Quantifying the Impact of the COVID-19 Pandemic on Steel Price and Production Trends Using Time Series Analysis

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Abstract. This paper aims to quantify the impact of the COVID-19 pandemic on steel price and production trends. By using time series analysis to identify discernible trends over the past decade, Seasonal ARIMA models and variations of Holt-Winters' methods were implemented. Historical data, from 2013 to 2020, was used as the training set for steel price and production values to forecast their post-pandemic predictions, from 2020 to 2023. The ratio of the mean absolute percentage errors (MAPEs) of the pre-pandemic model and its post-pandemic forecast against the actual data established a method of quantifying the observed impact on trends. Results showed that the post-pandemic test set values had MAPEs approximately 4-6 times greater than those for the pre-pandemic training set values when compared against the best-fitting forecast models. Providing quantitative evidence that the COVID-19 pandemic impacted steel price and production trends, this analysis enhances the understanding of the steel industry's resilience and responsiveness to external shocks.

Keywords: Steel price and production; seasonal ARIMA; Holt-Winters; time series analysis; forecasting.

1. Introduction

Steel is one of the most vital commodities in the world. From towering structures to automobiles and everyday gadgets, steel is a key resource with pervasive applications. Given the large-scale global impact of the COVID-19 pandemic, exploring its far-reaching effects on the steel industry is crucial. The overall aim of this investigation is to quantify the impact of the COVID-19 pandemic on steel price and production trends by conducting time series analyses over the past ten years.

Research into this topic is motivated by an attempt to compare forecasts derived from historical pre-pandemic trends to actual post-pandemic data. This will potentially be useful for industry analysts intending to anticipate changes in the steel market in the wake of external macroeconomic shocks. In this investigation, the focus was also expanded to include world production as well as the top three individual steel-producing countries – China, India, and Japan – to analyze potential nation-specific differences.

The outline of this paper will first review background literature on the steel industry, relevant industry impacts of the pandemic, and the suitability of time series analysis. Subsequently, data sources and explanations of the methodology used will be detailed. This is followed by the results of analyzing steel price and production trends over the past decade, quantifying the impact of the COVID-19 pandemic. This investigation's limitations will then be explained in the discussion. Finally, a summary of the results as well as conclusions from the analysis and industry implications will be detailed.

Regarding steel prices, the industry experienced an unprecedented surge in prices during the COVID-19 pandemic, surpassing records set during the 2008 boom. This dramatic rise was due in part to a sharp increase in the cost of raw materials, production processes, and energy [1]. Lockdowns and economic slowdowns severely impacted the manufacturing of steel-containing products, disrupting supply chains and reducing steel inventories [2]. Examining the historical context of this unique situation, the impact of the COVID-19 pandemic also caused significant supply chain disruptions, resulting in market dynamics that drove steel prices to record highs.

Regarding steel production, China has historically long strived to become the world's leading steel producer, being a goal that dates to the late 1950s. Consequently, China currently stands out as the most significant player in the global steel production landscape, having contributed to around half of the worldwide steel output over the past decade. However, the COVID-19 pandemic accelerated the dominance of Chinese steel production, boosting it to be responsible for 62% of total production at its peak, greatly impacting the post-pandemic global steel industry [3]. By contrast, global crude steel production outside of China fell sharply during the same period, especially in countries such as India and Japan. This disparity underscores China's substantial advantage in steel production during the pandemic, portraying a shift from pre-pandemic trends regarding the steel industry's dynamics on a global scale.

Lastly, the process of using time series analysis in forecasting commodity price and production trends is widely implemented across many industries [4]. This statistical approach involves analyzing data points observed at regular time intervals, providing a method of tracking the dynamic and often cyclical nature of the steel industry. In constructing models, it includes the detection of trends, autocorrelations, seasonal and random components, etc. Therefore, leveraging time series analysis enables the identification of patterns, seasonal variations, and long-term trends in steel price and production. It also facilitates the detection of external factors influencing these variables, highlighting valuable insights into anomalous or unexpected deviations from historical trends. Hence, this provides credible justification to the chosen methodology carried out for this investigation.

2. Methods

2.1.Steel Price

There are several unique variations of steel sold as a commodity on stock exchanges. Therefore, in order to remain representative, a focus on its most traded form – rebar steel – was chosen. More specifically, the RBT1 Rebar Steel COMB commodity denominated in U.S. Dollars, found through the Bloomberg Terminal market tracking software. This data is issued daily (on business trading days), leading to 2417 observations across the past ten years (30 September 2013 – 31 August 2023). Due to the raw data’s non-consecutive nature and high non-integer frequency, the data used to analyze and forecast steel price was chosen to be its monthly closing values. This resulted in a more manageable 120 total observations and enabled a more consistent dataset with a constant frequency of 12. By focusing on monthly closing steel price, this also reduced the required forecasting periods from 848 to 42, leading to potentially more accurate forecasts.

2.2.Steel Production

Regarding steel production, a focus on data from the International Iron and Steel Institute (IISI) of the World Steel Association was decided as it is the official international industry association for the sector. More specifically, the IISI Crude Steel Production sources for the total worldwide production and the top three individual producing nations (China, India, Japan). This data is issued monthly, leading to 120 observations over the past ten years (30 September 2013 – 31 August 2023) for each specified region.

2.3.Definition of Pre- and Post-Pandemic

For this investigation, the start of the COVID-19 pandemic has been defined as 11 March 2020, as COVID-19 was declared a pandemic by the WHO on this day [5]. Therefore, to simplify the segmentation of the data, values dated before 11 March 2020 are referred to as pre-pandemic. Similarly, values dated after 11 March 2020 are referred to as post-pandemic for semantic clarity. Though the spread of COVID-19 outbreaks differed in various regions across the globe, this definition provides consistency and comparability to the data analysis.

2.4.Forecasting Models

Seasonal ARIMA models and variations of Damped/Undamped Holt-Winters' Additive/ Multiplicative methods were fit against the pre-pandemic training set data for steel price and production respectively. These models were chosen for forecasting as they were the ones which produced the lowest training set mean absolute percentage errors and matched their respective training datasets the closest.

2.5.Mean Absolute Percentage Error

Mean Absolute Percentage Error (MAPE) is a commonly used metric to measure the accuracy of forecasting models. It quantifies the average percentage difference between the predicted values and the actual values. MAPE is particularly helpful for determining forecast accuracy in a form that is relatable to actual values and easy to understand. The formula for MAPE is as follows:

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{A_i - F_i}{A_i}\right| \times 100\% \quad (1)$$

Where A_i represents the actual values, F_i represents the forecasted values, and n is the number of data points. By displaying errors as a share of actual values, MAPE provides a comparative evaluation of prediction accuracy that makes it easier to comprehend the importance of variances, due to its percentage-based nature. MAPE is also scale-independent, enabling more straightforward comparisons for the errors in steel price and nations with different levels of steel production.

By selecting forecasting models based on the lowest training set MAPE, this ensures the chosen models are optimally accurate, enabling reliable predictions for the datasets in focus. Training set MAPE is calculated based on the model's performance when it is trained on the historical dataset, which is the pre-pandemic data. It assesses the accuracy of the model in capturing the patterns and fluctuations, due to inherent noise or minor variations, present in the training data. The model's corresponding test set MAPE is then calculated after using the trained model to forecast values. The model is applied to predict outcomes, and the accuracy of these predictions is evaluated by comparing them to the actual observed values during the post-pandemic period.

To quantifiably evaluate the impact of the COVID-19 pandemic on steel price and production trends, comparing the training set to the test set MAPE for each closest-fitting model is used, where the training set is the pre-pandemic data for forecasting, and the test set is the post-pandemic actual data. This can be illustrated by the following ratio:

$$\frac{Test\ Set\ MAPE\ (\%)}{Training\ Set\ MAPE\ (\%)} \quad (2)$$

If the ratio is significantly greater than 1, it indicates that the forecasting model performed worse on the test set compared to the training set. This could suggest that the post-pandemic actuals were not as accurately predicted by the chosen forecasting model as the pre-pandemic training set data. Therefore, this would provide quantitative evidence for the COVID-19 pandemic substantially impacting steel price and production trends. However, if the ratio is equal or close to 1, it could suggest that the model's performance on the training and test sets were similar, meaning that there were negligible differences in pre-pandemic and post-pandemic trends. If the ratio is less than 1, it implies that the forecasting model performed better on the test set than on the training set. Though it might seem counterintuitive, this outcome could indicate that the forecasting model was robust and generalized well to the new post-pandemic data.

3. Results and Discussion

3.1.Steel Price Plot

As Figures 1 & 2 show, the Seasonal ARIMA (1,1,3)(0,0,2)[12] model was chosen to forecast the pre-pandemic monthly closing values of steel price as it provided the most accurate training set MAPE among the fitted models.

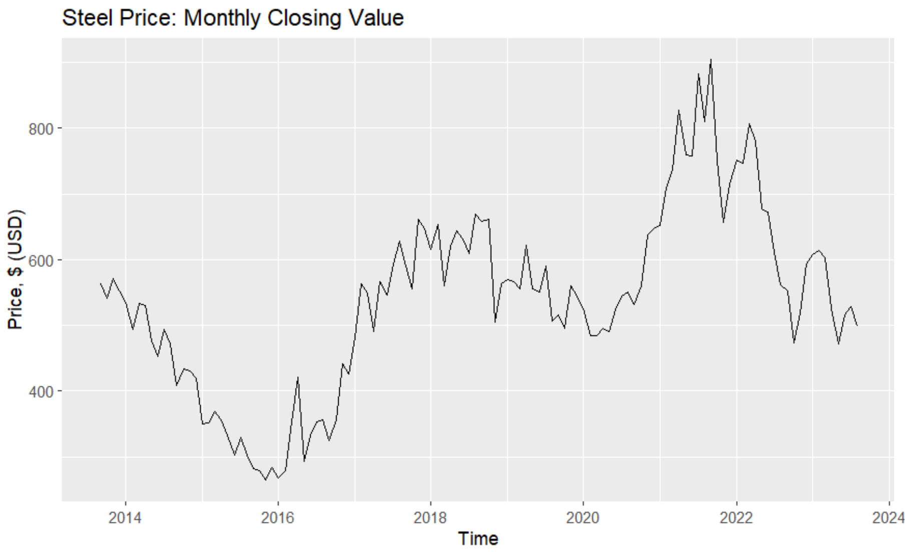

Fig. 1 Steel Price: Monthly Closing Values (Actuals)

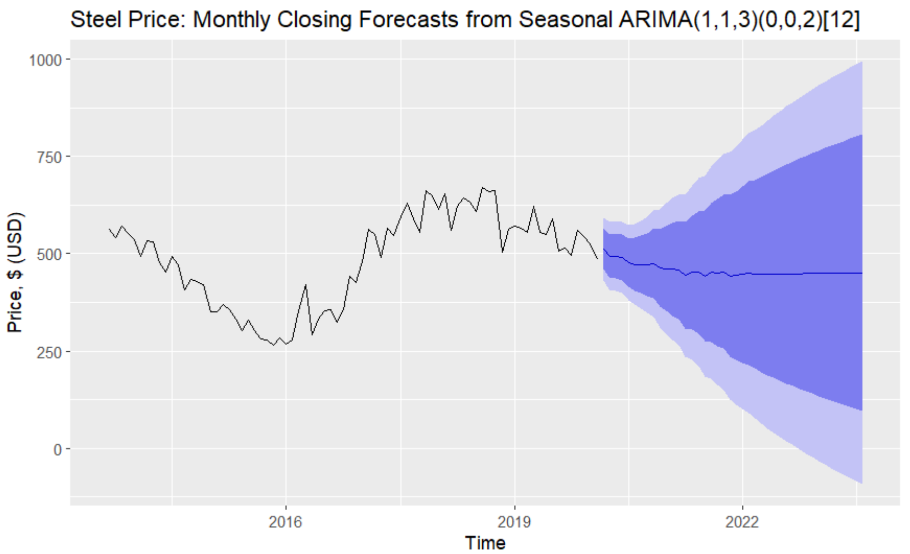

Fig. 2 Steel Price: Monthly Closing Values (Forecasts)

Autoregressive Integrated Moving Average (ARIMA) models are a versatile class of models which generally apply to stationary time series, whose statistical characteristics remain relatively constant [6]. Given that some seasonality is observable in the data, below is an explanation of the Seasonal ARIMA *(p,d,q)(P,D,Q)[m]* model parameters [7]:

p is the order of the autoregressive model (no. of lagged terms), *q* is the order of the moving average model (no. of lagged terms), and *d* is the number of differences required to make the time series stationary. Similarly, *P* is the order of the seasonal autoregressive model, *Q* is the order of the seasonal moving average model, and *D* is the number of seasonal differences applied to the time series. Lastly, the *m* parameter represents the seasonality or frequency of the model. For example, the differencing required to make the pre-pandemic monthly closing steel price training time series data stationary is found below:

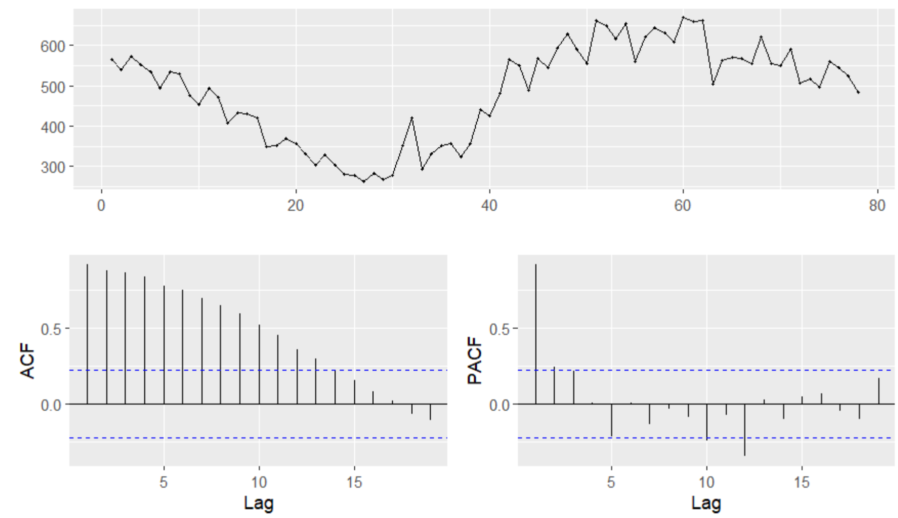

Fig. 3 ACF and PACF Plots of Pre-Pandemic Monthly Closing Steel Price

From Figure 3, there is a gradual decay in autocorrelations as the lags increase in the ACF plot, indicating that this is a non-stationary time series. This suggests that the past values are influencing the current values over a longer period. Since there is a gradual decrease in autocorrelations rather than a quick drop-off, this also indicates that the series has some underlying trend or seasonality that needs to be addressed.

In the PACF plot, there is a large spike at lag = 1, which indicates that there might be first-order differencing needed. Differencing involves subtracting the current value from the previous one. By differencing, this aims to remove the trend or seasonality that is causing the autocorrelation at lag = 1, making the series more stationary. The subsequent sharp decrease in the PACF for higher lags within the critical values suggests that after differencing, the direct influence of past observations diminishes quickly.

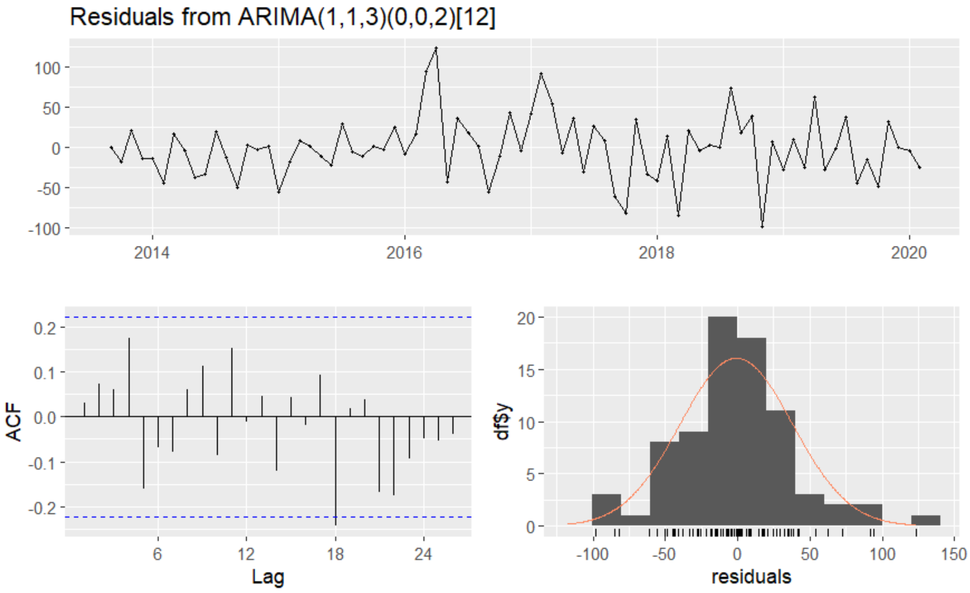

Fig. 4 Residuals from Seasonal ARIMA Model

Calculating for the other parameters as well, this led to the Seasonal ARIMA (1,1,3)(0,0,2)[12] model for forecasting monthly closing steel price. Furthermore, the p-value derived from the Ljung-Box test at 0.2417 is greater than the 5% significance level, meaning that the null hypothesis should not be rejected. Overall, observing Figure 4, this is determined to be the most effective forecasting model as its residuals also predominantly fall within the critical values and hence are consistent with white noise.

Table 1. Steel Price Forecast MAPEs

	Closest Fitting Training Set Model	Training Set MAPE (%)	Test Set MAPE (%)	$\frac{Test\ Set\ MAPE\ (\%)}{Training\ Set\ MAPE\ (\%)}$
Price	ARIMA(1,1,3)(0,0,2)[12]	6.043469	25.756041	4.261797

The ratio of test set to training set MAPE equal to approximately 4.26 is significantly greater than 1 and provides empirical evidence that the COVID-19 pandemic had a substantial impact on steel prices (Table 1). This is because the deviations between the model's forecasted steel prices and the actual prices were notably larger during the pandemic compared to the pre-pandemic period. This suggests that the pandemic disrupted the pre-existing trends and patterns in steel prices, making accurate predictions more challenging.

3.2.Steel Production

With regards to steel production, logarithmic transformations of base 10 were carried out to produce and analyze forecasts. By observation, the pre-pandemic steel production data was broadly found to be more closely following seasonal non-linear trends. Therefore, implementing logarithmic transformations aims to linearize the training data and stabilize the variance of the time series. As the training set pre-pandemic steel production data for different regions generally tended towards exponential growth or decay, applying a logarithmic transformation helped make the changes more proportional. This led to it being easier to interpret trends and patterns for the forecasts, as percentage changes became more uniform. Moreover, given that the pre-pandemic steel production values vary between thousands of "1000 metric tons" units, the standard logarithm of base 10 was decided to represent the data and forecasts.

3.2.1 World steel production

As Figures 5 & 6 show, the Damped Holt-Winters' Additive method was chosen to be the closest fitting training model for World steel production because it produced the lowest training set MAPE compared to the alternative methods.

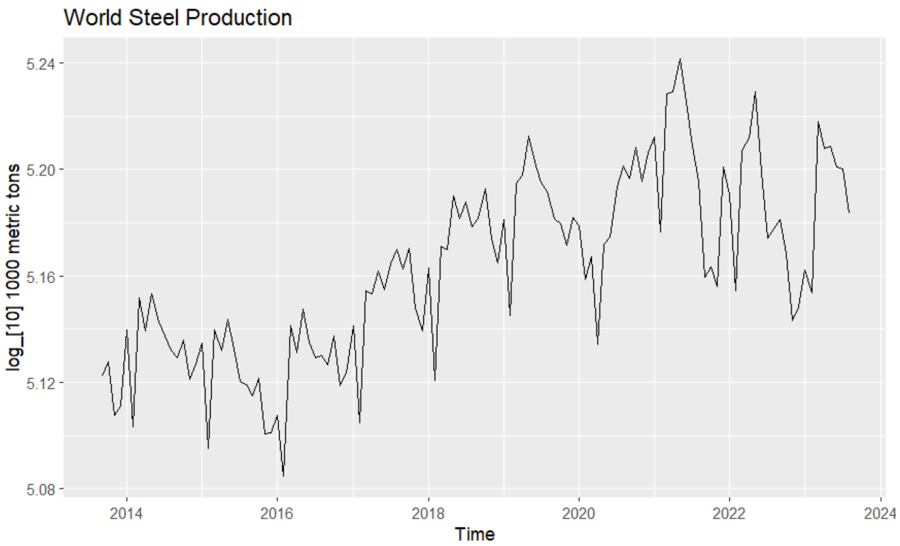

Fig. 5 World Steel Production (Actuals)

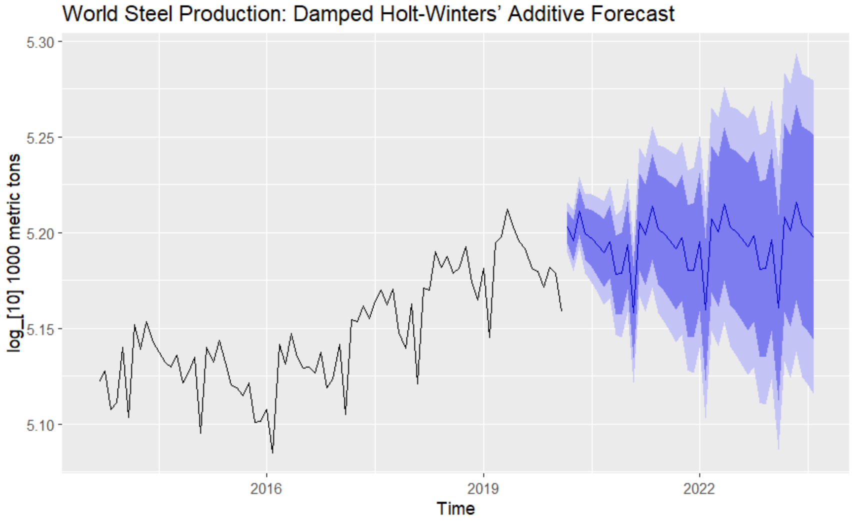

Fig. 6 World Steel Production (Forecasts)

The Damped Holt-Winters' Additive model expands upon Holt's exponential smoothing to capture seasonality, resulting in exponentially smoothed values for the trend of the forecast, with the additive method being more suited to approximately constant seasonal variations throughout the dataset. The seasonal component is expressed in absolute terms with the same units as the observed data. The level equation then corrects for seasonal variations by subtracting this seasonal component, which, over the course of a year, typically averages out to approximately zero [8]. Moreover, the damping parameter serves to lessen the impact of earlier observations as the prediction horizon moves further into the future.

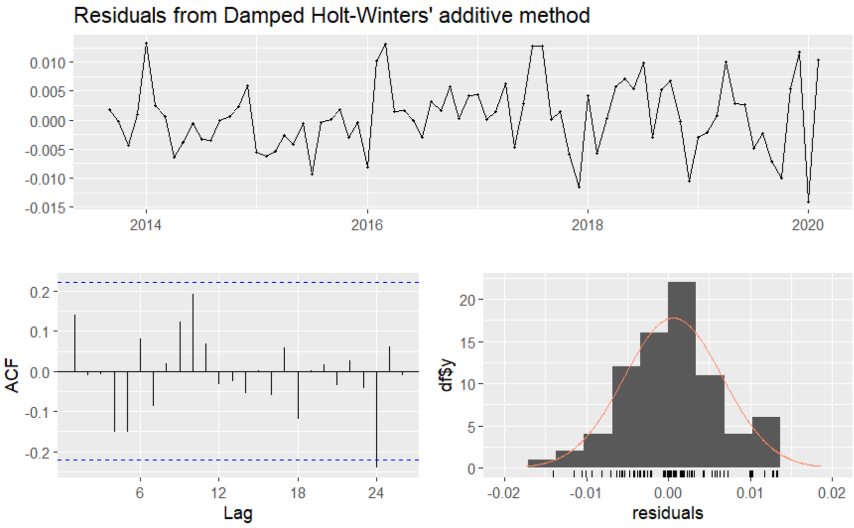

Fig. 7 Residuals from Damped Holt-Winters' Additive Model

From Figure 7, the ACF plot shows that its residuals generally fall within the critical values and are hence consistent with white noise. The histogram also shows that the residuals seem to be generally centered around zero. Moreover, the p-value of 0.6863 derived from the Ljung-Box test is greater than the 5% significance level, meaning that the null hypothesis should not be rejected and that this is a well-fitting model for world steel production.

Table 2. World Steel Production Forecast MAPEs

	Closest Fitting Training Set Model	Training Set MAPE (%)	Test Set MAPE (%)	$\frac{Test\ Set\ MAPE\ (\%)}{Training\ Set\ MAPE\ (\%)}$
World	Damped Holt-Winters' Additive	0.08909085	0.35791994	4.017471

The considerable rise in errors indicates a noticeable difference in the model's accuracy between pre- and post-pandemic values (Table 2). As the ratio of around 4.02 is substantially greater than 1, this suggests that the same model's ability to predict world steel production during/after COVID-19 significantly worsened compared to the pre-pandemic period. This, therefore, strongly implies that the pandemic had an influential impact on world steel production trends.

3.2.2 Chinese steel production

As Figures 8 & 9 show, the Holt-Winters' Additive method was chosen to be the closest fitting training model for Chinese steel production because it produced the lowest training set MAPE compared to the alternative methods.

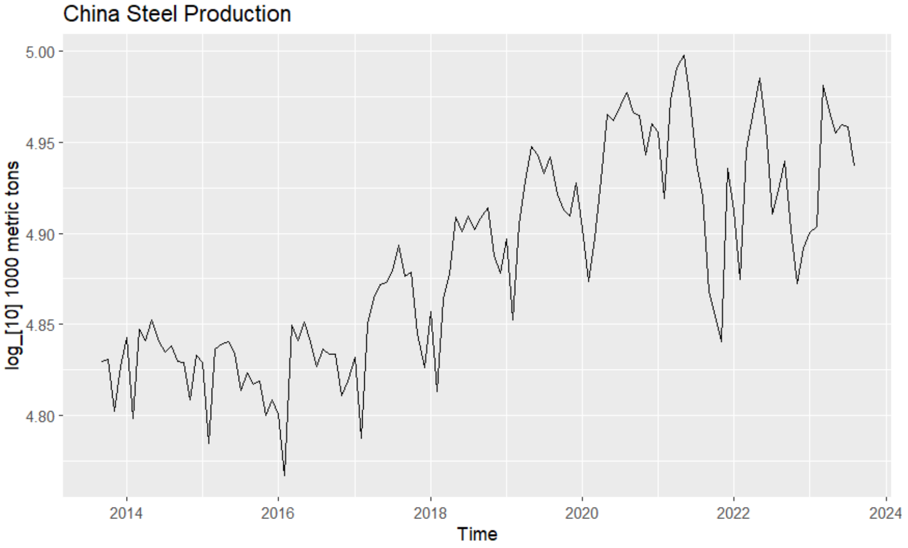

Fig. 8 Chinese Steel Production (Actuals)

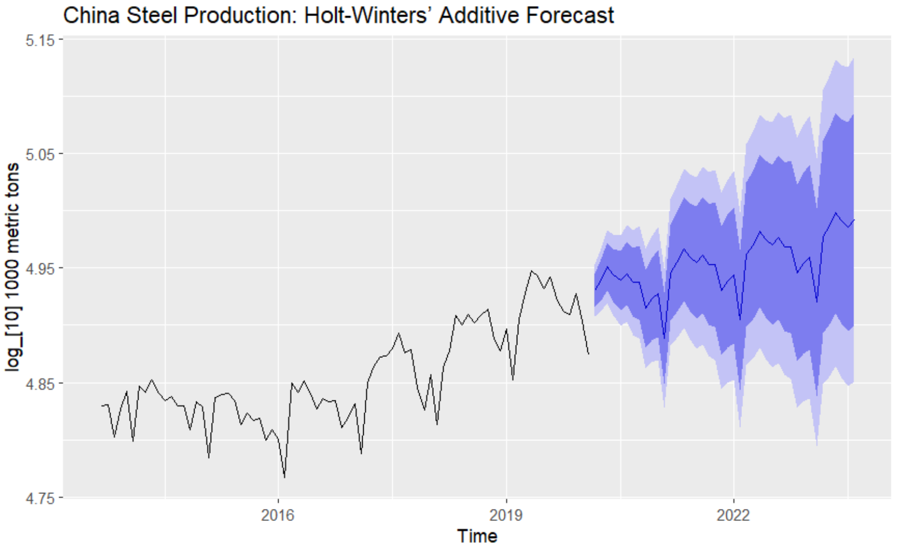

Fig. 9 Chinese Steel Production (Forecasts)

The Holt-Winters' Additive model expands upon Holt's exponential smoothing to capture seasonality, resulting in exponentially smoothed values for the trend of the forecast, with the additive method being more suited to approximately constant seasonal variations throughout the dataset [9]. This is in line with observing the shape of the pre-pandemic data for Chinese steel production.

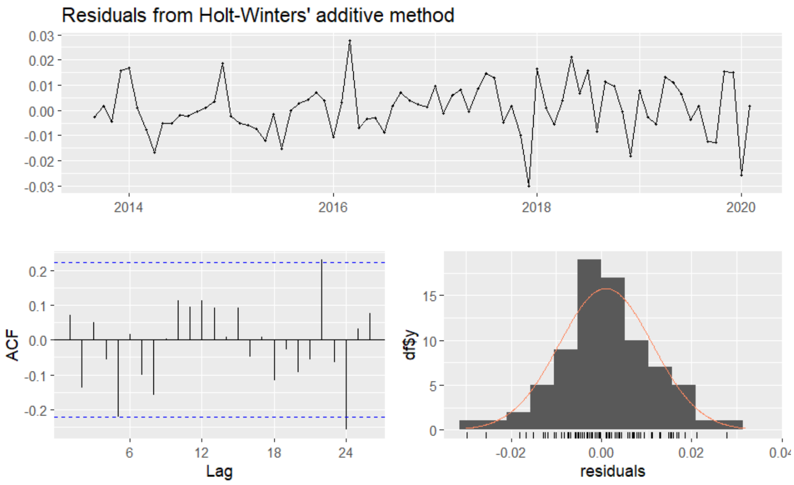

Fig. 10 Residuals from Holt-Winters' Additive Model

From Figure 10, the ACF plot shows that its residuals generally fall within the critical values and are hence consistent with white noise. The histogram also shows that the residuals seem to be generally centered around zero, showing a symmetrical distribution. Moreover, the p-value of 0.5347 derived from the Ljung-Box test is greater than the 5% significance level, meaning the null hypothesis should not be rejected and that this is a well-fitting model for Chinese steel production.

Table 3. Chinese Steel Production Forecast MAPEs

	Closest Fitting Training Set Model	Training Set MAPE (%)	Test Set MAPE (%)	$\frac{Test\ Set\ MAPE\ (\%)}{Training\ Set\ MAPE\ (\%)}$
China	Holt-Winters' Additive	0.1616134	0.7061366	4.369295

There is a significant rise in inaccuracies between the forecasting model and post-pandemic data for Chinese steel production (Table 3). By observing the test set to training set MAPE ratio of approximately 4.37, it is notably greater than 1, suggesting that there were significant disruptions to the actual post-pandemic data compared to the model's forecasted expectations based on pre-pandemic trends. Therefore, this strongly suggests that the COVID-19 pandemic significantly impacted pre-pandemic trends for Chinese steel production.

3.2.3 Indian steel production

As Figures 11 & 12 show, the Holt-Winters' Multiplicative method was chosen to be the closest fitting training model for Indian steel production because it produced the lowest training set MAPE compared to the alternative methods.

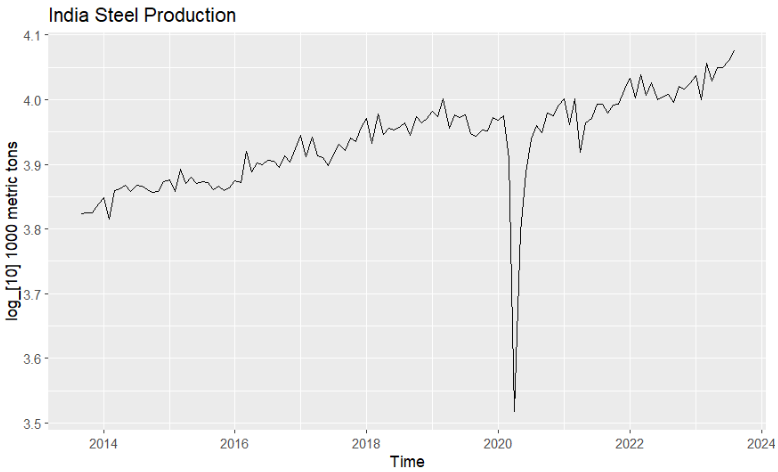

Fig. 11 Indian Steel Production (Actuals)

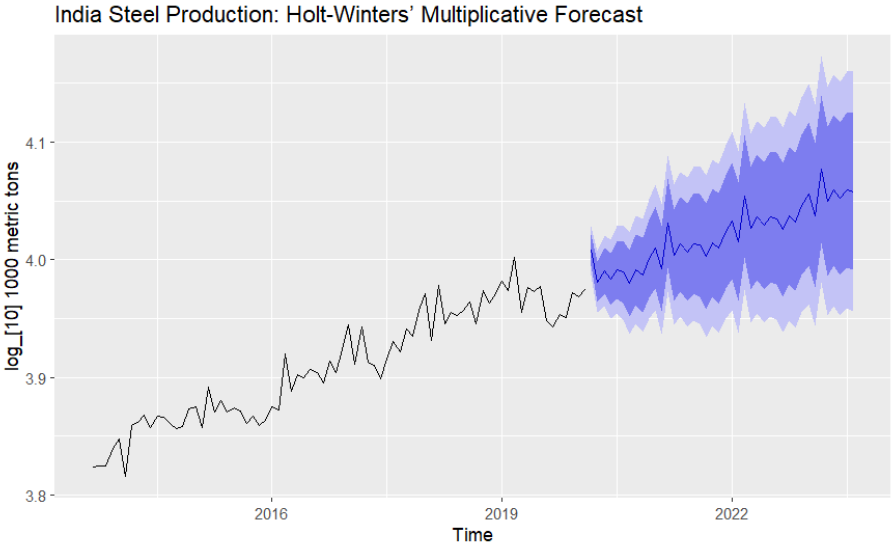

Fig. 12 Indian Steel Production (Forecasts)

The Holt-Winters' Multiplicative model is another widely used method for time series forecasting, particularly suited for data with a seasonal component with proportional changes in variations [10]. This aligns with the above observations of the pre-pandemic data for Indian steel production. Similar to the additive methods seen previously, this model also extends Holt's method to handle both broader trends and seasonality.

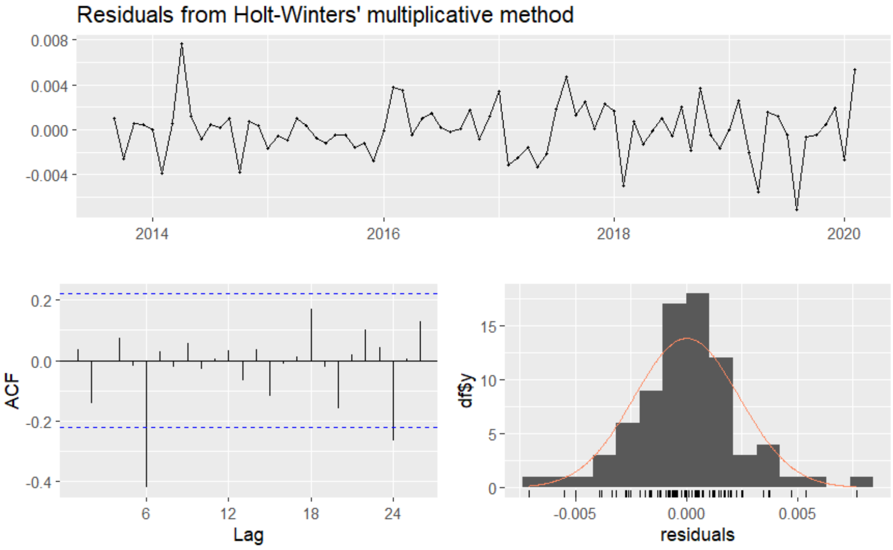

Fig. 13 Residuals from Holt-Winters' Multiplicative Model

From Figure 13, the ACF plot shows a spike at lag = 6, but it is not quite enough to be significant at the 5% level since the Ljung-Box test produces a p-value of 0.2091 > 0.05. Moreover, the histogram shows that the residuals seem to be centered around zero, which suggests it has a nearly symmetric distribution. Therefore, this is still a well-fitting model for Indian steel production.

Table 4. Indian Steel Production Forecast MAPEs

	Closest Fitting Training Set Model	Training Set MAPE (%)	Test Set MAPE (%)	$\frac{Test\ Set\ MAPE\ (\%)}{Training\ Set\ MAPE\ (\%)}$
India	Holt-Winters' Multiplicative	0.1721013	1.0669406	6.199492

Similar to the previous results, there is a substantial increase in MAPE between the actual post-pandemic data compared to its forecast, revealed by the test set to training set MAPE ratio equaling approximately 6.20, which is much greater than 1 (Table 4). Perhaps due to its much more drastic drop in production in mid-2020, the MAPE ratio for Indian steel production at 6.20 is also notably higher than that for world and Chinese steel production at 4.02 and 4.37 respectively, suggesting that post-pandemic deviations from the best-fitting Indian steel production model were more pronounced. Therefore, this strongly suggests that the COVID-19 pandemic significantly impacted pre-pandemic trends for Indian steel production.

3.2.4 Japanese steel production

As Figures 14 & 15 show, the Damped Holt-Winters' Multiplicative method was chosen to be the closest fitting training model for Japanese steel production because it produced the lowest training set MAPE compared to the alternative methods.

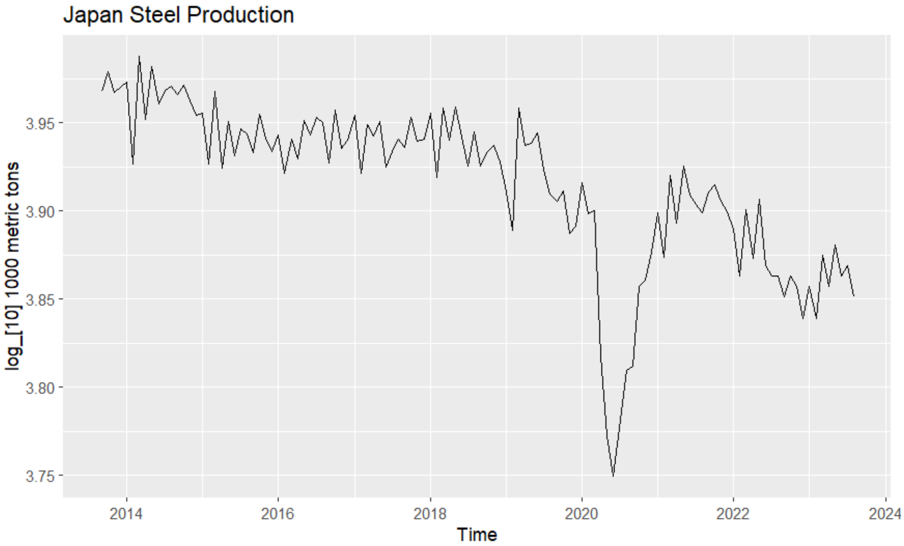

Fig. 14 Japanese Steel Production (Actuals)

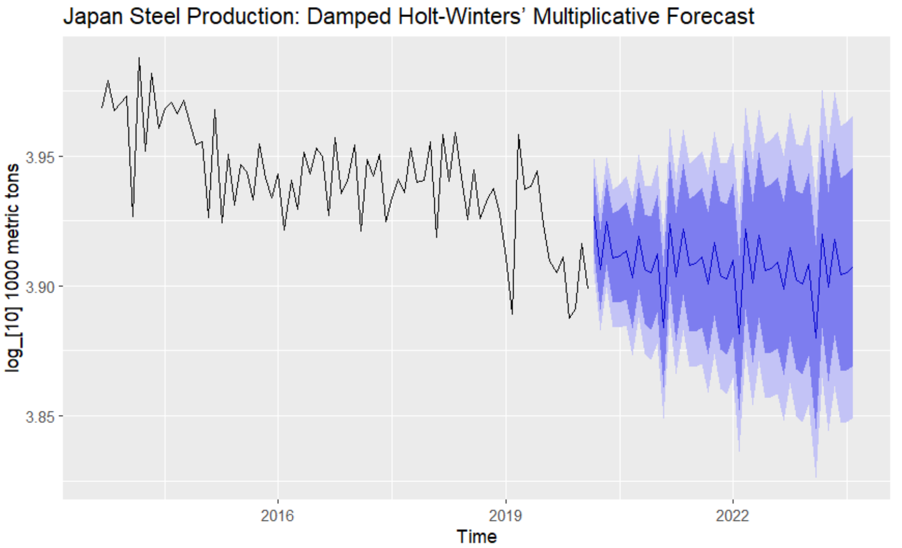

Fig. 15 Japanese Steel Production (Forecasts)

The Damped Holt-Winters' Multiplicative model is an extension of the Holt-Winters' Multiplicative Model seen previously, designed to provide more stable and damped forecasts by introducing a damping parameter [11]. This parameter aims to reduce the influence of past observations as the forecast horizon extends further into the future.

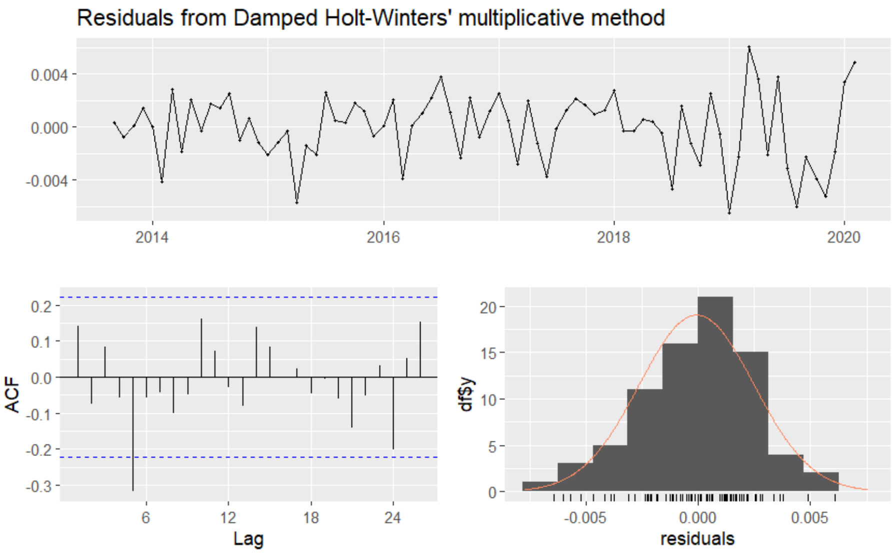

Fig. 16 Residuals from Damped Holt-Winters' Multiplicative Model

From Figure 16, the ACF plot shows a spike at lag = 5, but it is not quite enough to be significant at the 5% level since the Ljung-Box test still produced a p-value of 0.2512 > 0.05. Moreover, the histogram shows that the residuals seem to be centered around zero albeit with a slight negative skewness. Therefore, this is still an overall well-fitting model for Japanese steel production.

Table 5. Japanese Steel Production Forecast MAPEs

	Closest Fitting Training Set Model	Training Set MAPE (%)	Test Set MAPE (%)	$\frac{Test\ Set\ MAPE\ (\%)}{Training\ Set\ MAPE\ (\%)}$
Japan	Damped Holt-Winters' Multiplicative	0.2004086	1.0856630	5.417248

In line with the results of the previous analyses, there is once more a significant value for the test set to training set MAPE ratio at around 5.42 for Japanese steel production (Table 5). As this ratio is substantially larger than 1, this suggests that the actual post-pandemic values greatly differed from the model's forecasts based on pre-pandemic trends. With the second-highest MAPE ratio of 5.42 within the scope of this investigation, this could potentially be attributed to the similarly massive drop in production during mid-2020 seen in the Indian steel production data previously. Therefore, this strongly indicates that the COVID-19 pandemic had a quantifiable impact on existing trends in Japanese steel production.

3.3.Discussion

After analyzing how the COVID-19 pandemic affected steel price and production trends, it is crucial to evaluate the results critically and discuss other important factors, recognizing the complexity that underpins trends in the steel industry which go beyond the pandemic.

Though it was assumed that the global start date of the pandemic, for the purposes of this paper, occurred on 11 March 2020, this may not have accurately applied to all nations. For example, in November/December 2019, China reported earlier cases of COVID-19, underlining the need for careful consideration of regional variations [12].

Given that the analysis was mainly focused on changes in steel price and production trends during/after the COVID-19 pandemic, it's possible that factors such as cost variations, changes in policy, regulations, technological advancements, and taxation may have played an unconsidered role that also influenced the observed changes in trends. The dynamics of steel prices may have also been affected by shifts in the patterns of global demand, which are influenced by a wide range of economic and sociopolitical factors as well. As the nuances of these interactions were not closely considered, understanding these complex relationships might also be important to comprehend the trends and responsiveness of the industry.

Furthermore, steel production in some nations, namely Japan and India, showed precipitous drops in the middle of 2020 before quickly rebounding to largely follow previous pre-pandemic patterns after 2021. This reversal of early pandemic anomalies might illustrate the steel production industry's adaptability and resilience to deal with setbacks, emphasizing the credibility of the pre-pandemic forecasts.

This highlights the multifaceted and intricate nature of the steel industry, considering the potentially significant influence of factors other than the pandemic. This greater understanding may also be crucial for navigating the opportunities and challenges that lie ahead as the industry continues to change and adapt.

4. Conclusion

The results of this investigation provide ample evidence that the COVID-19 pandemic quantifiably impacted trends in steel price and production over the past decade. If trends were similar before and after 11 March 2020, then the ratio of actual post-pandemic test set MAPE to pre-pandemic training set MAPE should have been approximately equal to one. This is because the deviations between the forecast and the actual post-pandemic values would be similar to those between the model and the pre-pandemic training data. However, through this ratio, the analysis showcases that the post-pandemic test set values had Mean Average Percentage Errors (MAPEs) that were around 4-6 times greater than those for the pre-pandemic training set values when compared against the best-fitting forecast models. This provides quantitative evidence to suggest that post-pandemic data for steel price and production significantly differed from existing pre-pandemic trends.

Steel production across top-producing nations was also revealed to be highly seasonal, closely following variations of Holt-Winters' modelling methods in pre-pandemic data. This can be seen by the observation of their graphs, as well as their extremely low training set MAPEs of around 0.09-0.20%. Therefore, it can also be concluded that without the massive external shock of the COVID-19 pandemic, steel production was relatively predictable and consistent with annual seasonality but still followed long-term trends.

Steel price, however, was revealed to be more volatile. This can be noted by observing the greater randomness found in its graph as well as its significantly higher training set and test set MAPEs, compared to those for steel production, at approximately 6% and 26% respectively. This greater volatility could possibly originate from its nature as a publicly traded commodity that is more sensitive to immediate market reactions. Therefore, it can also be deduced that this higher volatility and randomness could imply greater unpredictability for accurate forecasts of steel prices.

Overall, based on the quantitative results of this paper, these implications could therefore provide a road map for continuing improvements in the post-pandemic environment for the steel industry by attempting to reduce risks and seize new opportunities with regards to time series forecasting.

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