

Comparison of Different Hubble Constant Measurements Approach

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Abstract. As a matter of fact, the Hubble constant is a fundamental parameter describing the universe's expansion rate as well as the relationship between celestial object distances and redshifts. In reality, precisely measuring is crucial for understanding the universe's evolution, age, and its fundamental components. The Hubble constant's value is determined through diverse methods including pulsating stars, supernovae, and cosmic microwave background radiation. However, these approaches have yielded conflicting results, resulting in the Hubble constant tension. With this in mind, this study first introduces two commonly used measurement methods of Hubble constant, including distance ladder and CMB, and then compares the observation data obtained by combining these two observation methods, that is, comparing the results obtained by different measurement methods of Hubble constant. Different results were obtained. At the same time, this phenomenon was analyzed, the possible reasons were obtained, and the future development of this field was summarized and prospected.

Keywords: Hubble constant; distance ladder; CMB; Hubble tension.

1. Introduction

The exploration of the Hubble constant's history harks back to the early 20th century, a time when the comprehension of the universe's expansion was in its early stages. Yet, as astronomical observation methods progressed, researchers began to observe the redshift phenomenon in the spectra of galaxies situated far from us. This redshift indicated that these galaxies were undergoing motion away from the vantage point. This discovery revealed the fact that the universe was undergoing expansion. In one of the most renowned and classic scientific papers in history, Edwin Hubble's 1929 article in PNAS introduced the Hubble Law and revealed the concept of an expanding universe. This groundbreaking discovery fundamentally altered the comprehension of the cosmos, marking the inception of observational cosmology. Hubble's contributions marked the commencement of a fresh chapter in the quest to understand the cosmos. The work revealed an astonishing truth: the universe is incredibly immense, and it has been undergoing continuous expansion and transformation for around 14 billion years. The cosmos, replete with billions of galaxies, also conceals enigmatic elements like dark matter and dark energy. These enigmas both challenge and enhance the comprehension of the universe's structure and behavior [1].

At the core of comprehending this cosmic phenomenon lies a critical factor known as the Hubble constant, conventionally represented a H_0 . Hubble's groundbreaking work unveiled a fundamental connection between the distance of galaxies and their redshift—the shift of light towards longer wavelengths due to motion away from an observer. He observed that galaxies situated farther away from us receded at greater velocities, a relationship that naturally falls into place if the entire Universe is undergoing expansion. This relationship between the velocity of recession and the distance to galaxies is particularly linear for nearby objects, which is a crucial aspect. It implies that the same dependency can be observed not only from the own Milky Way but from any other galaxy in the cosmos [2]. The precise measurement of the Hubble constant is paramount for comprehending the evolution of the universe, cosmic expansion [3], estimating its age [4]^[4] and gaining insights into enigmatic cosmic constituents like dark energy and dark matter [5].^[4]

However, perplexingly, different research teams employing various measurement methodologies have produced conspicuously disparate results when determining the Hubble constant [6]. These

discrepancies have sparked extensive debates and controversies, not only within academic circles but also in the public discourse. This situation has piqued the curiosity, motivating us to delve deeper into the issue to identify the underlying causes of these inconsistencies. The motivation for this study stems from the desire to comprehensively understand the strengths and limitations of different Hubble constant measurement techniques through comparison. This study aims to pinpoint factors that may contribute to these incongruous outcomes, whether they stem from systematic errors in observational techniques or assumptions within cosmological models. On this basis, it is aspired to gain a more profound insight into the true nature of cosmic expansion, providing more accurate data for the field of cosmology and laying a more solid foundation for future cosmological research.

In the first section, this study will provide fundamental descriptions of the Hubble's law, including the definition of H_0 and the cosmological background. This study will also introduce the basic principles and methods used to measure the Hubble constant. The second section will delve into the methods used to detect the Cosmic Microwave Background (CMB), the models employed, and the results obtained from these observations. This study will also discuss the instruments and techniques related to CMB measurements and their roles in Hubble constant research. In the third section, this study will elucidate the methods of the distance ladder approach, including the models used and the results obtained. This study will introduce the instruments and techniques used for the distance ladder method and explain how this approach is employed to measure the Hubble constant. The fourth section will involve a comparison of the Hubble constant measurement results obtained through the CMB and distance ladder approaches. This study will provide explanations for the discrepancies and explore their implications on the understanding of cosmic expansion. In the fifth section, this study will discuss the current limitations in the field and provide insights into future research directions. This study will examine the current constraints of existing methods and propose potential avenues for future studies. Finally, in the conclusion section, this study will summarize the key findings of this paper, emphasize the significance of the research, and present the potential value and directions for future studies.

2. Basic Descriptions

Hubble found in the early research and observation of galaxies in the range of hundreds of millions of parsecs: there is a strong correlation between the luminosity of galaxies and their redshifts, and it is deduced that the redshift of galaxies increases with the increase of distance. To explore whether the redshift continues to increase for more distant galaxies. Hubble began to cooperate with M. Humason, and they obtained the relationship between Doppler velocity (v) and redshift (z) as

$$1 + z = \sqrt{\frac{c+v}{c-v}} \quad (1)$$

Then the above formula can be simplified to

$$v = c \frac{z^2 + 2z}{z^2 + 2z + 2} \quad (2)$$

If the redshift is small, the above relationship can be simplified as $v = cz$. Hubble and Humason's observations show that the redshift of galaxies is proportional to their distance. They concluded that there is a linear relationship between $\log v$ and the magnitude m , the relationship is

$$\log v = 0.2m + C_1 \quad (3)$$

where C_1 is a constant. At the same time, for the galaxies studied, the redshift is small, so it is possible to use $v = cz$:

$$\log(cz) = 0.2m + C_1 \quad (4)$$

If the galaxy G emits L units of energy in a unit time, these energies pass isotropically through a sphere with G as the center and D as the radius, and the energy received per unit area on the sphere is f

$$L = \oint_S \mathbf{f}(x, y, z) \cdot d\mathbf{S} \quad (5)$$

From this it can be concluded

$$f = \frac{L}{4\pi D^2} \quad (6)$$

In this case, the magnitude is defined by the following logarithmic relationship

$$m = -2.5 \log f + C_2 \quad (7)$$

where C_2 is a constant. Combining the above formulas can be obtained

$$\log(cz) = \log d - 0.5 \log(L/4\pi) + C_3 \quad (8)$$

where C_3 is a constant. If the observed galaxies have the same luminosity, then $0.5 \log(L/4\pi)$ is a constant which leads to the equation

$$cz = H_0 \times d \quad (9)$$

where d is distance and H_0 is Hubble constant [4]. This leads us to establish the well-known Hubble's law, which reveals a linear relation between galaxy velocity (v) and its distance (d) [1, 6].

$$v = H_0 \times d \quad (10)$$

According to the observation results, Hubble concluded that $H_0 = 530 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$. Since then, astronomers have continuously revised the Hubble constant, greatly reducing the estimated value of the Hubble constant [7]. PCP13 reported a value of $H_0 = (67.3 \pm 1.2) \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$ for the base- Λ CDM cosmology [8]. SH0ES21 project: This is another research project that estimated H_0 to be $H_0 = (73.8 \pm 2.4) \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$, which is higher than the value reported by PCP13 [9]. R18: The SH0ES project, specifically R18, continued to refine their measurement of H_0 and reported a value of $H_0 = (73.48 \pm 1.66) \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$, which is consistent with their previous estimate but still higher than the PCP13 value [10, 11]. The SH0ES project used data from the Gaia mission to further improve their measurement, resulting in $H_0 = (73.52 \pm 1.62) \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$ [12]. This paper discusses even more refined measurements of H_0 using improved measurements of LMC Cepheids. They reported a value of $H_0 = (74.03 \pm 1.42) \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$ [13].

Next, the article will focus on the analysis of two common methods of measuring H_0 , the cosmic microwave background radiation measurement Hubble constant method and the Distance Ladder measurement Hubble constant method, and analyze the differences between the two methods. Radial velocities, which have been adjusted for the motion of the Solar System, are plotted against distances that have been estimated using data from stars and the average brightness of nebulae within a cluster. The black discs and solid line indicate the solution when considering the motion of individual nebulae; the circles and dashed line depict the solution when grouping nebulae together; the cross represents the average velocity corresponding to the mean distance of 22 nebulae for which individual distance estimates were not available [1].

3. Measurements Based on CMB

The leading cosmological model, known as the Hot Big Bang Theory, proposes that the universe's origin traces back approximately 15 billion years. During this epoch, the universe existed in a state of intense heat, high density, and near uniformity, occupying only a minuscule portion of its current volume, often described as an "infinitesimal singularity". Inflation is a phenomenon that took place in the first tiny fraction of a second following the Big Bang and is characterized by the universe expanding quickly and exponentially. This expansion led to a transition in the universe's dominance

from being radiation-driven to being governed by matter. Following inflation, there was a Reheating phase during which the energy of the inflaton field (responsible for inflation) decayed into radiation, leading to a return to radiation domination. Within the first second following the Big Bang, baryonic matter, which includes protons and neutrons, started to form. The creation of light elements like hydrogen and helium through nucleosynthesis, which occurred as the cosmos continued to expand and cool, took place in a matter of minutes. Recombination, a crucial occurrence, occurred roughly 300,000 years after the Big Bang. The universe had cooled to a temperature of about 3000 K at that time, allowing protons to grab free electrons and create atomic hydrogen. The universe's matter changed from a plasma state to a neutral state at this precise moment. Before Recombination, the universe remained opaque to electromagnetic radiation because free electrons scattered photons. However, as Recombination unfolded, the population of free electrons significantly dwindled. Consequently, this led to a state of Opacity and Decoupling between matter and radiation. As a result, the universe became transparent to light. During the era of decoupling, the release of Cosmic Background Radiation (CBR) occurred. This radiation has since manifested as the CMB, which one presently detects as a faint, pervasive microwave glow throughout the cosmos. The CMB is a remnant of the early universe and provides crucial insights into its early history [14].

Measurements of the CMB typically depend on cosmological models that are grounded in theoretical frameworks. Among these models, the most frequently employed one is the Λ CDM model. This model includes the dark energy (Λ) and cold dark matter (CDM) of the univers. By fitting the Λ CDM model with the CMB observation data, the value of the Hubble constant can be determined. The best-fitting value for the Hubble constant obtained from the analysis of temperature spectrum, lensing power spectrum, and polarization spectrum data by the Planck satellite in 2018 is $H_0 = (67.36 \pm 0.54) \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$. On the other hand, a combination of measurements of the Cosmic Microwave Background (CMB) using the Atacama Cosmology Telescope (ACT) and the Wilkinson Microwave Anisotropy Probe (WMAP) yields a calculated Hubble constant of $H_0 = (67.9 \pm 1.5) \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$ [7]. These estimations are exclusively derived from scrutinizing the historical expansion of the universe and examining measurements related to the temperature and polarization of the CMB. [15]. Table 1 presents marginalized values and 68% confidence intervals for the Λ CDM cosmological model, along with its one-parameter extensions.

Table 1. Regarding the estimation of cosmological parameters, including the Hubble constant. [15]

		Ω_{DE}	$H_0[\text{km/s/Mpc}]$	Ω_k	w	$\sum m_\nu [\text{eV}]^a$
Λ CDM	BAO	$0.701^{+0.017}_{-0.015}$	—	—	—	—
	CMB T&P	0.6836 ± 0.0084	67.29 ± 0.61	—	—	—
	CMB T&P + BAO	0.6881 ± 0.0058	67.60 ± 0.43	—	—	—
	CMB T&P + SN	0.6856 ± 0.0078	67.43 ± 0.57	—	—	—
	CMB T&P + BAO + SN	0.6890 ± 0.0057	67.67 ± 0.43	—	—	—
ϕ CDM	BAO	$0.636^{+0.085}_{-0.070}$	—	$0.079^{+0.083}_{-0.10}$	—	—
	CMB T&P	$0.561^{+0.050}_{-0.041}$	$54.5^{+3.3}_{-3.9}$	$-0.044^{+0.019}_{-0.014}$	—	—
	CMB T&P + BAO	0.6884 ± 0.0059	67.62 ± 0.62	0.0000 ± 0.0018	—	—
	CMB T&P + SN	0.670 ± 0.017	65.2 ± 2.2	$-0.0061^{+0.0062}_{-0.0054}$	—	—
	CMB T&P + BAO + SN	0.6891 ± 0.0057	67.67 ± 0.60	0.0000 ± 0.0018	—	—
w CDM	BAO	$0.728^{+0.017}_{-0.038}$	—	—	-0.69 ± 0.15	—
	CMB T&P	$0.801^{+0.057}_{-0.022}$	—	—	$-1.58^{+0.16}_{-0.35}$	—
	CMB T&P + BAO	0.695 ± 0.013	$68.5^{+1.3}_{-1.6}$	—	$-1.035^{+0.062}_{-0.052}$	—
	CMB T&P + SN	0.692 ± 0.010	68.3 ± 1.1	—	$-1.035^{+0.037}_{-0.037}$	—
	CMB T&P + BAO + SN	0.6929 ± 0.0076	68.23 ± 0.83	—	$-1.027^{+0.033}_{-0.033}$	—
ν CDM	CMB T&P	$0.680^{+0.016}_{-0.0087}$	$67.0^{+1.2}_{-0.67}$	—	—	$< 0.268(95\%)$
	CMB T&P + BAO	0.6892 ± 0.0065	67.72 ± 0.50	—	—	$< 0.129(95\%)$
	CMB T&P + SN	$0.686^{+0.011}_{-0.0083}$	$67.47^{+0.83}_{-0.65}$	—	—	$< 0.174(95\%)$
	CMB T&P + BAO + SN	$0.6898^{+0.0065}_{-0.0056}$	$67.76^{+0.49}_{-0.44}$	—	—	$< 0.124(95\%)$

4. Distance Landar

Through Hubble's law, one can know that measuring the Hubble constant first requires an accurate determination of the receding speed and distance of the celestial body. The receding speed of a celestial body can be measured by its redshift. One can know that the recession speed and red shift have the following relationship

$$1 + z = \sqrt{\frac{c+v}{c-v}} \quad (1)$$

As the universe undergoes expansion, the light emitted by galaxies situated at a distance from us experiences a stretching effect. This stretching is a consequence of these galaxies moving away from the vantage point, causing their spectral lines to noticeably shift toward the red region of the spectrum. This redshift is quantified using a parameter called "z":

$$z = \frac{\lambda}{(\lambda)_0} - 1 \quad (2)$$

where λ is observed wavelength and $(\lambda)_0$ is rest wavelength.

The technique employed to measure immense distances in the universe relies on the notion of "standard candles." Standard candles are celestial entities characterized by their well-documented intrinsic brightness or luminosity. Once one determines the luminosity of a standard candle, one can use its observed brightness to calculate its distance from mankind. The standard candles frequently employed in astronomical measurements encompass Cepheid variable stars and Type Ia supernovae. Cepheid variables represent stars that experience recurrent fluctuations in luminosity caused by their inherent instability. These stars display a firmly established correlation between their luminosity and the duration of their brightness oscillations. By measuring the period-luminosity relationship of Cepheids, astronomers can accurately determine their intrinsic luminosity. Comparing this intrinsic luminosity with their observed brightness allows us to calculate their distance. Type Ia supernovae are the result of a violent thermonuclear explosion at the end of a massive star's evolution. These supernovae have nearly uniform maximum luminosities, making them excellent standard candles. Their luminosity is exceptionally high, about 5 billion times that of the Sun. The small variations in luminosity can be corrected using the Phillips relationship, which describes the relationship between the speed of luminosity decay over time. This correction enables researchers to precisely measure the total luminosity of Type Ia supernovae and subsequently calculate their distance.^[7] In order to observe these distant celestial objects, astronomers usually use a variety of advanced astronomical observation instruments and technologies, including telescopes, optical and infrared optical instruments, radio telescopes, infrared telescopes, etc.

The Carnegie-Chicago experimental team found a value of $H_0 = 69.8 \pm 0.8 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$, and its accuracy is 1.1% [16]. Seen from Fig. 1, The black filled circles represent galaxies that are part of both the TRGB and Cepheid samples. A slope of $H_0=70\text{km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1}$ is shown in black. Combined with the most well-known and current measurements of the Milky Way's tip and foreground extinction, John Soltis et al. find $M_{\text{I,TRGB}} = -3.97 \pm 0.06 \text{ mag}$ for the I-band luminosity of the blue edge. Using the TRGB luminosity calibrated from the Gaia EDR3 parallax of $\omega \text{ Cen}$ to calibrate the luminosity of Type Ia supernovae results in a value for the Hubble constant of $H_0 = 72.1 \pm 2.0 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$ [17].

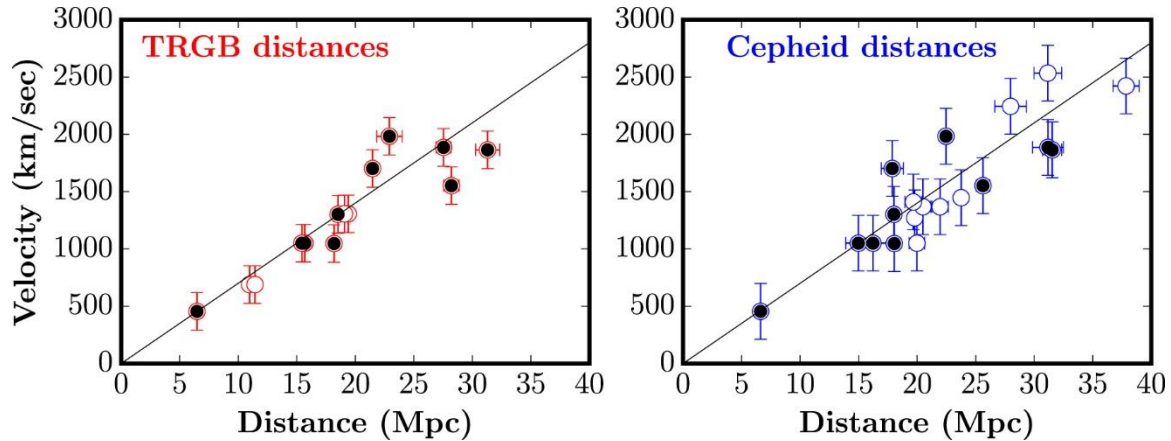


Fig. 1 Nearby galaxies using the Hubble relationship, two panels are presented: one based on TRGB (Tip of the Red Giant Branch) distances (on the left), and the other using Cepheid distances (on the right).

5. Comparison

The Hubble tension, also known as the Hubble constant tension, is a perplexing issue in the field of cosmology that revolves around discrepancies in the measured values of the Hubble constant H_0 .

The problem arises from the fact that different measurement methods and observational datasets yield distinct values for the Hubble constant. On one hand, measurements derived from observations of CMB and other early universe data tend to yield lower. On the other hand, measurements obtained through observations of supernovae, the distance ladder method, and other cosmological distance probes tend to produce higher Hubble constant values. Seen from Fig. 2, The data points in black, as well as the shaded region, are obtained through measurements of the CMB. The blue data points correspond to calibrations of the local value of H_0 using Cepheid variables, while the red data points represent calibrations using the TRGB. The red star indicates the best-fit value determined within the scope of this research paper [16].

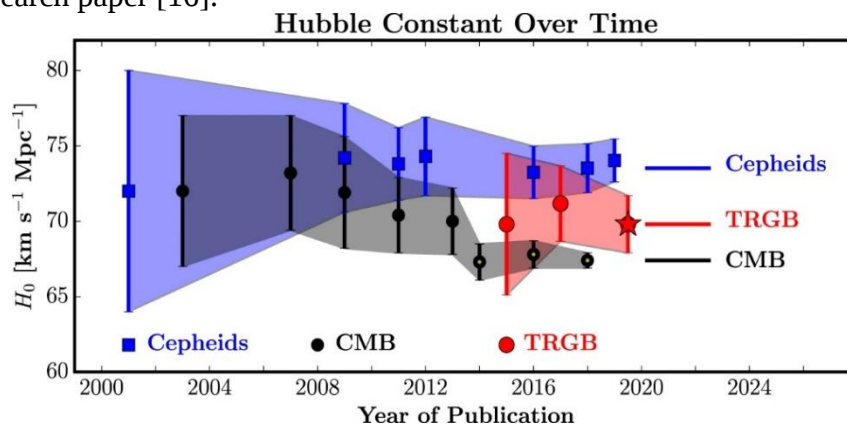


Fig. 2 Plot of H_0 values as a function of time.

Several factors contribute to this tension. Firstly, the universe's expansion is not uniform. Unlike the simplified cosmological models, the Universe is not perfectly homogeneous and isotropic. It exhibits inhomogeneities and anisotropies, which means that the idealized assumptions of the FLRW metric do not fully capture its intricacies. This departure from a perfectly uniform Universe introduces uncertainties in the measurements of H_0 . This leads to various regions exhibit variations in their expansion rates, introducing complexities in measurements. Peculiar velocities, often considered as random motions, have also been proposed as variations in the expansion rate of the Universe. These peculiar velocities are not fully understood, and their impact on H_0 measurements remains a topic of investigation. Additionally, the corrections applied to account for peculiar velocities in measurements are not always well-defined and can introduce further uncertainties [18].

Additionally, the standard cosmological model, Λ CDM, assumes a simple and static dark energy component, but the existence of extra energy components, such as phantom energy and early dark energy, has been proposed to address the tension. Some researchers posit that this might be indicative of unidentified errors within the standard cosmological model, necessitating the development of novel physics. In other words, they propose modifications to the Λ CDM model to address this issue, which implies the presence of new physics during the evolution of the universe. These modifications to the Λ CDM model can be broadly categorized into three classes: the first involves adjustments to dark matter, the second entails modifications to dark energy, and the third combines dark matter and dark energy as a unified entity, often referred to as the "dark sector." Furthermore, there are various theoretical amendments, such as alterations to gravitational theories, aimed at mitigating the Hubble constant crisis [7]. Moreover, different measurement methods and data sources, including gravitational wave events, HII regions, surface brightness fluctuations, and secondary distance indicators, have yielded a wide range of H_0 values. These variations highlight the challenges in achieving precise and consistent measurements across diverse cosmological probes [19].^[19]

6. Limitations & Future outlooks

The field of measuring the Hubble constant currently faces several limitations and challenges. Firstly, different measurement methods and datasets often yield inconsistent values for the Hubble constant, leading to the "Hubble tension." Discrepancies between H_0 values obtained from distance gradient observations and analyses of CMB data have raised concerns about the consistency between measurement methods and data sources. This discrepancy may arise from systematic errors, differences in observational techniques, or unknown physical factors. Secondly, Hubble constant measurements typically rely on observations of celestial objects within the local universe. However, the complexity of local cosmic structures and gravitational fields can influence H_0 measurements, especially when attempting to extrapolate these measurements to the broader universe. The complexity of cosmological models is another challenge. These models encompass multiple components, including dark matter, dark energy, and ordinary matter, each with their own properties and interactions. The uncertainties associated with these components can introduce biases into H_0 measurements. Furthermore, systematic errors are a significant concern in Hubble constant measurements. These errors can stem from instrument precision, parallax corrections, intrinsic luminosities of stars, and more. Failing to accurately estimate and correct for these errors can result in inaccurate measurements. Additionally, the accuracy of Hubble constant measurements depends on the understanding of the early universe, which can be complex and subject to unknown physical phenomena, such as early dark energy or interactions between dark components. These factors can affect the interpretation of CMB data and subsequently impact H_0 measurements.

In contemporary times, scientists have put forth numerous approaches to address the Hubble tension. Thanks to advancements in observational capabilities, the refinement of cross-validation techniques, the sophistication of models, the mitigation of systematic errors, and the advent of novel testing methods, astronomers are growing more attentive to the issue of Hubble tension. Confidence in overcoming current challenges and obtaining more precise determinations of the Hubble constant. These advancements will enrich the comprehension of the universe's evolutionary processes and the underlying principles of physics that dictate its expansion.

7. Conclusion

To sum up, this study analyzes and compares the CMB and Distance Ladder methods for measuring the Hubble constant. Combining experimental data, one finds that Hubble constant values derived from early universe data, including CMB observations, generally result in lower measurements. Conversely, Hubble constant values obtained through the observation of supernovae, utilizing the distance ladder method, and employing various cosmological distance probes typically

yield higher results. This phenomenon could arise from a variety of factors such as non-uniform expansion of the universe, the presence of dark matter and dark energy, incompleteness in the models of the universe, and numerous other variables. The author aims for readers to gain insights into potential causes behind the Hubble tension through this article and to stimulate innovative approaches to measuring the Hubble constant. It is anticipated that as observational capabilities continue to enhance, astronomers will make further strides in unraveling the intricacies of the universe's evolution and its expansive nature.

References

- [1] Bahcall N A. Hubble's Law and the expanding universe. *Proceedings of the National Academy of Sciences*, 2015, 112(11): 3173-3175.
- [2] Jackson N. The Hubble constant. *Living reviews in relativity*, 2015, 18: 1-51.
- [3] Kirshner R P. Hubble's diagram and cosmic expansion. *Proceedings of the National Academy of Sciences*, 2004, 101(1): 8-13.
- [4] Hoyle F, Narlikar K. *The physics-astronomy frontier*. None Physics, 1980.
- [5] Peebles P J E, Ratra B. The cosmological constant and dark energy. *Reviews of modern physics*, 2003, 75(2): 559.
- [6] Hubble E. A relation between distance and radial velocity among extra-galactic nebulae. *Proceedings of the national academy of sciences*, 1929, 15(3): 168-173.
- [7] Shuyin Yang, Yuhang Tang, Aide Sha, et al. The Hubble Constant Tension in Cosmology. *Astronomy and Astrophysics*, 2023, 10: 25.
- [8] Aghanim N, Akrami Y, Ashdown M, et al. Planck 2018 results-VI. Cosmological parameters. *Astronomy & Astrophysics*, 2020, 641: A6.
- [9] Riess A G, Macri L, Casertano S, et al. A 3% solution: determination of the Hubble constant with the Hubble Space Telescope and Wide Field Camera 3. *The Astrophysical Journal*, 2011, 730(2): 119.
- [10] Riess A G, Macri L M, Hoffmann S L, et al. A 2.4% determination of the local value of the Hubble constant. *The Astrophysical Journal*, 2016, 826(1): 56.
- [11] Riess A G, Casertano S, Yuan W, et al. New parallaxes of galactic cepheids from spatially scanning the hubble space telescope: Implications for the hubble constant. *The Astrophysical Journal*, 2018, 855(2): 136.
- [12] Riess A G, Casertano S, Yuan W, et al. Milky Way Cepheid standards for measuring cosmic distances and application to Gaia DR2: implications for the Hubble constant. *The Astrophysical Journal*, 2018, 861(2): 126.
- [13] Riess A G, Casertano S, Yuan W, et al. Large Magellanic Cloud Cepheid standards provide a 1% foundation for the determination of the Hubble constant and stronger evidence for physics beyond Λ CDM. *The Astrophysical Journal*, 2019, 876(1): 85.
- [14] Gawiser E, Silk J. The cosmic microwave background radiation. *Physics Reports*, 2000, 333: 245-267.
- [15] Alam S, Aubert M, Avila S, et al. Completed SDSS-IV extended Baryon Oscillation Spectroscopic Survey: Cosmological implications from two decades of spectroscopic surveys at the Apache Point Observatory. *Physical Review D*, 2021, 103(8): 083533.
- [16] Freedman W L, Madore B F, Hatt D, et al. The Carnegie-Chicago Hubble program. VIII. An independent determination of the Hubble constant based on the tip of the red giant branch. *The Astrophysical Journal*, 2019, 882(1): 34.
- [17] Soltis J, Casertano S, Riess A G. The parallax of ω Centauri measured from Gaia EDR3 and a direct, geometric calibration of the tip of the red giant branch and the Hubble constant. *The Astrophysical Journal Letters*, 2021, 908(1): L5.
- [18] Rameez M, Sarkar S. Is there really a Hubble tension. *Classical and Quantum Gravity*, 2021, 38(15): 154005.
- [19] Mörtzell E, Dhawan S. Does the Hubble constant tension call for new physics. *Journal of Cosmology and Astroparticle Physics*, 2018, 2018(09): 025.