Analysis of the Principle and Applications of Sharping and Smoothing of Graphs Based on Fourier Transform

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Abstract. As a matter of fact, Fourier transform is regarded as one of the most fundamental tools in image processing in recent years, which has been implemented in various applications including signal processing, acoustic generation and filtering as well as image processing. With this in mind, in this article, the principle of this transformation is introduced and discussed. At the same time, some related filter techniques is introduced to transform the image space into frequency space. The specific process of four different filters is shown in this study when applied in smoothing or sharping the graph. To demonstrate the significance of this technology, many real-world applications will be presented in the mean time. According to the analysis, the current limitations as well as future prospects are demonstrated. As the demand for digital image processing in various domains such as medical, telecommunication, and environment, the requirement for the Fourier transform has increased.

Keywords: Fourier transform; image processing; image sharping and smoothing; filter.

1. Introduction

Digital Image Processing (DIP) has emerged as a crucial field at the intersection of computer science and image analysis. With the increasing prevalence of digital images in various domains such as medical imaging, geological prospecting, and fingerprint, the requirement for effective image processing techniques is under high expectation. This research focuses on a fundamental aspect of DIP—image enhancement, with a specific emphasis on image sharpening and smoothing techniques by further applying Fourier transform. The corresponding strategies of DIP, entailing the application of mathematical procedures to deal with pictures, mainly derived from two areas: enhancing pictorial data or better storage, transforming some machine information [1]. Noise in transportation will make a huge difference on the quality of image. Due to noise, some digital photographs are blurry, images are supposed be upgraded for additional processing. Thus, the process of filtering is often used to improve the photographs by lowering the noise and sharpening them further. The filters are used to sharpen images and decrease noise in images using digital image enhancing techniques [1]. Many approaches have been applied to deal with it. In this study, sharpening and smoothing techniques with the application of Fourier transform will be introduced.

Fourier transform replicates the periodic elements of the image's frequencies, eliminating unwanted frequencies by performing the inverse of this change. Another name for this procedure is filtering. Filters could be used to deal with a large variety of image processing task, which mainly has two kinds, low-pass filters and high-pass filters, playing roles in smoothing and sharping respectively. The former affect high frequencies but have no effect on low frequencies, while the latter do not affect high frequencies [2]. Both sharpening and smoothing techniques are indispensable tools in image processing, enabling tailor images for numerous specific applications. These two operations are closely dependent because they are both able to deal with image’s high spatial frequencies: smoothing can reduce small signals or information, while sharpening is able to find out and highlight the image noise [3]. The Fourier transform is a mathematical concept that aids in converting data across several domains, which was named after Jean Baptiste Joseph Fourier who made a huge contribution to this area. The first form of the transform is discrete Fourier transform, was developed for better address a variety of issues, particularly those related to DIP. In order to solve the problem of large sample size, fast Fourier transform (FFT) has been created based on discrete transform to computing in a convenient process [4]. For a long time, FFT has been crucial to image processing. One of the potent tools used to improve, restore, encode, and characterize the images is the 2D Fourier Transform. There is also the FFT variant, which is utilized in many image processing applications to cut down on computing costs. By using the consecutive doubling technique, the FFT is simple to implement and therefore plays a significant role in image processing applications [1]. Discrete fast Fourier transform could be applied to telecommunication networks, playing a role as compressing both audio and image signals [5]. The fast form (FFT) could be utilized to handle the continuous wave radar data with the determination for the real-time location of the moving item. Due to the integration of the Fourier transformation and 4-D (4 Dimensional) image segmentation processes, diagnosis of brain cancer could be improved [6]. FT can assist with microplastic in water, which ruins the ecosystem and is a great pollute for human-beings. Fourier transform infrared (FTIR) spectroscopy is used to deal with the microplastic to measure the intensity of the water and could detect very small size particles [7, 8]. Such a kind of spectroscopy could also be applied to determine mineral phases. With the help of X-ray diffraction, mineral's unique absorption pattern can determine the mineral concentrations [9]. Some domains like angiography use the inverse version of Fourier transformation. With this method, the vessel connectivity is better to visualize, with Fourier convolution images [10].

The main objective of this study is to build the foundations of picture sharpening and smoothing based on Fourier transformations. This entails a thorough investigation of the mathematics and computational underpinnings of image processing methods. Understanding these approaches' pixel-level operations and how they make use of ideas like convolution and high-pass filtering is essential for recognizing their strengths and weaknesses. This research aims to contribute to the broader field of DIP by providing insights into the principles and applications of image sharpening and smoothing. Hence, this paper is written to introduce the principle of Fourier transform, give some related formula and use filters based on graph sharpening and smoothing to give a concise view on image processing.

2. Fourier Transform in Digital Image Processing

Fourier transform is a useful tool in Digital Image Processing. The image is transferred from physical space to frequency space, then the image can be processed and analyzed by using Fourier spectrum characteristics [11]. The intensity of the change of image signal or gray level in physical plane is a reflection of the frequency of an image. For an image, the region with slow signal change corresponds to lower frequency. Low frequencies always reflect the general picture of an original image and are always located in the interior of the image's outline. Accordingly, the region with sharp gray level change corresponds to higher frequency. High frequencies of an image always show the details of the image and converge on the boundary and noise of the image [11].

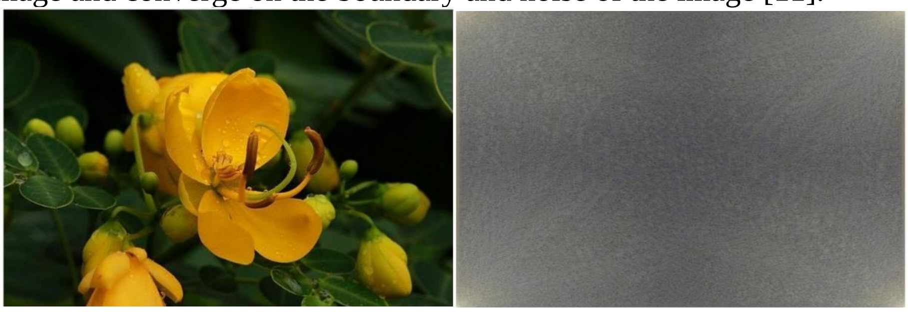

Fig. 1 The original image and Fourier transform spectrogram.

The image's energy is mainly concentrated in regions with low frequency. After processing an image with two-dimensional Fourier transform, it can be found that it is the brightest in the four corners. This reflects that low frequent image signals are concentrated in the four corners. This is shown in Fig. 1. A translation can be done to adjust the brightest part of the Fourier transform spectrogram to the middle part. After the translation, low frequencies and the image energy will be concentrated in the middle part (as shown in Fig. 2). The Spectrum centralization can make it easier and more convenient to exert a filter on the frequency space.

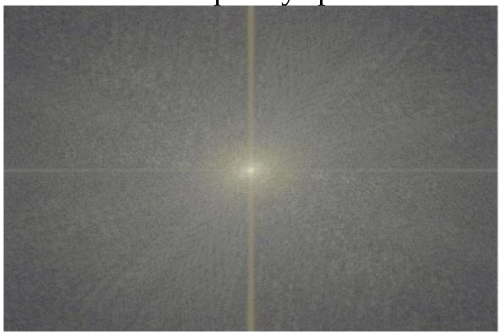

Fig. 2 The Fourier transform spectrogram of Fig. 1 after Spectrum centralization

3. Definition and Properties of 2D Discrete Fourier Transform

Particularly, two-dimension discrete Fourier transform is used in Digital image Processing. For an image with the dimension M*N, the Fourier transform and inversion formula are defined as follows:

$$F(u,v)=\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}f(x,y)e^{-j2\pi(\frac{ux}{M}+\frac{vy}{N})} \tag{1}$$

with $u=0,1,\dots,M-1, v=0,1,\dots\dots,N-1$:

$$f(x,y)=\frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}F(u,v)e^{j2\pi(\frac{ux}{M}+\frac{vy}{N})} \tag{2}$$

with $x=0,1,\dots.,M-1, y=0,1,\dots\dots,N-1$. Here, x, y represent the position coordinates of the pixels, u,v present the frequency information; f(x,y) presents the physical plane of digital images and F(x,y) is the images in the frequency plane. For convenience, it is helpful to breviate formula Eq. (1) with DFT and Eq. (2) with IDFT. Based on translation, one derives:

$$f(x-a,y-b)\leftrightarrow e^{\frac{-j2\pi(au+bv)}{N}}F(u,v) \tag{3}$$

$$F(u-c,v-d)\leftrightarrow e^{\frac{j2\pi(cx+dy)}{N}}f(x,y) \tag{4}$$

Particularly,

$$F(\frac{u-M}{2},\frac{v-N}{2})\leftrightarrow(-1)^{x+y}f(x,y) \tag{5}$$

Above equations show that F(u,v) can be translated from the origin to the central point (M/2,N/2) in the frequency space by exerting Fourier transform to f(x,y) multiplied by $(-1)^{x+y}$. By convolution, we have

$$f(x,y)\otimes g(x,y)\leftrightarrow F(u,v)G(u,v) \tag{6}$$

$$f(x,y)g(x,y)\leftrightarrow F(u,v)\otimes G(u,v) \tag{7}$$

which shows the convolution of two functions in the image space is the product of their Fourier transform in the frequency space.

4. Steps of Frequency Space Filtering

After giving the needed formulas, the steps of frequency space filtering will be shown as follows. For a given image f(x,y) with dimension M*N, it is first multiplied with $(-1)^{x+y}$ and exerted with Fourier transform, the result is recorded as F(u,v). F(u,v) is the Fourier transform spectrogram with low frequencies concentrated in the central point. When a filter H(u,v) is given, the product $G(u,v)=D(u,v)H(u,v)$ is the image signals in frequency space after filtering.Then exert IDFT on G(u,v) to transfer it back to the image space, and let $g(x,y)=IDFT(G(u,v))*(-1)^{x+y}$, the result g(x,y) shows the picture removed of unwanted image signals [12]. In the frequency plane, when the high frequency is filtered, the noise will be filtered in the physical plane accordingly, thus the image can be smoothed. In the following part, four typical filters will be introduced. In the formulas below, $D(u,v)$ is the distance between (u,v) and (M/2,N/2). For ideal low-pass filter (ILPF)

$$H(u,v) = \begin{cases} 1, D(u,v) \leq D_0 \\ 0, D(u,v) > D_0 \end{cases} \quad (8)$$

Here, D_0 is given number that can be set according to needs. When exert ILPF H(u,v) on the frequency space, low frequencies smaller than or equal to D_0 are remained and frequency signals that larger than D_0 are removed totally. As shown in Fig. 3, ILPF is a sharp filter. A serious drawback of ILPF is that the smaller D_0 is, the more serious Ringing phenomenon is, as shown in Fig. 4.

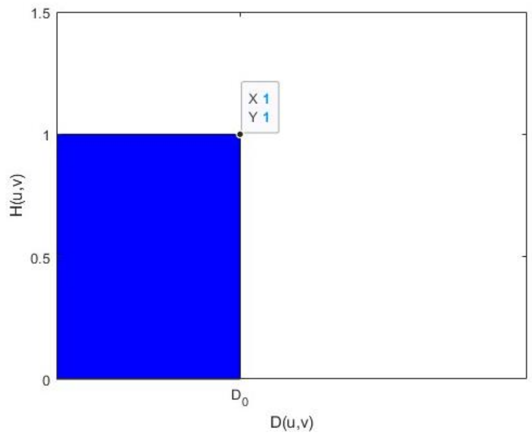

Fig. 3 Filter radial cross sections of ILPF.

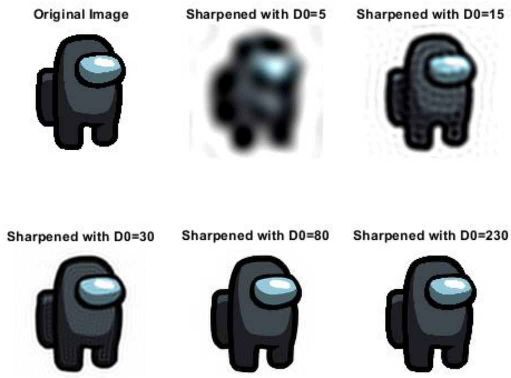

Fig. 4 Outcomes of Ideal filter

With the usage of butterworth low-pass filter (BLPF)

$$H(u,v) = \frac{1}{1+[D(u,v)/D_0]^{2n}} \quad (9)$$

where D_0 and n can be set freely, one obtains Fig. 5. It is shown clearly that there is no frequency cutoff point in BLPF, so BLPF is smoother than ILPF and has alleviated Ringing phenomenon. When n is large, BLPF is close to ILPF; the smaller n is, the smoother BLPF is. Fig. 6 shows that when D_0=5, BLPF with n=5 and n=20 have serious ringing phenomenon and in BLPF with n=1 there is no ringing phenomenon; and the processed image through BLPF with n=20 is like that through ILPF.

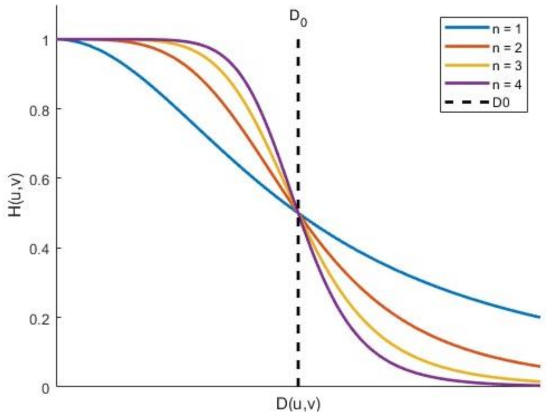

Fig. 5 Filter radial cross sections of orders 1 through 4

Fig. 6 Outcomes of BLPF when D_0=5

Based on Gauss low-pass filter (GLPF):

$$H(u,v) = e^{-D^2(u,v)/2D_0^2} \tag{10}$$

we obtain Fig. 7 where GLPF is the smoothest in the three filters. Ringing phenomenon can be eliminated totally in GLPF.

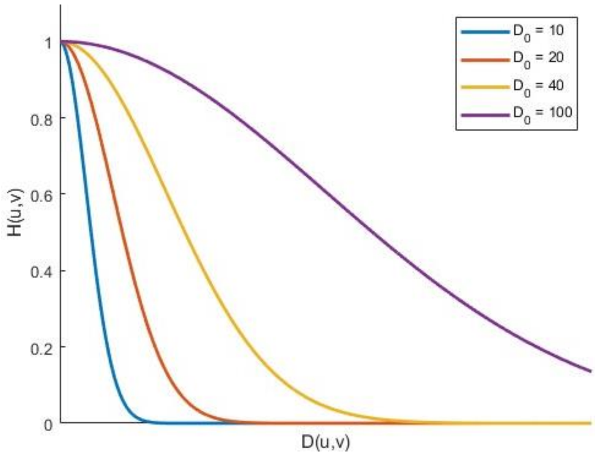

Fig. 7 Filter radial cross sections for various values of D_0.

Sharpening an image is enhancing the edge and details of the image. To sharpen an image, a high-pass filter is needed to remove low frequencies in frequency space. A high-pass filter can be achieved by the formula:

$$H_{HP}(u,v) = 1 - H_{LP}(u,v) \tag{11}$$

Here, $H_{LP}(u,v)$ is a low-pass filter. The image of a sharpened edge is shown in Fig. 8 and Fig. 9.

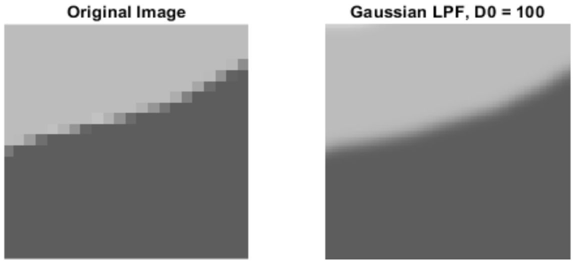

Fig. 8 The sharpening of certain edge by GLPF.

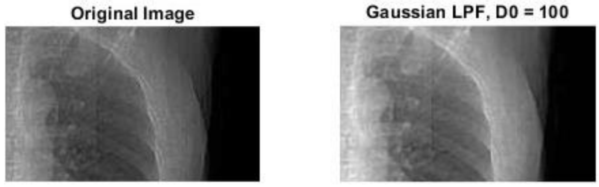

Fig. 9 An original CT image and the image through GLPF.

5. Applications

Medical image processing has become an important branch of digital image processing. The complexity of human tissue can have adverse effects on the quality of medical images, one example is blurred edge details. The unclear edges will greatly affect the diagnosis and treatment of diseases by doctors. So medical image enhancement is an extremely important part of image processing. Through a high-pass filter, the edge details can be made clearer and thus help the diagnosis of diseases. The image from a satellite shows the details of a region. Through a low-pass filter, the insignificant details can be weakened and the general characteristics like the outline of the image are left clearly, so the analysis of important characteristics can be simplified. The photo of a person's face often has wrinkles and spots. A low-pass filter can help clear flaws on the face thus make a sharp original image smooth and soft. When printing and copying texts, there are always broken characters. Low-pass filter can help do the pretreatment of recognition of broken characters, by making the edge of characters smoother, the Fracture point can be continued thus the characters will be easier to recognize.

6. Limitations & Future Outlooks

As it mentioned before, a significant drawback of ILPF is that ringing phenomenon. In Fig. 4, the issue remains very pronounced when D0=30. In addition, this problem persists when D0=80 and D0=230 after zooming into the image. Therefore, ILPF approach is not suitable for application in most scenarios. As for BLPF, ringing start to show up as the order of the filter increase. According to Fig. 6, the outcomes from Butterworth filter with n=20 is like that of ideal filter, both have relatively large ringing outline. Furthermore, after keeping n=2 and increasing the value of D0, the edges of the object inside the image have become more smoothed and there is no ringing phenomenon. In this case, it is possible to apply BLPF to sharpen the images to remove the edge aliasing. However, parameters D0 and n should be chosen adequately to obtain good performance and this approach could be subjective. In this case, GLPF could be the best way to process the images as there is only one D0 to modify and ringing phenomenon could be eliminated completely. Meanwhile, computational complexity is another issue to consider about. It would take longer time to process the image if the order of BLPF increase because calculating the filter response intricate the mathematical operations. What's more, in real-world application, the images are often rich in detail and complexity. It would usually be difficult to apply traditional filter in these conditions. For instance, while filters aim to enhance an image or reduce noise, they may remove important details of the image, especially in the sharpening process. The current study explored the effectiveness and limitations of various filters in image processing, which lay the foundation of advanced technologies to improve the performance of image sharpening and enhancement. For example, with the development of quantum computing and neural networks, the accuracy and speed of image processing will have a significant improvement, which make it possible to compute the complex mathematical operations of BLPF. However, as images and videos continue to grow in resolution and data density, greater challenges like real-time processing of virtual 3D object surfaces.

In the future, artificial intelligence and deep learning can also be applied to determine the parameters of BLPF and GLPF automatically so that the optimal result can be obtained. In addition, the improvement in image processing procedure can also be achieved by integrating Butterworth and Gaussian filters together. An appropriate filter and parameters can be chosen based on the local characteristics of the image, for instance, sharpening filters may be more efficiency to edge regions, while smoothing filters might be better suited for uniform areas, which can incorporate the strengths of both filters to their maximum potential.

7. Conclusion

In essence, this research has carried out a thorough exploration into the fundamental aspects of Fourier transform and related application into image smoothing and sharping, which are the essential parts of DIP. The Fourier transform is delved deeply into including its definition, translation and convolution. Simultaneously, the study has comprehensively examined the steps of frequency space filters, revealing the characteristics and applicability of both low-pass filters and high-pass filters in noise elimination. The implications of this research extend beyond theoretical understanding, directly influencing the optimization of image processing in domains like medical diagnostics and satellite picture. As one acknowledges the limitation of these picture processing techniques, this study lays the groundwork for future research and provides a concise guide for practitioners navigating the intricate landscape of image enhancement in the digital era.

Author Contribution

All the authors contributed equally and their names were listed in alphabetical order.

References

[1] Makandar A, Bhagirathi H. Image enhancement techniques using highpass and lowpass filters. International Journal of Computer Applications, 2015, 109(14): 12-15.

[2] Dogra A, Parvinder B Image sharpening by gaussian and butterworth high pass filter. Biomedical and pharmacology journal, 2014, 7(2): 707-713.

[3] Cristina P B, Cristina J, Alberto C, Samuel M. Graph-based methods for simultaneous smoothing and sharpening of color images. Journal of Computational and Applied Mathematics, 2019: 380-395.

[4] Ghani H A, Malek M R A, Azmi M F K, et al. A review on sparse Fast Fourier Transform applications in image processing. International Journal of Electrical & Computer Engineering (2088-8708), 2020, 10(2).

[5] Shutko V, Tereshchenko L, Shutko M, et al. Application of spline-fourier transform for radar signal processing. 2019 IEEE 15th International Conference on the Experience of Designing and Application of CAD Systems (CADSM). IEEE, 2019: 1-4.

[6] Saeed S, Abdullah A, Jhanjhi N Z. Implementation of fourier transformation with brain cancer and CSF images. Indian Journal of Science and Technology, 2019, 12(37): 37.

[7] Veerasingam S, Ranjani M, Venkatachalapathy R, et al. Contributions of Fourier transform infrared spectroscopy in microplastic pollution research: A review. Critical Reviews in Environmental Science and Technology, 2021, 51(22): 2681-2743.

[8] Baker M J, Trevisan J, Bassan P, et al. Using Fourier transform IR spectroscopy to analyze biological materials. Nature protocols, 2014, 9(8): 1771-1791.

[9] Hahn A, Vogel H, Andó S, et al. Using Fourier transform infrared spectroscopy to determine mineral phases in sediments. Sedimentary geology, 2018, 375: 27-35.

[10] Ciulla C. Inverse Fourier transformation of combined first order derivative and intensity-curvature functional of magnetic resonance angiography of the human brain. Computer Methods and Programs in Biomedicine, 2021, 211: 106384.

[11] Zhao S. Research on objective evaluation method of image quality based on Fourier transform domain. Shandong University of Finance and Economics, 2014.

[12] Li F. Improvement and application of X-ray medical image algorithm based on Retinex theory. Capital Normal University, 2014.