

Modeling Historical Volatility of 10-year Chinese Treasury Bond Futures: A Comparative Analysis of MLE and MCMC Approaches

Shujun Zhao*

Department of Statistics, University of Warwick, Coventry, UK

*Corresponding author: Shujun.Zhao@warwick.ac.uk

Abstract. Chinese treasury bond futures are gaining significance in the market. Volatility is an important metric for measuring the fluctuations in asset prices. This paper provides historical volatility modeling for 10-year Chinese treasury bond futures through the Student-t distribution GARCH (1,1) model. The data encompasses the entire historical daily prices, which are used for calculating logarithmic returns. Before modeling, plots and hypothesis tests such as Augmented Dickey-Fuller testing and Lagrange Multiplier testing, are performed to assess the underlying assumptions. In pursuit of better outcomes, a comparative analysis is conducted on parameter estimation, by using both the Maximum Likelihood Estimator (MLE) and Markov Chain Monte Carlo (MCMC) with the Metropolis-Hasting algorithm. The results are performed by “fGarch” and “bayesGARCH” packages in RStudio. Two methods give different estimates, but both accurately fit the dataset. The forecast errors show the MLE is better. However, a generalizable conclusion remains elusive, primarily due to disparities in data sources, model configurations, and the default settings in R packages.

Keywords: 10-year Chinese Treasury Bond Futures; Historical Volatility; GARCH (1,1) Model; MLE; MCMC M-H Algorithm.

1. Introduction

With the growing maturity of China's treasury bond futures market, this relatively “new” futures variety has gained substantial attention within the financial sector. Consequently, extensive research efforts have been devoted to this field in recent years. Chinese treasury futures were introduced in December 1992, but all trading ceased in 1995. 18 years later, the China Financial Futures Exchange (CFFEX) relaunched 5-year treasury bond futures in September 2013, followed by 10-year and 2-year treasury bond futures in 2015 and 2018 accordingly. The essential features of treasury futures such as price discovery and hedging risks were spotted in the Chinese market by Tang et al. (2018) and Ruan et al. (2021) [1, 2]. According to the 2022 Annual Report on Chinese Treasury Bond Futures authored by Cao et al. (2023), the ratio of average daily trading volume to open interest for Chinese treasury bond futures was 0.45, which increased by 0.03 compared to 2021, placing it on par with the level of developed markets such as the United States and Germany [3]. This indicates that trading has become active in the Chinese treasury bond futures market.

However, price fluctuation might cause considerable uncertainty in the asset valuation during transactions. Volatility is introduced as one of the metrics to evaluate the instability in futures prices. Understanding volatility provides crucial insights for risk management and asset pricing [4]. Therefore, the significance of models that possess the ability to precisely forecast volatility is indisputable. One of the most common models is constructed on historical volatility, defined as the conditional standard deviation of futures' logarithmic return. The idea is based on the dependence between the changes in prices found by Mandelbrot (1997) in the financial market [5]. This phenomenon results in the successive clustering of volatility. Engle (1982) proposed the autoregressive conditional heteroscedastic (ARCH) model to capture variations in the conditional variance of forecasting residuals [6]. Bollerslev (1986) extended the ARCH model to the generalized autoregressive conditional heteroscedastic (GARCH) model, considering the effect not only from past errors but also from their lagged conditional variances [7]. Various alternative models have been

introduced to address more specific scenarios, including EGARCH, GJR-GARCH, and AGARCH [8-11]. The GARCH (1,1) model is widely used because of its simplicity and effectiveness.

Two primary approaches are commonly employed to estimate model parameters: frequentist inference and Bayesian inference. In the realm of frequentist inference, the Maximum Likelihood Estimator (MLE) is a widely utilized method that exclusively depends on empirical evidence via observations. Gao and Lu (2021) applied the MLE technique to estimate Copular-GARCH model parameters, yielding significant results to volatility and correlation in the Chinese and American Soybean futures markets [12]. Djeddour and Kerar, L. (2021) employed quasi-likelihood estimators for modeling GARCH and conducted an empirical analysis on S&P500 stocks, achieving a favorable estimation result as reflected by the Root Mean Square Error (RMSE) [13].

In the Bayesian inference framework, parameters are incorporated with assumptions. The posterior distribution is derived from both prior conceptions and observed data. When faced with complex posterior distributions that are challenging to determine explicitly, the Markov Chain Monte Carlo (MCMC) method serves as a valuable tool for simulating these distributions. In the study conducted by Shiferaw (2023) on the impact of volatility resulting in East African tea crop prices, the MCMC approach was applied to obtain parameters in the Markov-switching GARCH [14]. Extended MCMC algorithms were used by Livingston Jr. and Nur (2022) on multivariate-GARCH-BEKK models, enabling the estimation when the parameter space was varying in dimensions [15].

For 10-year Chinese treasury futures, few compared the estimated performance of the two methods on their model parameter estimation. Bayesian inference offers advantages when confronting models characterized by intricate structures, in which the maximum likelihood estimation method may prove to be inadequately effective in exploration. However, in cases where both methodologies are feasible, one would expect to compare their performance. This research encompasses the entire historical data to model the volatility of this type of future. In addition, this work aims to acquire two sets of parameters for the Student-t distribution GARCH(1,1) model and assess their respective goodness of fit by using forecasting error measures. The MLE and MCMC methods will be employed using the R studio. The result indicates that the MLE gives more minor forecasting errors than MCMC.

2. Data Collection

2.1. 10-year Chinese Treasury Bond Futures

This study uses daily closing price data about 10-year Chinese Treasury Bond futures traded on the China Financial Futures Exchange (CFFEX). The dataset covers the period commencing from March 20, 2015, through July 21, 2023, encompassing a total of 2029 data points, each corresponding to consecutive trading days. The time series representing these daily closing prices is denoted as P_t .

2.2. Logarithmic Return

The daily logarithmic return series is used for modeling the historical volatility, defined as

$$r_t = \ln \left(\frac{P_{t+1}}{P_t} \right), \quad (1)$$

where is the return on day t and P_t is the closing price on day t. The logarithmic return is also known as the continuously compounded return because the number of time subintervals takes the limit to infinity; using log return allows the summation of returns in a period, which resolves the issue associated with multiplying the numerical value close to zero [16]. To facilitate the processing of data within a consistent scale, the daily log return is converted into annual by multiplying $\sqrt{252}$.

3. Methodology

3.1. Models

3.1.1 The Mean Function

Before examining its conditional heteroscedasticity, the return required to be described based on previous data. The mean function is defined as:

$$r_t = \mu_t + \varepsilon_t = E(r_t | F_{t-1}) + \varepsilon_t \quad (2)$$

where F_{t-1} is the filtration including all information before day t ; the residuals are denoted as ε_t . As a result, modeling the conditional variance of residuals is sufficient to capture the volatility within the return series:

$$\text{Var}(r_t | F_{t-1}) = \text{Var}(E(r_t | F_{t-1}) + \varepsilon_t | F_{t-1}) = \text{Var}(\varepsilon_t | F_{t-1}). \quad (3)$$

3.1.2 The GARCH Model

The general GARCH(p,q) model has the form

$$\varepsilon_t = \sigma_t u_t, \quad (4)$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2, \quad (5)$$

where $\alpha_0 > 0$, $\alpha_i \geq 0$, $i = 1, \dots, q$ and $j = 1, \dots, p$, $0 < \sum_{i=1}^q \alpha_i + \sum_{j=1}^p \beta_j < 1$ to satisfy the stationary condition; u_t are i.i.d. innovations having a distribution with mean 0 and variance 1 for $t = 1, \dots, T$. The GARCH(1,1) model was proven to be effective in various datasets [17].

3.2. Estimation

In this section, the parameter vector is defined as $\theta = (\alpha_0, \alpha_1, \beta_1, \nu)$ and the observations are $\varepsilon = (\varepsilon_1, \dots, \varepsilon_T)$.

3.2.1 The Maximum Likelihood Estimator

When the innovations u_t are i.i.d. standard Student-t distribution with mean 0 and variance 1, the marginal density of the GARCH(1,1) model is given by:

$$f(\varepsilon_t | F_{t-1}) = \frac{1}{\sigma_t \sqrt{(\nu-2)\pi}} \frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2})} \left(1 + \frac{\varepsilon_t^2}{(\nu-2)\sigma_t^2} \right)^{-\frac{\nu+1}{2}}, \quad (6)$$

where $\Gamma(\cdot)$ stands for the Gamma function; $\nu > 2$ is the degree of freedom of Student-t distribution. The joint likelihood function of can be expressed as follows:

$$L(\theta | \varepsilon) \propto \left[\frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2}) \sqrt{\nu-2}} \right]^T \prod_{t=1}^T \frac{1}{\sigma_t} \left(1 + \frac{\varepsilon_t^2}{(\nu-2)\sigma_t^2} \right)^{-\frac{\nu+1}{2}}. \quad (7)$$

Using the log-likelihood function can derive the maximum likelihood estimators for the parameters. This can be implemented by the function `garchFit()` in the R package “rugarch” from the RStudio [18].

3.2.1 Bayesian inference

The Bayesian inference framework involves the process of updating previous assumptions about unknown parameters via the use of observed evidence. The concept is based on Bayes' theorem:

$$p(\theta|\varepsilon) = \frac{p(\varepsilon|\theta)p(\theta)}{\int_{\theta'} p(\varepsilon|\theta')p(\theta') d\theta'}, \quad (8)$$

with posterior distribution $p(\theta|\varepsilon)$, likelihood function $p(\varepsilon|\theta)$ and prior distribution $p(\theta)$. Unlike maximum likelihood estimation, it further integrates a prior distribution for unknown parameters. However, the normalizing factor in the denominator is often difficult to determine in most realistic situations, which means one can only know the posterior distribution up to a constant:

$$p(\theta|\varepsilon) \propto p(\varepsilon|\theta)p(\theta). \quad (9)$$

(Note that the likelihood function $p(\varepsilon|\theta)$ may differ from the one mentioned above; therefore, I use different expressions.)

3.2.3 Markov Chain Monte Carlo (MCMC)

The MCMC method is applied to tackle the concern above, enabling the simulation of the posterior distribution through a certain process of sampling. The aim is to design an irreducible and aperiodic Markov chain whose invariant distribution $\pi(\theta)$ is the posterior distribution $p(\theta|\varepsilon)$. Once the Markov chain converges, every walk within the chain could be regarded as a sample from the target distribution. The Metropolis-Hastings (M-H) algorithm was introduced:

1) Set proposal distribution Q , sample size n , initialize state θ_0 at $t=0$;

2) While $t \leq n$,

Sample $\theta_* \sim Q$, $u \sim \text{Uniform}(0,1)$;

Define $\alpha(\theta_t, \theta_*) = \min \left\{ \frac{p(\theta_*|\varepsilon)Q(\theta_t, \theta_*)}{p(\theta_t|\varepsilon)Q(\theta_t, \theta_*)}, 1 \right\}$;

If $u < \alpha(\theta_t, \theta_*)$, set $\theta_{t+1} = \theta_*$; Otherwise, set $\theta_{t+1} = \theta_t$;

3) Output $\{\theta_0, \dots, \theta_n\}$;

Using the acceptance rate α in this form avoids the unknown of the full posterior distribution [19]. The MCMC M-H algorithm for estimating the Student-t GARCH(1,1) model was implemented by the R package “bayesGARCH” [20]. Ardia and Hoogerheide (2010) presented a detailed usage of bayesGARCH; they wrote out the posterior density in terms of the likelihood function and the joint prior distribution according to an alternative expression of Student-t innovations [20].

4. Empirical Results

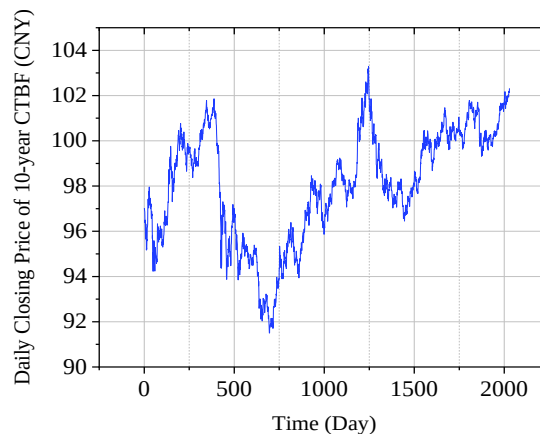


Fig. 1 Daily Closing Price

This investigation illustrates modeling procedures and conducts a comparative analysis of estimations concerning the log-return series associated with 10-year Chinese treasury bond futures, spanning from March 20, 2015, to July 21, 2023. Fig. 1 visually represents the daily closing prices of 2029 consecutive trading days, with an overall upward trend. Prior to stepping into the volatility modeling, it is imperative to perform several hypothesis tests.

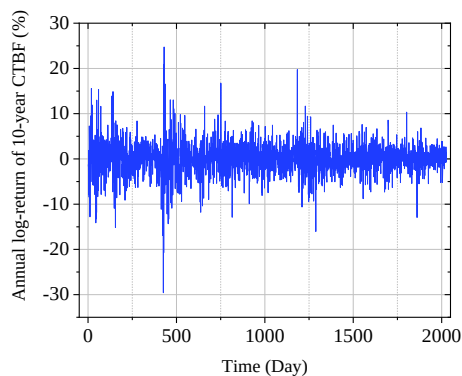


Fig. 2 Annual log-return

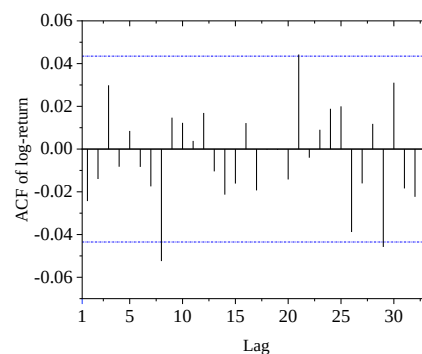


Fig. 3 ACF plot

4.1. Hypothesis Testing

4.1.1 The Augmented Dickey-Fuller Testing

The GARCH model is employed under the prerequisite of stationarity to ensure accurate results [6,7]. Fig. 2 demonstrates the annual log-return time series, appearing to exhibit stationarity. To formally assess stationarity, the Augmented Dickey-Fuller (ADF) test is utilized. The result is illustrated in Table 1. The p-value falls below 0.01, the null hypothesis is rejected under the 1% significance level, showing that the yield series demonstrates stationarity.

Table 1. ADF test results

ADF test statistic	Lag	p-value
-12.705	20	< 0.01

The Autocorrelation Function (ACF) figure in Fig. 3, illustrates the absence of autocorrelations within the dataset, by observing that most of spikes lie between the critical values. This research employs the constant mean function as a viable approach: $r_t = \mu + \varepsilon_t$, where μ is the sample mean with value 0.000416.

4.1.2 The Lagrange Multiplier Testing

The Lagrange Multiplier test (LM test) is used for testing the presence of ARCH effect: the conditional heteroscedasticity between lagged squared residuals ε_t^2 [6]. The null hypothesis is consistent with no ARCH effect. From Table 2, the p-value significantly falls below the 1% significance threshold, providing compelling evidence to reject the null hypothesis. The GARCH model is applicable to the dataset.

Table 2. LM test results

Chi-squared statistic	Degree of Freedom (df)	p-value
303.1	12	< 2.2e-16

4.2. Estimation Results

The quantile-quantile (Q-Q) normal plot in Fig. 4 illustrates the data has a fat tail so the normality is not met. Hence, the standard Student-t innovations are suggested in this study.

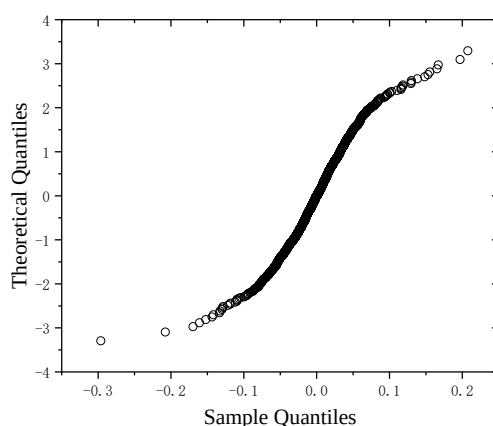


Fig. 4 Daily Closing Price

The maximum likelihood estimates are generated by R function “garchFit()” and the summary output gives the numerical values. The estimates using MCMC simulation are conducted by “bayesGARCH()” with $\alpha_1 + \beta_1$ as a prior condition. The prior distributions are set as default [20].

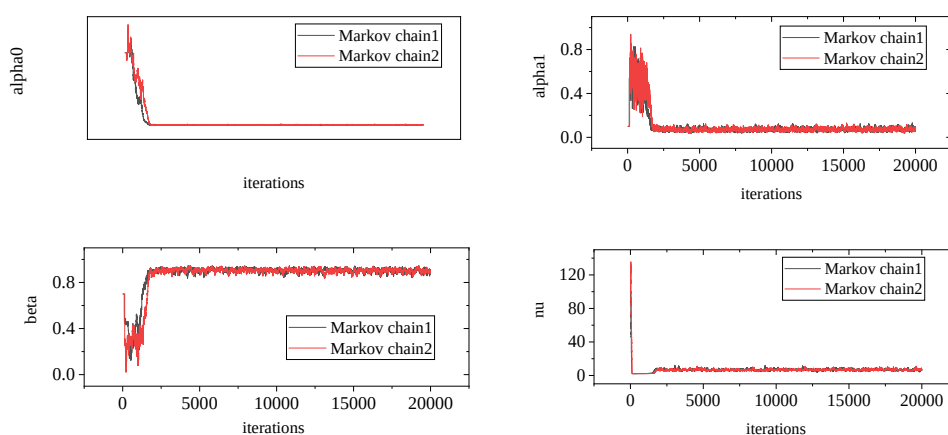


Fig. 5 Daily Closing Price

Figure 5. illustrates two Markov chains each with length 20000 and both converges; the histograms are displayed where the first 5000 iterations in each of the chain are excluded. Figure 6. combines two chains as one sample, using histograms to simulate the marginal density of parameters.

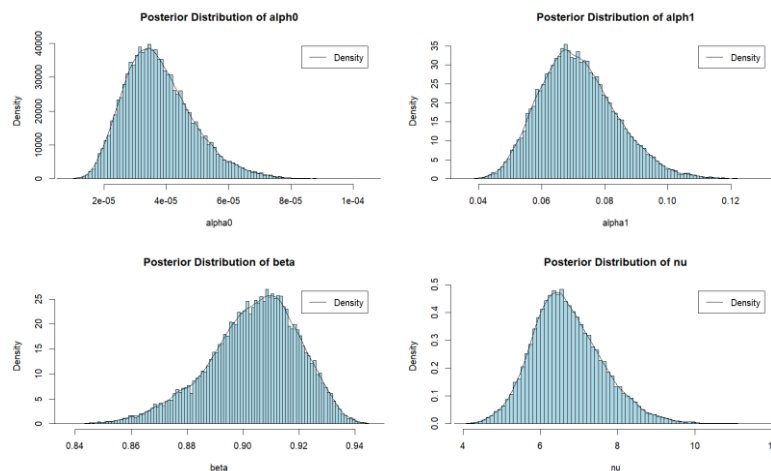


Fig. 6 Marginal densities of parameters

Table 3. Estimate of parameters in Student-t GARCH(1,1) by MLE and MCMC

	MLE	MCMC M-H	
	Maximum Likelihood Estimate	Mean of Posterior	Maximum a Posteriori
α_0	0.00001232	0.00003762	0.00003430
α_1	0.04346	0.07157	0.06764
β_1	0.9469	0.9037	0.9095
ν	7.215	6.706	6.427

Table 3. presents the estimates and corresponding standard errors of the Student-t GARCH(1,1) model using two different techniques. In the context of Bayesian inference, both the mean of posterior distributions and the maximum a posteriori (MAP) are taken into consideration.

To conduct a comparative analysis of estimates, one can assume the absolute value of observed residuals as the realized volatility and using forecast errors to examine the model fit. This paper includes metrics: Mean Absolute Error (MAE), Mean Square Error (MSE), Root Mean Square Error (RMSE). Table 4. displays the result. In the GARCH(1,1) model for 10-year Chinese treasury bond futures, MLE yields an average volatility of 3.6%; MCMC estimation results in an average volatility of 3.67%.

Table 4. The forecast errors

	MSE	MAE	RMSE
MLE	0.0007101	0.02059	0.02665
Mean	0.0007181	0.02097	0.02680
MAP	0.0007169	0.02094	0.02677

1.1 Discussion

All three sets of parameters yield commendable forecasting outcomes for volatility, as evidenced by the minimal errors in Table 4. For comparing the estimation, the MLE method assigns greater significance to the impact of conditional standard deviation. Specifically, the MLE estimates exhibit higher values for β_1 , while the estimates for α_0 and α_1 are relatively minor compared to those obtained through the MCMC approach. The structure of the prior distribution in the algorithms may cause this result. The estimates from the Mean and MAP are very close, indicating that unimodality can be found in the joint posterior distribution of parameters, with a clear peak in the parameter space. Although their model parameters differ, both approaches are consistent with nearly the same average volatility level.

The MLE is preferable to MCMC by measuring forecast errors on estimates despite no significant difference in magnitude from models. However, this outcome must provide conclusive evidence across other datasets and algorithms. Gu (2011) used the Metropolis-Hastings method to estimate parameters in the GARCH(1,1) model, assuming standard expected innovations. Based on data from the SSE Composite Index, the empirical results indicate that MCMC outperforms MLE in model fitting [21]. Therefore, the choice between these methods may depend on the specific characteristics within the data. Moreover, the selection of prior distributions and likelihood functions can directly affect the simulation of the posterior distribution. Finally, employing different R packages can also introduce challenges in result interpretation due to variations in default algorithms and settings.

5. Conclusion

In summary, based on historical data, this study undertakes a volatility analysis of 10-year Chinese treasury bond futures. The results of the hypothesis testing conducted on the logarithmic returns indicate that the data exhibits stationarity and lacks autocorrelation. The observed data is assumed to be distributed according to a Student-t distribution. The GARCH(1,1) model accurately fits historical volatility when using both MLE and MCMC approaches. The study reveals that the two methods provided different numerical values for the model's unknown parameters, but the average volatility was very close. The MLE method performs better than the MCMC method in terms of error.

Nevertheless, it is essential to recognize the constraints inherent in this study since it focused on specific data sources, model settings, and the use of R packages. Therefore, definitive judgments about the superiority of the two methods cannot be drawn. Future research can expand in more aspects, such as including macroeconomic influences or using other machine learning approaches to analyze Chinese treasury bond futures' volatility. These endeavors aim to enhance risk assessment and provide valuable insights for financial practitioners and regulators, fostering the development of the Chinese futures market.

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