry growth, which is the combination of mathematical theory and application, to help people solve more basic and life problems. In the future, people can try to apply Taylor series to more other fields, such as school management and statistics, which can help schools reduce pressure. Of course, Taylor series also has many limitations, when we need to quadrature a series of continuous non-cyclic and infinite numbers, there is no way to use this aspect of the theory, that is, Taylor series cannot be used in the quadrature formula. To sum up, in this study, we mainly discussed the use of Taylor series expansion in chemistry and daily life, and obtained the freezing point reduction formula, which is not a small challenge for us. However this expansion of Taylor's series is limited in many other areas of daily life and not all areas and similar problems can be solved. The significance of this paper is to help Taylor series in the freezing point, Taylor series to a certain extent to solve the problem of freezing point in mathematics.

Author Contribution

All the authors contributed equally and their names were listed in alphabetical order.

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