Optimal Investment in Electricity Generation under Mean-Variacne Criteria

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Abstract. This study introduces an application of mean-variance optimization, a common risk management technique in financial markets, to the New England area electricity market. The research optimizes portfolio selection in energy markets, treating 24 hourly intervals as distinct assets, and utilizes historical data from ISO New England hourly locational prices from 2022 to 2023. The study demonstrates the application of mean-variance optimization model across cases for hydroelectric and gas turbine combined power plants, providing generation companies with strategies to optimize their portfolio selection. This approach allows companies to make informed decisions about when and how to sell their power, aligning with their risk and return preferences. The research findings suggest that this optimization method can offer a systematic framework for generation companies to navigate the volatile electricity market and manage the risk and return.

Keywords: Mean-variance; portfolio theory in energy sector; locational marginal pricing (LMP).

1. Introduction

In the intricate scope of the electricity market, generation companies face the challenge of navigating significant price volatility. Electricity differs a lot from other commodities due to its unique characteristics: it is produced and consumed instantaneously, cannot be economically stored on a large scale, and its transmission is subject to the physical constraints of the grid infrastructure. These distinctive characteristics lead to an unmatched degree of price volatility comparing with other commodity markets, making efficient financial planning and risk management not merely advantageous but vital for the sustainability and prosperity of these enterprises.

The nature of the electricity market has necessitated the adaptation and application of a range of risk management techniques that are traditionally utilized within the financial sector. These methodologies, which encompass hedging, portfolio optimization, risk measurement, and asset valuation, have become indispensable tools for managing the complexities inherent in the market. The literature in this domain is rich, with numerous studies dedicated to applying these financial techniques to the electricity sector's unique requirements. Notably, Liu and Wu have investigated strategies for financial risk management to enhance economic returns and diminish associated risks [1], while Jain and Srivastava have integrated risk assessment and simulation models into negotiation strategies [2]. Additionally, Zhou et al. have applied portfolio theory to develop decision systems for energy purchases in spot markets [3].

Among the various risk management approaches, portfolio optimization refers to a strategic process for selecting efficient portfolios that aim to maximize returns while concurrently minimizing risk, grounded in the principles of Modern Portfolio Theory (MPT). The groundbreaking work of Harry Markowitz in 1952, which introduced the mean-variance optimization and later earned him a Nobel Prize, has been instrumental in shaping financial markets. Markowitz's efficient frontier is a cornerstone concept that guides investors in crafting portfolios that judiciously balance expected returns with associated risks [4]. Following in these footsteps, researchers such as Fazıl Gökgöz and Mete Emin Atmaca have worked to reconcile traditional financial optimization models with the specific demands of electricity generation companies [5]. This paper endeavors to extend the application of mean-variance optimization to meet needs of electricity generation companies in the New England area in the U.S. By scrutinizing real-time hourly locational marginal prices (LMPs), it aims to construct an optimization model that adeptly determines the weights of hourly risky assets and develops optimal portfolios, providing a basic approach to portfolio construction in the volatile context of electricity trading.

Section 2 presents an overview of the basic electricity market background in the New England area, including its pricing mechanisms. Section 3 delves into the core principles of Markowitz mean-variance optimization model, setting the stage for its application to the New England area electricity market. This section also explains the adaptation of the model within the context of the electricity market. Section 4 illustrates the results of applying the mean-variance model to the electricity market, using graphical and tabular representations to convey findings. Finally, Section 5 draws conclusions from the study.

2. Market structure of the New England area electricity market

The New England electricity market operates on a structure that integrates both capacity and energy markets to ensure a reliable supply and efficient trading of electricity. The capacity market, particularly the Forward Capacity Market (FCM), is designed to guarantee future electricity availability by compensating power generators for maintaining a certain capacity to produce a specified quantity of electricity, irrespective of actual generation [6]. This system primarily relies on mechanisms such as capacity auctions and bilateral contracts to establish pricing.

Conversely, the energy market is responsible for the actual trading of electricity and is divided into the day-ahead and real-time markets, forming a comprehensive "multi-settlement" system. The Day-Ahead Energy Market enables participants to hedge against price fluctuations by committing to electricity transactions a day in advance. Meanwhile, the Real-Time Energy Market responds to the immediate demands of the operating day, where electricity is traded dynamically.

A critical component of the energy market is the Locational Marginal Pricing (LMP) system. LMP reflects the cost of delivering electricity to different locations, taking into account various factors such as demand patterns, generation capacity, and transmission network limitations. In an unconstrained and lossless network, LMPs would be uniform, representing the cost to supply the next increment of load. However, real-world constraints like transmission and reserve constraints lead to diverse LMPs across the grid. Additionally, even in scenarios where the lowest-cost electricity is widely accessible, the marginal costs due to physical losses can cause LMP disparities. Given these complexities, it is crucial to note that all contracts carry inherent risks, and a robust risk management strategy should be consistently applied across all pricing systems.

3. Methodology

3.1. Foundations of Modern Portfolio Theory and Mean-Variance Optimization

The field of financial risk management primarily focuses on two principal strategies to reduce investment risk: hedging and diversification. Diversification operates on the premise that a portfolio comprising a wide array of assets is inherently less risky than one focused on a single asset class. This could involve diversifying across various sectors, incorporating a mix of assets, or holding multiple foreign currencies. The essence of diversification is that it can potentially maintain a portfolio's expected return while diminishing its risk.

The Modern Portfolio Theory (MPT) builds on this concept by revolutionizing the principle of diversification in investment. Developed by economist Harry Markowitz in his seminal 1952 paper, MPT formalizes the approach to investment diversification, underscoring the importance of considering the relationships (correlations, co-movements) between portfolio assets [4]. Markowitz's framework, known as the mean-variance framework, became a cornerstone in the field of financial economics.

Elaborating on his theoretical insights, Markowitz's "Portfolio Selection" elucidated the proposition of employing past variance of asset prices as a predictive metric for future risk. This paper also introduced the concept of diversifying investments to optimize returns while minimizing risk which culminated in the concept of Mean-Variance Optimization. For his pioneering contributions, Markowitz was awarded the Nobel Memorial Prize in Economic Sciences in 1990, sharing the accolade with distinguished economists Merton H. Miller and William F. Sharpe.

3.2. Portfolio Management in Mean-Variance Model

The mean-variance optimization is a foundational mathematical framework designed for constructing a portfolio of assets. Its central aim is to either maximize expected return for a preset level of risk or minimize risk for a desired expected return. Through this objective, a set of all possible portfolios can be obtained to build efficient portfolios for investors seeking to optimize their investment portfolios. This goal can only be achieved under the basic assumption that investors with their risk-aversion level merely select their portfolios based on the mean-variance criteria. The model's efficacy is contingent on certain foundational assumptions:

Investors are risk-averse, preferring less risk for a given level of expected return.

The expected returns, variances, and covariances of the assets are known and remain constant over the investment horizon.

There are no taxes or transaction costs.

To implement this model, investors can define the expected return and variance of the portfolio calculated based on the asset weights, formalized as:

$$E(r_p) = \sum_{i=1}^{N} W_i r_i, \tag{1}$$

$$\sigma_p^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} W_i W_j \sigma_{ij.} \tag{2}$$

Here, N represents the number of assets in the portfolio, W_i represents the weight, and r_i and σ_{ij} denoting the expected return and covariance between assets i and j, respectively. σ_p^2 is the variance of the portfolio.

The next step is to identify efficient portfolios, which are the ones that offer the least risk for a given level of expected return. This can be achieved by the minimization of portfolio variance (risk), including finding the combination of asset weights that results in the lowest possible variance or standard deviation of the given expected portfolio's returns. This is achieved by minimizing the portfolio variance (or risk) subject to certain constraints:

Ensuring the portfolio meets the targeted return.

The sum of asset weights equals one, implying the full budget is invested.

Asset weights remain non-negative, eliminating the possibility of short-selling.

Under the above objective and constraints, the basic mean-variance optimization model can be presented in basic equation as:

$$Min.(\sigma_p^2) = \sum_{i=1}^{N} \sum_{j=1}^{N} W_i W_j \sigma_{ij}, \tag{3}$$

s.t.

$$E(r_p) = \sum_{i=1}^{N} W_i r_i = r_e\ , \tag{4}$$

$$\sum_{i=1}^{N} W_i = 1, \tag{5}$$

$$W_i \geq 0\ \ \forall i \in [1,2, \dots, N]. \tag{6}$$

The portfolios derived from these constraints form the efficient frontier, representing the best possible risk-return combinations. Investors can further refine their choices by employing a utility function that matches their risk tolerance and financial objectives. This function expresses the trade-off between risk and return:

$$U = E(r_p) - \frac{1}{2} A \sigma_p^2. \tag{7}$$

In this equation, U represents the portfolio's utility, R its return, and σ its variance. The parameter A indicates the investor's risk aversion; a higher value implies a greater aversion to risk.The last term, represents the risk penalty, where coefficient ½ as a scaling convention. As risk (variance of returns) increases, the utility decreases. The risk penalty is proportional to the variance of returns, reflecting the idea that higher risk reduces the investor's satisfaction or utility. Generally, $A = 3$ is used to represent an average risk aversion. Combining information from the utility function, the optimum portfolio for investors with risk aversion parameter A can be found through the following computation:

$$U = E(r_p) - \frac{1}{2} A\sigma_p^2, \tag{8}$$

s.t.

$$\sum_{i=1}^{N} W_i = 1, \tag{9}$$

$$W_i \geq 0 \ \ \forall i \in [1,2, \dots, N], \tag{10}$$

where

$$E(r_p) = \sum_{i=1}^{N} W_i r_i, \tag{11}$$

$$\sigma_p^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} W_i W_j \sigma_{ij}. \tag{12}$$

Through these constructions the mean-variance optimization model is an effective tool in financial planning, aiding investors in crafting portfolios that align with their financial goals and risk appetite.

3.3. Identification of Risky Assets in Electricity Market

The adaptation of mean-variance optimization within the energy sector necessitates the determination of risky assets first. This research is based on an analysis of real-time hourly locational marginal prices (LMPs) sourced from Salem, Massachusetts, commencing in August 2022. We may utilize this data, along with the average operating expenses for different power plants (hydroelectric power plant and gas turbine power plant), to build risky assets. The LMPs' frequencies are assumed to have a normal distribution around the mean.

In the complicated energy markets, a strong correlation exists between prices and electricity consumption (demand), particularly in the real-time segment. This consumption typically follows a diurnal pattern, where demand is lower during the night when most people are asleep, and peaking in evening when people return home from work and use appliances, lighting, and heating or cooling systems. Despite certain oddities such as holiday seasons, weekends, and seasonal fluctuations, this consumption pattern stays largely consistent, which can be seen in Fig.1. The study used hourly demand characteristics of consumers to capture this pattern. As a result, the locational marginal prices (LMPs) of 24 hours a day are used to choose 24 separate risky assets for the portfolio.

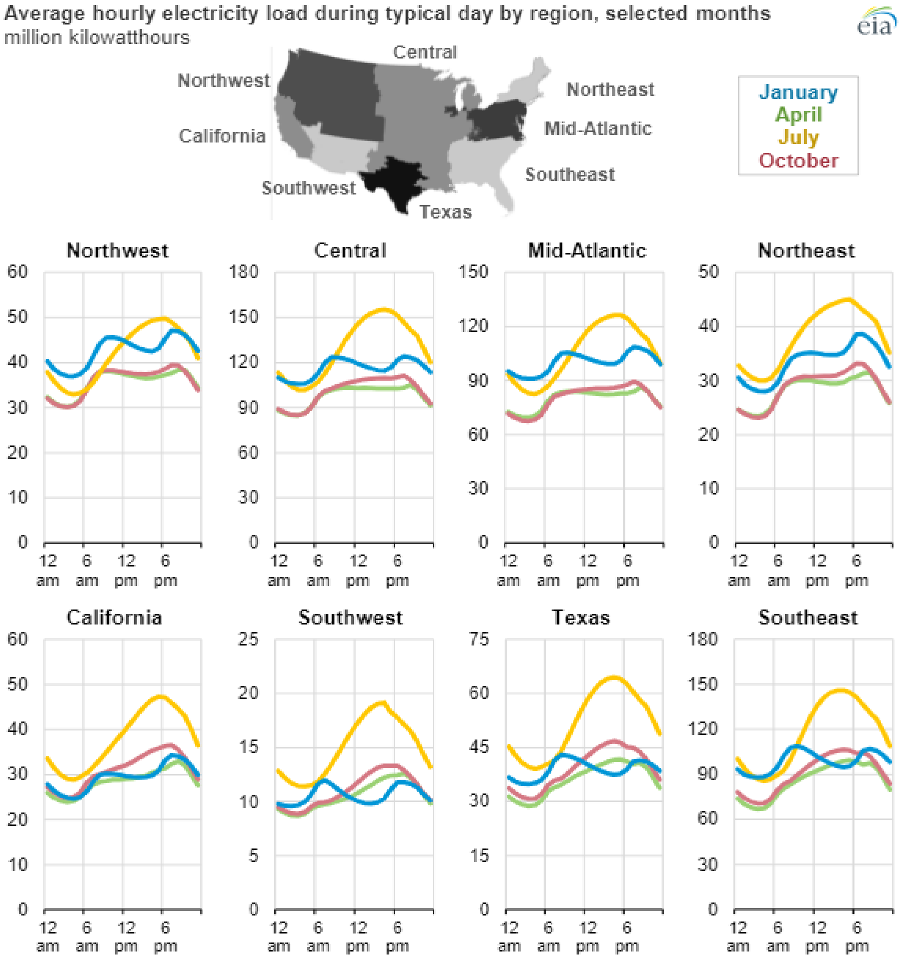

Fig. 1 Average hourly electricity load pattern

From: U.S. Energy Information Administration, U.S. Hourly Electric Grid Monitor

The analysis utilizes real-time hourly locational marginal prices (LMPs) spanning from August 2022 to August 2023. Datasets comprise 395 data points for each hour of the day.

$$\overrightarrow{f_1} = \begin{bmatrix} p_{1,1} \\ p_{1,2} \\ \vdots \\ \vdots \\ p_{1,395} \end{bmatrix}, \overrightarrow{f_2} = \begin{bmatrix} p_{2,1} \\ p_{2,2} \\ \vdots \\ \vdots \\ p_{2,395} \end{bmatrix}, \dots \dots, \overrightarrow{f_{24}} = \begin{bmatrix} p_{24,1} \\ p_{24,2} \\ \vdots \\ \vdots \\ p_{24,395} \end{bmatrix}. \tag{13}$$

Eq.(8) represents price vectors for each hour, denoted as $\overrightarrow{f_n}$ for $n \in [1,2, \dots 24]$ and $p_{n,m}$ corresponds to the LMPs, with $n = 1,2,\dots,24, m = 1,2,\dots,395$.

While traditional financial models for the rate of return might not seamlessly align with the electricity market's characteristics, Min Liu and Felix Wu have tailored a definition suitable for the Real-Time Energy Market, defining the rate of return as the ratio of spot prices minus generation cost to generation cost, adjusted for different power plants [1].

For the purposes of this study, a methodical approach of keeping the generation cost consistent is adopted. The author utilizes the average operational expenses of power plants as the cost variable in the calculations. Consequently, Eq.(9) furnishes the method to find out the rate of return. It's worth to acknowledge that diverse dynamic methodologies could be integrated into this optimization model, provided the rate of return formula remains unambiguous. The rate of return vectors, derived from the price vectors in Eq.(8), are then used to calculate the rate of return for each asset as showcased in Eq.(10).

$$r_{n,m} = \frac{a_{n,m} - C}{C}, for\ n \in [1,2, \dots,24], and\ m \in [1,2, \dots, 395] \tag{14}$$

$$\overrightarrow{r_1} = \begin{bmatrix} r_{1,1} \\ r_{1,2} \\ \vdots \\ \vdots \\ r_{1,395} \end{bmatrix}, \overrightarrow{r_2} = \begin{bmatrix} r_{2,1} \\ r_{2,2} \\ \vdots \\ \vdots \\ r_{2,395} \end{bmatrix}, \dots\dots, \overrightarrow{r_{24}} = \begin{bmatrix} r_{24,1} \\ r_{24,2} \\ \vdots \\ \vdots \\ r_{24,395} \end{bmatrix}. \tag{15}$$

where C denotes the average generation cost. For the scope of this study, generation costs associated with two distinct power plant categories are chosen: combined Gas Turbine with small-scale power plants, and hydroelectric power plants. The study derived the real average operational expenses from data presented by the Federal Energy Regulatory Commission for major U.S. investor-owned electric utilities in 2022. These expenses are shown in Table 1. It's pertinent to note that due to the renewable essence of hydroelectric power plants, fuel costs are inconsequential.

Table 1. Average Power Plant Operating Cost in 2022

Power Plant Type	Operation ($/MWh)	Maintenance ($/MWh)	Fuel ($/MWh)	Total ($/MWh)
Hydro-electric	7.68	4.76	--	12.44
Gas Turbine combined and small-scale	2.20	2.36	38.72	43.28

Given the construction of our set of 24 risky assets, the average for both the rate of return and variance for each asset are calculated as shown in Eq.(11) and Eq.(12) respectively. For the purpose of constructing a mean-variance optimization model, it is also essential to establish the Variance-Covariance Matrix, which can be achieved utilizing Eq.(13) and Eq.(14).

$$\bar{r} = \frac{1}{395}\left(\sum_{m=1}^{395} r_{n,m}\right), \tag{16}$$

$$\sigma_n = \sqrt{\left(\frac{1}{395}\sum_{m=1}^{395}(r_{n,m} - \overline{r_n})^2\right.}\ , \tag{17}$$

$$\sigma_{xy} = \frac{1}{395}\sum_{m=1}^{395}(r_{x,m} - \overline{r_x})(r_{y,m} - \overline{r_y})\ \ , \tag{18}$$

$$\begin{bmatrix} \sigma_1^2 & \sigma_{1,2} & \cdots & \sigma_{1,24} \\ \sigma_{2,1} & \sigma_2^2 & & \\ \vdots & & \ddots & \\ \sigma_{24,1} & & & \sigma_{24}^2 \end{bmatrix}. \tag{19}$$

3.4. Construction of mean-variance optimization model

In this study, the standard mean-variance model is applied, signifying that electricity generation companies are positioned to exclusively sell electricity to the real-time hourly market. That is, there are only 24 distinct risky selling opportunities. Converting Eq.(3)-(6), the model can be mathematically represented as:

$$Min.\left(\sigma_p^2\right) = \sum_{i=1}^{24}\sum_{j=1}^{24} W_i W_j \sigma_{ij}, \tag{20}$$

Subject to the following constraints:

$$E\left(r_p\right) = \sum_{i=1}^{24} W_i r_i = r_e\ , \tag{21}$$

$$\sum_{i=1}^{24} W_i = 1, \tag{22}$$

$$0 \le W_i \le 1\ \ \forall i \in [1,2,\dots,24]. \tag{23}$$

It is ensured that weights remain nonnegative, considering that negative weights aren't inherently feasible since power generators cannot produce a negative amount of electricity. Within this model's framework, there's no upper boundary for electricity generation, implying that generators can sell an unlimited amount of electricity. However, in a practical setting, generation companies may have additional constraints, which are also applicable for further development of this standard model. Equipped with this data, we can illustrate an efficient frontier for each power plant, showcasing the possible portfolio selection.

Subsequently, to formulate the optimal portfolio across all feasible selections and factor in the risk aversion level of different generation companies, the utility function defined earlier in Eq.(7) is used. Here, the study aims to find the portfolio that yields the best utility value. The portfolio that yields the highest utility value is deemed optimal for the investor because it provides the best trade-off between expected return and risk, given the investor's risk aversion level. In a graphical context, utility functions can be represented by indifference curves on a risk-return plane. The ideal portfolio is identified at the point where the highest attainable indifference curve touches the efficient frontier. This approach facilitates the assembly of the optimization set through a series of equations:

$$Max. U = E(r_p) - \frac{1}{2} A\sigma_p^2, \tag{24}$$

such that

$$\sum_{i=1}^{24} W_i = 1, \tag{25}$$

$$0 \leq W_i \leq 1 \ \forall i \in [1,2,\dots,24], \tag{26}$$

where:

$$E(r_p) = \sum_{i=1}^{24} W_i r_i, \tag{27}$$

$$\sigma_p^2 = \sum_{i=1}^{24} \sum_{j=1}^{24} W_i W_j \sigma_{ij}. \tag{28}$$

In the scope of this study, the risk-aversion coefficient, denoted as “A”, signifies the mean risk-aversion level. While individual preferences cause “A” to fluctuate among investors, it's conventionally acknowledged to lie between 2 and 4. For the purposes of this study, an average value of 3 for “A” is adopted. Utilizing the specified utility function, this study computes the optimal portfolio for generation companies looking to sell across 24 risky assets.

4. Main Results

The mean-variance optimization model developed in the prior section is applied to two specific scenarios: Hydro-electric power plants with an average cost of C_H=12.44$/MWh and the combined gas turbine and small-scale power plants at a cost of C_G=43.28$/MWh.

4.1. Efficient Frontiers and Optimum Portfolios

The efficient frontier stands as a pivotal concept in modern portfolio theory. In the context of mean-variance optimization, the efficient frontier graphically represents all the possible portfolios that yield the least amount of risk for a given level of expected return. The efficient frontier is derived by plotting the expected returns of portfolios against their corresponding risk (standard deviations). The resulting curve, the efficient frontier, represents the boundary of feasible portfolios. Efficient frontiers possess distinct characteristics:

Portfolios on the efficient frontier are deemed "efficient" because they provide the best expected return for their risk level. There is no alternative portfolio that offers a higher expected return for the same or less risk.

In contrast, portfolios located inside the efficient frontier are considered "inefficient" because they result in lower returns for equivalent risk or involve greater risk for the same expected return.

The “vertex” point of the efficient frontier represents the portfolio with the lowest possible risk (variance). This portfolio is constructed without regard to expected returns, focusing solely on minimizing variance.

On the efficient frontiers, indifference curves visually depict utility functions. The intersection of the highest attainable indifference curve with the efficient frontier determines the optimal portfolio.

In this study, the efficient frontiers reveal the outcomes from applying the basic mean-variance model to the 24 risky assets. This model doesn't place any production limitation on hourly output. The "Global Min-risk Portfolio" represents the portfolio with the lowest variance (risk). The efficient frontiers and optimal portfolios of Hydroelectric power plants and Gas Turbine combined and small-scale power plants are depicted in Fig. 2 and Fig.3 respectively.

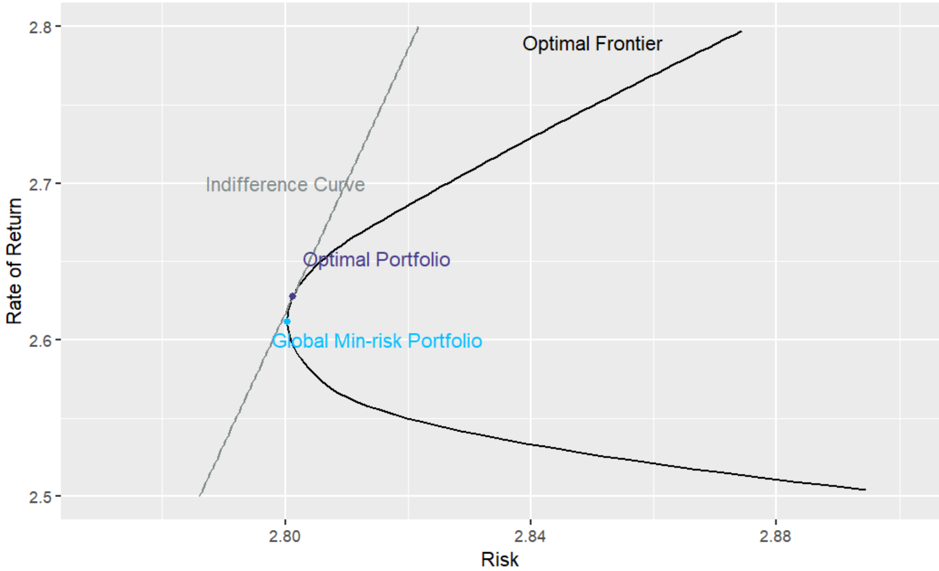

Fig. 2 Efficient frontiers with indifference curve for Hydro-electric power plant

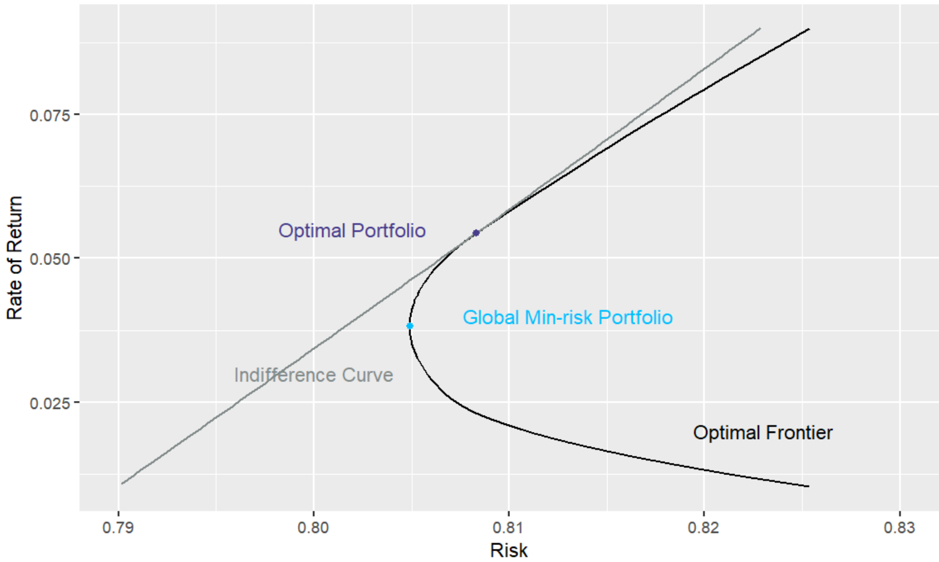

Fig. 3 Efficient frontiers with indifference curve for Gas Turbine combined and small-scale power plants

Armed with insights gleaned from these efficient frontiers, electricity generators can grasp the delicate balance between risk and return for the 24 risky assets. This understanding empowers them to formulate portfolios (in terms of electricity generation) that resonate with their risk aversion preferences and investment (profit) objectives by identifying points on the efficient frontiers.

4.2. Analysis

The results indicate that, despite differences in specific allocations, both Hour 1 and Hour 24 command significant weights in the optimal asset selection for both power plants. Under typical market conditions, Hours 1 and 24 can be considered the most profitable times for both power generator types. As generation costs rise for the gas turbine combined generator, its allocation for Hour 1 sees a corresponding increase.

Hydroelectric power plants offer more promising returns, but these come with higher volatility, as indicated by the greater standard deviation. On the other hand, gas turbine plants, though yielding lower returns, are characterized by smaller volatility. The reason behind this outcome can be deciphered by analyzing the intrinsic properties of these two power generation methods. Hydroelectric generators, despite their potential, have inherent stability issues and face more constraints when compared with gas turbine generators. This observation explains the prevalent use of gas turbines, even though they offer lower returns.

The result of our standard Markowitz mean-variance model proves the effectiveness of its application for all electric generators in the market and provides insightful results.

Table 2. Results of the optimal portfolio from standard mean-variance optimization model by different power plants

Optimal Portfolio	Hydroelectric power plant	Gas Turbine combined and small-scale power plants
Generation cost ($/MWh)	12.44	43.28
Hour 1	18.69821%	31.90102%
Hour 24	41.92169%	42.00237%
Expected rate of return	2.627921	0.05437548
Standard deviations	2.801296	0.8083079

The study constructs the standard mean-variance optimization model without imposing any added constraints. Given the unique nature of the electricity market, traditional short-selling strategies are not applicable since negative electricity production is infeasible. In reality, power generation companies have the flexibility to integrate additional constraints, transforming negative allocations into profit-making opportunities. One such strategy involves demand response programs, where consumers are paid to reduce their electricity usage during peak demand periods, effectively "selling" their negative consumption back to the grid. This system enables power generation firms to hedge against uncertainties, ensuring guaranteed returns. In addition, generators have the opportunity to introduce risk-free assets, such as contracts to provide a guaranteed return. In real-life scenarios, generators may also be confined under an upper production limit for the amount of electricity generated in each hour. As market conditions, asset returns, and investor preferences change over time, investors can adjust the utility function and the portfolio's risk-return profile to ensure that the portfolio remains optimal. Based on the basic model and construction in this study, electricity generators are able to further develop the mean-variance optimization model in order to fit their real and best situations.

5. Conclusion

This research has successfully adapted the principles of mean-variance optimization, a cornerstone of Modern Portfolio Theory, to the dynamic and unpredictable realm of the electricity market. Through a detailed analysis of real-time hourly locational marginal prices in the New England area, the study has crafted an optimization model that ascertains the weights of risky assets for two types of power plants: hydroelectric and gas turbine combined. This model is designed to reflect the diverse risk tolerances of various electricity generation companies, empowering them to forge optimal portfolios that resonate with their unique risk-return objectives.

The application of this financial optimization model to the electricity sector represents an advancement in risk management strategies for energy generation enterprises. It provides a systematic approach to balance the trade-off between expected returns and associated risks, a crucial consideration given the inherent price volatility in the electricity market. The findings of this research underscore the potential for sophisticated financial tools to enhance decision-making processes in energy generation and trading. Each hourly price occurrence is treated as a distinct risky asset, presenting 24 selling opportunities for electricity firms. Generators are thus equipped to adjust their strategies and to align their sales across different hours to optimize returns in accordance with their risk aversion preferences.

Looking to the future, this foundational model sets the stage for more intricate and nuanced applications. Factors such as fluctuating fuel prices, transmission congestion, maintenance considerations, capacity market contracts, transaction costs, and the broader financial market are ripe for incorporation into an expanded framework. Additionally, the exploration of other risk management methodologies, including hedging strategies, can further improve the risk management method for the New England area electricity market. In essence, this study not only demonstrates the practical application of mean-variance optimization to the New England area electricity market but also validates its efficacy, paving the way for continued innovation and enhancement in risk management practices within the energy sector.

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