

Analysis of Principle and Applications of FFT in Medical Imaging Dataset

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Abstract. The outbreak of novel infectious diseases usually resulted in the high number of death and infectious. It is always in urge to develop a method that can detect and diagnose the disease rapidly and accurately. This article provides a method based on Fast Fourier Transform that can process the medical image dataset and perform the following processing procedure. FFT plays an irreplaceable role in modern data and signal processing. The chest X-ray represents that this method accomplishes significant result. The differences between different characteristics of pneumonia have been significantly enhanced and sharpened. It becomes available to tell the difference from bacterial pneumonia, viral pneumonia and Covid-19 pneumonia while leaving the normal lung still clear. These results can be applied to help to screen out the suspicious case and extract the symptom of novel infectious diseases when it is still contained at early stage. It brings strategic significance to the isolation, prevention and control measures of novel infectious diseases.

Keywords: FFT; medical image process; harmonic analysis; pneumonia; Covid-19.

1. Introduction

Fast Fourier Transform, usually abbreviated as FFT, has the revolutionary impact in various fields and subjects. The thoughts can be traced back to 1753, when Euler provided the formula that a real variable function can be written into Fourier series with certain coefficients [1]. The idea of Discrete Fourier Transform (abbreviated as DFT) originated when Carl Friedrich Gauss calculated the certain asteroids' orbit from sample locations in 1805 [2]. Although some electronic equipment companies in the United States and Japan began designing and producing digital Fourier analyzers in the mid-1970s, DFT didn't been used widely because it require N^2 arithmetic operation. It was until in 1965, when Cooley and Tukey turned out that the new algorithm can replace the former with several operations to $N \log_2 N$ to compute Fourier transform of N data points [3]. This has been seen as a revolutionary effect of FFT and DFT had been more and more applied in various fields since the calculation became acceptable [4, 5]. Moreover, the velocity increases as the data points increases, for example, when $N = 1024$, $N^2/(N \log_2 N)$ is approximately 100 times; and it will be 50000 times if there are one million points.

Later, in 1984, Dohamel and Hollamann raised the "split radix" FFT algorithm, which improved the operational efficiency further [6]. There are still some new algorithms be proposed till now, however, they never improve the efficiency of FFT operations by more than 50%, which is far less than the upgrade of FFT to DFT. In this sense, the FFT algorithm is a milestone since it cut off the operation time greatly and reduces the cumulative quantization error caused by DFT operations, which is often neglected by people. The study of FFT never stops [7]. It is appropriate to say that FFT laid a practical foundation for signal and system analysis. The computational speed of Fourier analysis in spectrum analysis has been increased by more than a hundred times, thus achieving the goal of using electronic computers for spectrum analysis. This has formed the basic methods and concepts of digital signal processing centered on digital filtering and FFT [4, 8]. The specialized term "digital signal processing" has not emerged in the field of technology until in the 1970s, just a few years after. Cooley and Tukey gave the FFT.

In recent years, FFT has been used extensively in many fields beyond signal processing. It has been introduced to physical geodesy to deal with heterogeneity of data, presenting complex surfaces of data, uneven spatial distribution, and non-uniformity of data noise [9]. Moulinec and Suquet

purposed the FFT methods for the homogenization of composites in 1994 [10-14]. Since then, FFT has been applied to solve the nonlinear homogenization problem in computational micromechanics [15]. In digital protection system of building, FFT has also been applied to form a unique and improved algorithm to collect and process data [16]. It was also identified that time and frequency parameters could be applied to the cyclical phenomenon on the housing construction market so that FFT could be used to decision-making process of demand and supply in the housing construction market, and also helps to evaluate the expected price of the housing market and the formation of speculative foam [17]. Paul Adams, who is the leader of the Department of Molecular Biophysics and Integrated Bioimaging at the Lawrence Berkeley National Laboratory in California. In 1995, he improved the structure of bacterial protein gel. Even if he used the FFT and supercomputers, it still took him many hours, even several days. It was hard to imagine if there's no FFT at that time, how would Paul Adams complete his research.

In this paper, FFT is applied to process the medical imaging data. The medical imaging data, including the X-ray film, which is non colored and with low resolution is just a suitable application condition. After introducing the data and model that used in this paper briefly, it is expected that FFT can sharpen the image without affecting the image itself, so that some details can be seen more clearly. In the research, it is intended to judge whether the patient is infected with bacteria pneumonia or viral pneumonia by the processed X-ray simply.

2. Data and Method

2.1. Data

The data used in the research are downloaded from Kaggle, a platform where enterprises or researchers will publish data, problem descriptions, demands, etc., and solicit solutions from many data scientists. Chest X-ray images (anterior-posterior) were selected from retrospective cohorts of pediatric patients of one to five years old from Guangzhou Women and Children's Medical Center, Guangzhou. All chest X-ray imaging was performed as part of patients' routine clinical care. For the analysis of chest x-ray images, all chest radiographs were initially screened for quality control by removing all low quality or unreadable scans. The dataset is already organized into normal, bacterial pneumonia and viral pneumonia. Since X-ray imaging devices cannot focus like optical lenses, the resulting image generally tends to be slightly blurred. While radiologists diagnose X-ray images with their own eyes, so it is determined that the black/white ratio and shape (blood vessels, lungs) are the criteria, which are determined by the edge. Therefore, this paper intends to aggravate and sharpen the image edges to make the details more clearly.

2.2. Model

All Fourier transforms towards continuous signals are sampled through discrete points and dealt with discrete Fourier transforms in the computer.

$$\widehat{x(k)} = \sum_{n=0}^{N-1} x(n) W_N^{nk}, \quad k = 0, 1, \dots, N - 1 \quad (1)$$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} \widehat{x(k)} W_N^{-nk}, \quad n = 0, 1, \dots, N - 1 \quad (2)$$

$$W_N^{nk} = e^{-i\frac{2\pi}{N}kn} \quad (3)$$

If DFT is used for calculation with N points, then N times of multiplication are required and the complexity will become N^2 . FFT is considered one of the "top ten algorithms of the 20th century" for it greatly reduce a significant amount of computation while maintaining the same computational results as DFT. FFT utilizes the following properties to reduce complexity. It is very easy to verify these properties through the properties of complex numbers.

$$W_N^{n(N-k)} = W_N^{k(N-n)} = W_N^{-nk} \quad (5)$$

$$W_N^{n+\frac{N}{2}} = -W_N^n, \overline{W_N^{N-n}} = W_N^n \tag{6}$$

$$W_N^{nk} = W_{mN}^{mnk}, W_N^{nk} = W_{N/m}^{nk/m} \tag{7}$$

Based on these characteristics, a long DFT operation can be decomposed into a combination of several short sequence DFT operations, thereby reducing the computational complexity. The classification of FFT is basically divided into time extraction method (DIT-FFT) and frequency extraction method (DIF-FFT). This paper takes time extraction method as an example to illustrate. The main idea is that each recursive stage divides DFTs of size N into interleaved DFTs of size $N/2$. To start with, it is hypothesized that N is an integer power of 2, which means $N = 2^M$. If not, the computer will replenish 0 to make sure it is an integer power of 2. Then, separate $x(n)$ to the odd sequence and the even sequence.

$$x(2r) = x_{even}(r) \quad x(2r + 1) = x_{odd}(r) \quad r = 0, 1, \dots, \frac{N}{2} - 1 \tag{8}$$

DFT will also process as the two parts.

$$\widehat{x(k)} = \sum_{r=0}^{\frac{N}{2}-1} x_{even}(r) W_N^{2rk} + W_N^k \sum_{r=0}^{\frac{N}{2}-1} x_{odd}(r) W_N^{2rk} \tag{9}$$

The equation can be simplified by the properties mentioned before:

$$\widehat{x(k)} = \sum_{r=0}^{\frac{N}{2}-1} x_{even}(r) W_{\frac{N}{2}}^{rk} + W_N^k \sum_{r=0}^{\frac{N}{2}-1} x_{odd}(r) W_{\frac{N}{2}}^{rk}, \quad k = 0, 1, \dots, \frac{N}{2} - 1 \tag{10}$$

It is converted into two discrete Fourier transforms of $N/2$ points with a length of $N/2$. From here, it is feasible to use the periodicity of complex numbers to find the other $N/2$ points.

$$\widehat{x(k + \frac{N}{2})} = \sum_{r=0}^{\frac{N}{2}-1} x_{even}(r) W_{\frac{N}{2}}^{rk} - W_N^k \sum_{r=0}^{\frac{N}{2}-1} x_{odd}(r) W_{\frac{N}{2}}^{rk} \tag{11}$$

So, through this decomposition, each $N/2$ point DFT only needs $(N^2)/4$ complex multiplication calculations, which significantly saves computational complexity. Furthermore, it is still available to separate the $N/2$ point DFT into $N/4$ point DFT, and keep this process until there is one point left. The inverse transform is the same process.

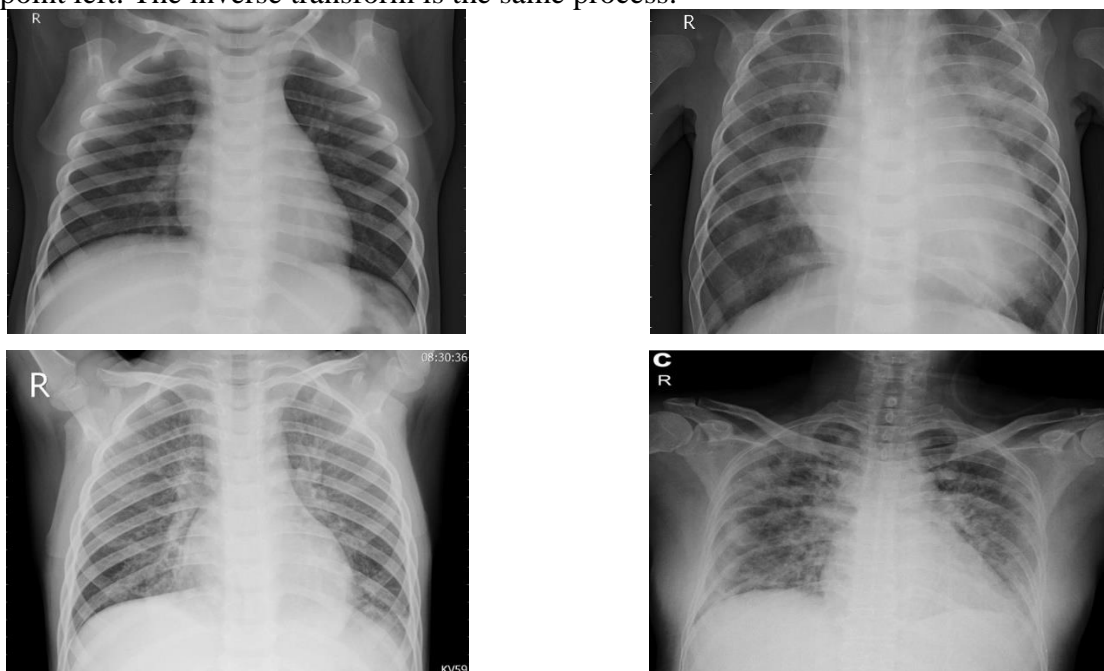


Fig. 1 The Chest X-ray for normal (upper left), Bacterial Pneumonia (upper right), Viral Pneumonia (lower left) and Covid-19 Pneumonia (lower right).

3. Results and Discussion

Chest X-ray is what this research mainly focused on as an example to illustrate the effect of the medical image process based on FFT. Some typical results are shown in Fig. 1. Since X-ray usually tends to be slightly blurry and fuzzy, it makes sense to sharpen the image so that radiologists are able to observe the details more easily. Although X-rays are often used as auxiliary examinations, they can supply many signals for judging diseases. For instance, pneumonia is just a typical example. The normal chest X-ray depicts clear lungs without any areas of abnormal opacification in the image. Bacterial pneumonia typically exhibits a focal lobar consolidation, whereas viral pneumonia manifests with a more diffuse “interstitial” pattern in both lungs. As for Covid-19 pneumonia, the lung markings are much more thickened and disordered, with spotted and grid shaped lesions distributed around them. These characteristics can actually determine what kind of pneumonia is. However, as the X-rays are not clear enough, doctors usually need to depend on other means and instruments to make the diagnosis. Fourier transform provides a platform to convert the image information of grayscale pixel mode into frequency domain which is convenient to perform further processing. The frequency domain image by FFT is shown Fig. 2. The closer the center position is, the lower the frequency is, and the brighter the position (with higher grayscale values) represents a larger signal amplitude at that frequency.

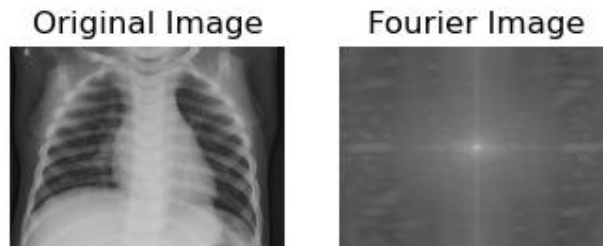


Fig. 2 Original image & frequency domain image

Then, it is feasible to set a high pass filter which only allows high-frequency spectrum to pass through. The high-frequency spectrum in the image means significant changes in pixel values.

$$H(u, v) = \begin{cases} 0 & \text{if } D(u, v) \leq D_0 \\ 1 & \text{if } D(u, v) > D_0 \end{cases} \quad (12)$$

Here, D_0 is a reasonable constant, and $D(u, v)$ is the distance between a point (u, v) in the frequency domain and the center of the frequency domain rectangle.

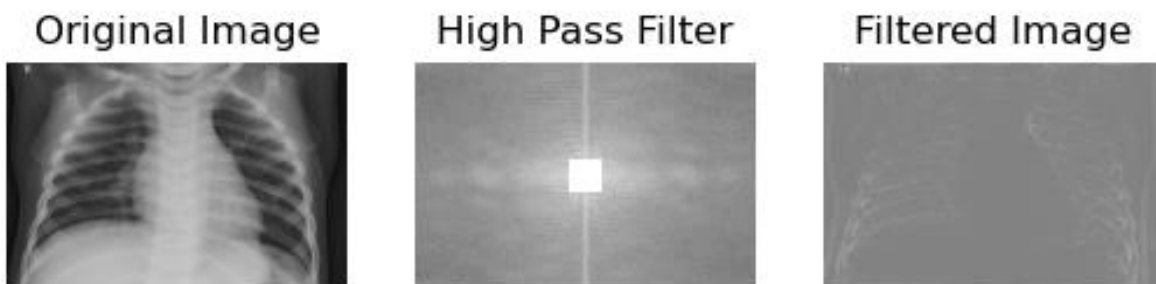


Fig. 3 High Pass Filter.

The edge information in the image is extracted, and the high-frequency part extracted by the high-pass filter is covered to the original image with weights to enhance the edge information of the image and achieve the effect of image sharpening (seen from Fig. 3). Combining high-frequency enhancement with histogram equalization is an effective method for obtaining edge sharpening and contrast enhancement. High frequency enhancement should be performed first, followed by histogram equalization. High frequency enhancement is aimed at the overall darkening of the image after high pass filtering. Therefore, by increasing the brightness of the average gray level, the visual discrimination ability of the image is improved. Then, through histogram equalization, the narrow

band dynamic range of the image is transformed into a broadband dynamic range, thereby achieving the effect of improving contrast. If histogram equalization is performed first and then high-frequency enhancement is performed, for example, when the image brightness presents a strong bipolar phenomenon, most pixels are mainly distributed in extremely dark areas, while a few pixels exist in extremely bright areas. Firstly, histogram equalization will cause the image to be bleached, and then high-frequency enhancement will be performed, resulting in less prominent edges and poor contrast of the image. The processed images are shown in Fig. 4, in contrast to the original image.

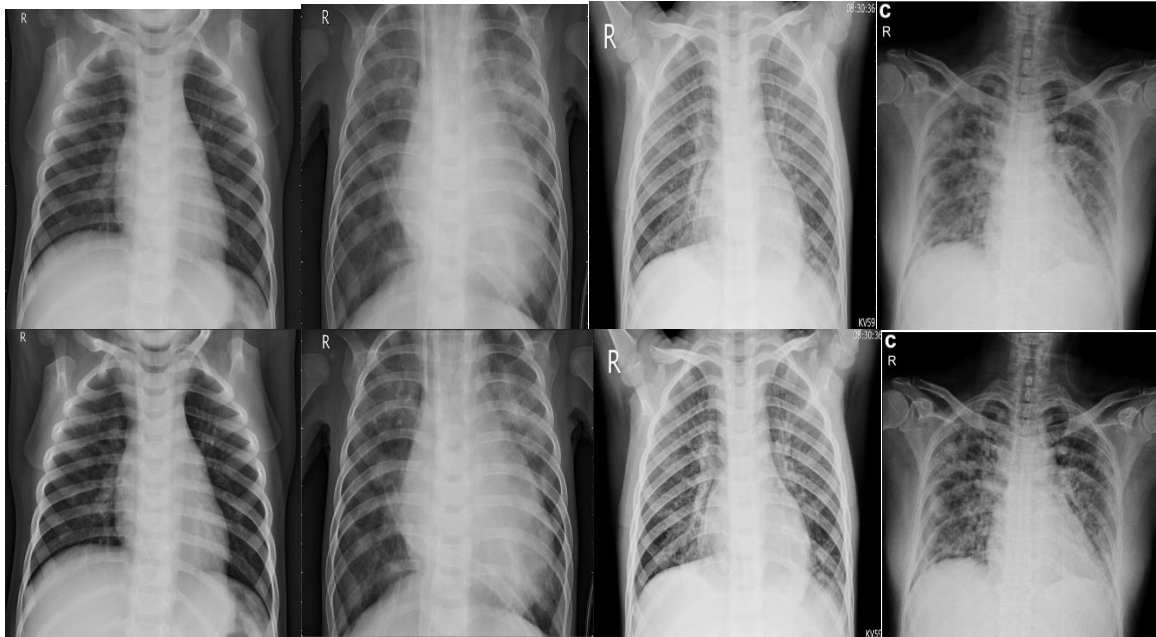


Fig. 4 The unprocessed (upper) and processed (lower) for normal, Bacterial Pneumonia, Viral Pneumonia and Covid-19 Pneumonia (from left to right).

It is not difficult to see that the processed images have better recognition ability. The normal lungs become clearer, removing some unnecessary white matter. The bacterial pneumonia in which consolidation of the entire left lower lobe becomes more distinguished than other. Viral pneumonia characteristic interstitial pattern and lung markings are much more obvious than the original image. While for the Covid-19 pneumonia, the patchy shadows with blurry boundaries that are difficult to distinguish still remain in the processed image, not be removed during the process. The symptoms of different pneumonias are much more significant and distinguished.

Although there already exists some other methods to process and analysis the medical image, they are mainly based on machine learning like deep learning [18-20]. Therefore, this study has high research value and significance. On one side, machine learning depends on a huge amount of data set to train and test the model, which has high medical costs that are not accessed easily. Even if it is possible to use other methods like transfer learning to replace the high-cost medical imaging data with other existing datasets [21], this very method is still far from mature. Hence, the method used in this article demonstrates the low-cost effect while supplying the high-resolution images for diagnose.

Moreover, any methods of machine learning have the critical demerit that they highly depend on the provided data to train and test. It cannot reflect value when the dataset is not able to be supplied. A large-scale epidemic or pandemic is just the suitable example. At the beginning of the Covid-19 pandemic, there were not so much imaging data available as the number of patients is not very high yet. However, the most needed time of this means for medical image processing and analysis to recognize and diagnose the disease is just during the outbreak of large-scale epidemics. The time when there are not enough data to apply the machine learning. Rather, the method in this paper provides a way that does not depend on the existing data set, which is also applicable when there is short of data set. From this perspective, this study has high research value because it is able to play a role at the beginning of an epidemic which is the most critical time. Screening of cases as early as

possible can effectively prevent the spread of epidemic. The method can also be applied to other medical images, e.g., CT Scan and MRI illustrated in Fig. 5.

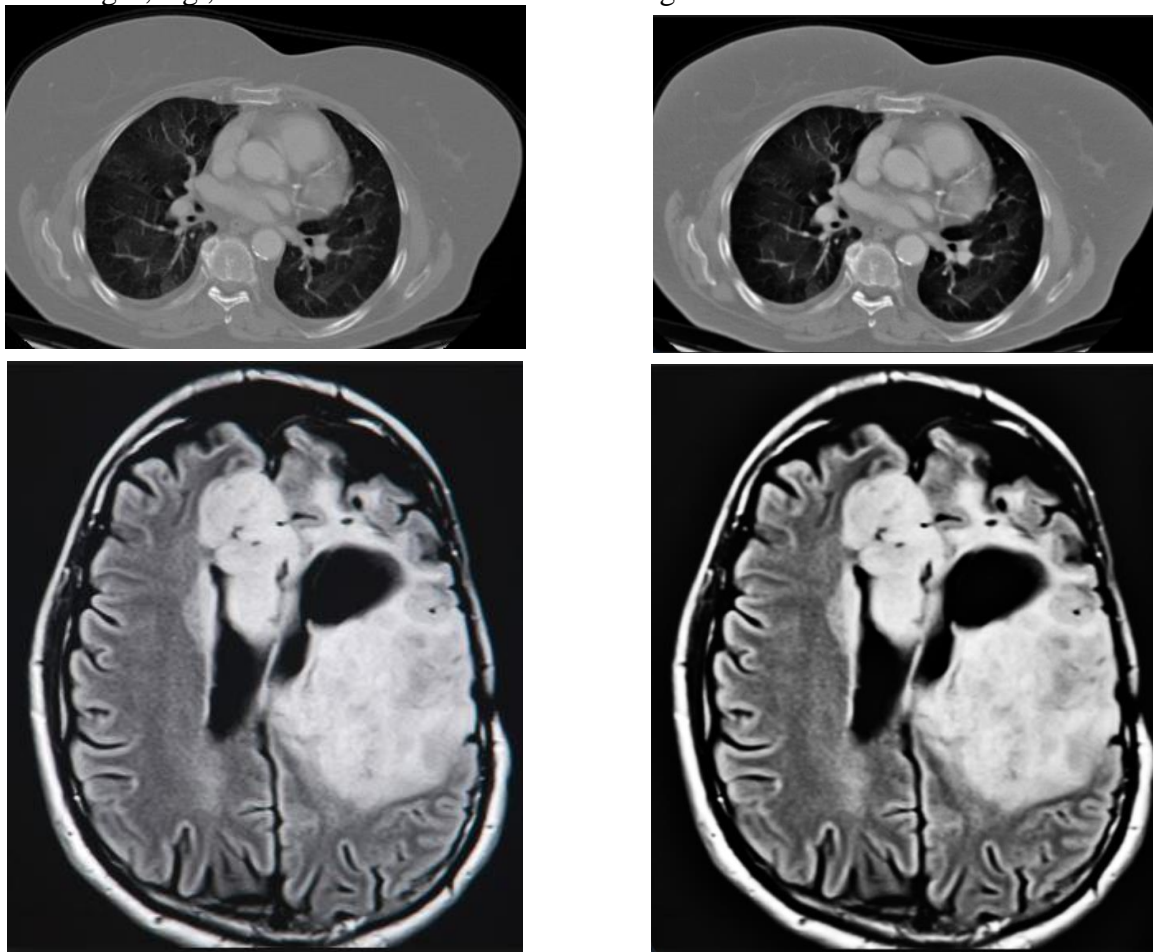


Fig. 5 The Chest CT-Scan (upper left), Chest CT-Scan (upper right), Brain MRI (lower left) and Brain MRI processed (lower right).

4. Limitations & Future outlooks

As with the majority of studies, the design of the current study is subject to limitations. Firstly, although this study is intended to apply FFT to process the medical image data, it only effects on the X-ray significantly. As for CT or MRI which already has high imaging resolution, this method cannot give too much help. For example, Fig. 5 is the image of brain tumor occurring in the left upper. The process only polishes the boundary of the tumor while leaving the continuous changes in color reflected by tissue components unprocessed. Another limitation in the X-ray involves the issue that this method fails to processed the images with high blurriness. Fig. 3 is much blurrier than others that the lower spine is almost invisible. However, it does not change obviously in Fig. 4. If it is aimed to observe the spine X-ray, then this method also loses its effect.

In the further research, there are several points that needed to be improved. As the method is be intended to process the medical image data, it should also be available to the high-resolution image like CT and MRI. There ought to be more models with detailed parameters fitting the different medical image. In this way, the subtle change of color in the image can also be observed in order that more information can be obtained from the medical image. By the same time, there also should be more than one model that available to X-ray. When the image is too blurred, it will be able to restore more information by setting the high pass filter's parameter. Apart from that, this study uses the pneumonia patients' chest X-ray as an example to operate, but what actually serves the information is the lung instead of the sternum. The biggest drawback of X-ray is that this two-dimensional image

with overlapping organizational structures usually hides and shadows the tissue which is need to be observed. If the sternums can be removed when the lungs are being examined, or get rid of the impression brought from the soft tissue on bones when examining fractures, the X-ray film will be further cleared. However, it is not easy for the computer to distinguish the sternum from the lesioned pulmonary consolidation. After all, when doing the image process, they both serve as the white color and few essential differences can be told from. Under this circumstance, if the image features of sternums are input and stored in program in advance, then it's feasible to extract the sternums from the original image. Since the image is not complete, to avoid the inaccuracy, it's possible to set a low pass filter after the high-frequency part has been extracted by the high pass filter. The low pass filter can increase the smoothness of the extracted image. After that, cover the high-frequency part to the image that processed with the low pass filter and complete the rest procedure just like what have been done before. Then the sternum can be removed from the image while not bringing other inaccuracy as much as possible, which will make the examined organs much clearer.

5. Conclusion

To sum up, FFT with high-frequency enhancement and histogram equalization have been applied to process with the medical image dataset. FFT provides the connection between the original image and the frequency domain. The edge information in the image has been enhanced by the high-frequency enhancement and histogram equalization. Chest X-ray films become much clearer than the unprocessed. It will be possible to recognize the symptoms of the normal lung, bacterial pneumonia, viral pneumonia and Covid-19 pneumonia. This method is also able to work at the very time when the epidemic breaks out while machine learning especially the deep learning cannot work as there is not enough data set. However, only X-ray films are sensitive to this method, while the image with high resolution like CT and MRI does not change obviously. Those highly blurry X-ray also remains blurry after the process. Besides, it would be instrumental if it is possible to remove the sternums while using the low pass filter to guarantee the smoothness. This article provides a method based on FFT that can make the details in medical imaging more apparent. The pneumonia, for example, becomes feasible to tell the difference from the bacterial, viral and Covid-19. It can help rapidly and accurately detecting the new infectious disease when it is about to break out.

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