Strategic Decision-Making in Swiss-System Chess Tournaments

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Abstract. This study delves into the decision-making strategies of chess players in the final round of Swiss-system chess tournaments. This paper first introduces the basic rules and characteristics of Swiss-system tournaments, as well as the objectives of players in the competition. Subsequently, this paper analyzes the strategies adopted by players in the final round under different circumstances, including scenarios without tied players, with tied players but leading or trailing in rankings, and situations where players are tied and have similar rankings. Through theoretical analysis and validation with empirical data, this paper derives optimal strategies for players in different situations to ensure achieving the highest possible ranking in the final round. Starting with a simple four-player championship model, the study provides important insights and guidance for player decision-making in Swiss-system chess tournaments, while also offering valuable exploration into understanding the decision-making process of players. The strategies are well discussed in Section 3.

Keywords: Swiss system, tournament, ranking, player decision-making, game theory.

1. Introduction

Chess competitions, with their strategic complexity and intense competition, provide a rich field for studying the decision-making processes of players. The Swiss-system tournament format is widely employed in competitive games, including most e-sports, badminton, and chess-the focus of this paper [1, 2]. This involves two-player zero-sum games and Nash equilibrium [3, 4]. In this study, the author delves into the Swiss-system chess tournament, where players aim to achieve the best possible ranking over multiple rounds of competition. The emphasis is on understanding the strategies adopted by players in the crucial final round to ensure attaining the highest possible ranking based on the superiority in game theory [5].

This article assumes that p players participate in an r-round Swiss chess tournament (where $p \geq 20$ and is even, $r \geq 7$), and all players aim to achieve the highest possible ranking. In each round, the p players engage in pairwise matches, with the winner receiving 2 points, the loser receiving 0 points, and a draw resulting in each player earning 1 point. After each round, the pairing for the next round is determined according to the Swiss system. The final ranking is determined by comparing the cumulative tiebreak points (the total points at the end of each round) in case of equal scores [6]. If the point difference is greater than or equal to two points, it can be approximated by comparing the scores from the previous round (i.e., if the current point difference between two players is greater than or equal to 2 points, the lower-scoring player has a very low probability of having a higher opponent tiebreak). The ultimate goal is to explore the strategies, as Mo mentioned, adopted in the final round to achieve the goal of "achieving the highest possible ranking": aggressive (hoping to win the game) or conservative (prioritizing unbeaten and avoiding losses) [7].

As the players are assumed to have equal strength, this paper assumes that the probability of winning or losing in each chess game is the same (ignoring the impact of the color), denoted as m, and the probability of a draw is n. Therefore, $2m + n = 1$. If a player chooses an aggressive approach to strive for victory, the probability of a draw decreases by t $(0 < t < 0.5n)$. Since choosing an aggressive strategy often involves opting for suboptimal, more complex variations to maintain the complexity of the position, the increase in the probability of losing is greater than the increase in the probability of winning [8, 9]. Therefore, the reduced draw proportion is proportionally distributed to the probabilities of winning and losing in the ratio of $r{:}s$ $(r < s)$, i.e., the probability of winning increases by $\frac{r}{r+s}$, and the probability of losing increases by $\frac{s}{r+s}$.

First, the author delves into the championship battle in the final round (where all players with a chance to win the championship strive for it. Even though there is no probability to win the champion, he will choose to be aggressive). Starting from relatively simple cases, this article assumes that only players at the top two board have theoretical hopes of winning the championship and investigate a four-player model, in which the players are with equal strength [10].

Let the player currently holding the highest real-time ranking be denoted as A, with a score of a. A will definitely appear on the first board in the final round. Let the other player on the first board be denoted as B, with a score of b. On the second board, the player with the higher ranking is denoted as C, with a score of c, and his opponent with a score of d is denoted as D. Therefore, $0 \le b, c, d \le a$, and $d \le c$. Furthermore, A's cumulative tiebreak points are higher than C's, which is higher than D's.

2. Basic Classification Settings

2.1. A has No Opponents with the Same Score

Situation 1. If player A is leading the second-place player by no less than 2 points, i.e., $\min\{a - b, a - c, a - d\} = 2$. Since A has the highest cumulative tiebreak points, securing 1 point in the final round is enough to ensure winning the championship, and therefore, A chooses a conservative strategy. B, C, and D, if unable to win, cannot surpass A and are destined not to win the championship, so they all choose an aggressive approach.

Situation 2. If the difference between player B and player A is only 1 point, and player C is trailing by more than 2 points, i.e., $a - b = 1, a - c > 1$. Due to the point disadvantage, if B does not win, they cannot surpass A and are destined not to win the championship. Hence, B chooses an aggressive strategy. Since both A and B are guaranteed to score, and B's points are higher than those of C and D, and A has the highest cumulative tiebreak points, C, D also choose an aggressive approach. In this case, the only way A loses the championship is if A loses the game. If A chooses a conservative strategy, the probability of losing the championship is $m + t \times \frac{r}{r+s}$, whereas if A chooses an aggressive strategy, the probability is $m + t \times \frac{r}{r+s} + t \times \frac{s}{r+s} = m + t$. Obviously, choosing a conservative strategy result in a smaller probability of losing the championship. Therefore, A is conservative, and B, C, D are aggressive.

Situation 3. If both B and C are trailing A by only 1 point, i.e., $a - b = 1, a - c = 1$. In this case, if BD do not win, they will be ranked lower than their opponents and are destined not to win the championship, so BD choose an aggressive strategy. If A remains unbeaten, A wins the championship. Therefore, A doesn't need to win and chooses a conservative strategy. If B wins, it can prevent A from winning the championship, so for C to have a chance to catch up with B, C must win. Thus, C chooses an aggressive strategy. In this case, A is conservative, and B, C, D are aggressive.

Situation 4. If A is ahead of B by two points and ahead of C by one point, i.e., $a - b = 2, a - c = 1$. In this case, if B does not win, they will be ranked lower than A. If D does not win, they will be ranked lower than C. Hence, B, D choose an aggressive strategy. If A remains unbeaten, A wins the championship, so A chooses a conservative strategy. For C to have a chance to win, B must defeat A. Meanwhile if C wins, he become the champion. If C loses, he is eliminated. If C draws, B, C have the same score. If C's cumulative tiebreak points is higher, C becomes the champion. And only in this way, can C win the champion. So, C chooses a conservative strategy despite he cannot control the result of the first board. Therefore, AC are conservative, B, D are aggressive. If B's cumulative tiebreak points are higher, B becomes the champion, so C chooses an aggressive strategy. Therefore, A is conservative, and B, C, D are aggressive in this case.

2.2. A has Opponents with the Same Score

Situation 5. If A and B have the same score which is higher than C's, i.e., $a = b, a - c > 0$. Since A is ranked first, it indicates that A's cumulative tiebreak points are higher than B's. In the final round, if A and B draw, A wins the championship due to the higher tiebreak points. Therefore, A adopts a conservative strategy, and B adopts an aggressive strategy. As A, B are guaranteed to score, C, D must win. Thus, A is conservative, and B, C, D are aggressive.

Situation 6. If A, B, and C have the same score which is higher than D's, i.e., $a = b = c > d$. If B does not win, they will be ranked lower than A, so B chooses an aggressive strategy. If D does not win, A, B are guaranteed to score, and D is destined not to win the championship, so D also chooses an aggressive strategy. Since C has lower tiebreak points, the conditions for winning the championship are either (1) A draws and C wins or (2) A loses, C wins, and C's tiebreak points are higher than B's. Therefore, C must strive for a win, and C adopts an aggressive strategy. A's conditions for winning the championship are either A wins or A draws and CD draw. If A chooses a conservative strategy, the probability is $m + t \cdot \frac{s}{r+s} + (n - t)(n - 2t)$. If A chooses an aggressive strategy, the probability is $m + t + (n - 2t)(n - 2t)$. The difference is $t \cdot (n - 2t - \frac{r}{r+s})$. Therefore, when $n > 2t + \frac{r}{r+s}$, A should choose a conservative strategy, and when $n < 2t + \frac{r}{r+s}$, A should choose an aggressive strategy.

Situation 7. If A and C have the same score and are higher than D, and B is trailing A, C by 1 point, i.e., $a = c > d, a - b = 1$. In this case, if B, D do not win, their rankings will definitely be lower than A, C, so they must adopt an aggressive strategy.

Case 1. If B's cumulative tiebreak points are lower than C's. Since B's cumulative tiebreak points are lower than C's, and C's cumulative tiebreak points are also lower than A's, it follows that B's tiebreak points are lower than A's. In this scenario, A's conditions for winning the championship are either A wins or A draws and C does not win. C's conditions for winning the championship are either A draws and C wins or A loses and C does not lose. The probabilities are as follows in Table 1.

Table 1. The probabilities of A and C winning the champion of Case 1

	conservative	aggressive
conservative	$m + t \cdot \frac{s}{r+s} + (n - t)(m + t \cdot \frac{r}{r+s} + n - t)$; $(n - t)(m + t \cdot \frac{s}{r+s}) + (m + t \cdot \frac{r}{r+s})(m + t \cdot \frac{s}{r+s} + n - t)$	$m + t \cdot \frac{s}{r+s} + (n - t)(m + t + n - 2t)$; $(n - t)(m + t) + (m + t \cdot \frac{r}{r+s})(m + t \cdot \frac{s}{r+s} + n - t)$
aggressive	$m + t + (n - 2t)(m + t \cdot \frac{r}{r+s} + n - t)$; $(n - 2t)(m + t \cdot \frac{s}{r+s}) + (m + t)(m + t \cdot \frac{s}{r+s} + n - t)$	$m + t + (n - 2t)(m + t + n - 2t)$; $(n - 2t)(m + t) + (m + t)(m + t + n - 2t)$

Case 2. If B's cumulative tiebreak points is lower than C's. For C to win the championship, A must either draw and C wins or A loses and C wins. Therefore, C adopts an aggressive strategy. A's conditions for winning the championship are either A wins or A draws and C does not win. If A chooses a conservative strategy, the probability is $m + t \cdot \frac{s}{r+s} + (n - t)(m + t + n - 2t)$ .If A chooses an aggressive strategy, the probability is $m + t + (n - t)(m + t + n - 2t)$. If $m + n > t \cdot (\frac{r}{r+s} + 1)$, A should choose an aggressive strategy; otherwise, A should choose a conservative strategy.

Situation 8. If A and C have the same score, A is 2 points ahead of B, and at least 2 points ahead of D, i.e., $a = c, a - b = 2, a - d > 1$. In this case, A's conditions for winning the championship are either A wins or A draws and C does not win, or A loses and C loses. C's conditions for winning the championship are either A draws and C wins, or A loses and C does not lose. The probabilities are as follows in Table 2.

Table 2. The probabilities of A and C winning the champion of Situation 8

	conservative	aggressive
conservative	$m + t\cdot\frac{s}{r+s} + (n-t)(m + t\cdot\frac{r}{r+s} + n - t) + (m + t\cdot\frac{r}{r+s})(m + t\cdot\frac{r}{r+s})$; $(n-t)(m + t\cdot\frac{s}{r+s}) + (m + t\cdot\frac{r}{r+s})(m + t\cdot\frac{s}{r+s} + n - t)$	$m + t\cdot\frac{s}{r+s} + (n-t)(m + t + n - 2t) + (m + t\cdot\frac{r}{r+s})(m + t)$; $(n-t)(m + t) + (m + t\cdot\frac{r}{r+s})(m + t\cdot\frac{s}{r+s} + n - t)$
aggressive	$m + t + (n-t)(m + t + n - 2t) + (m + t)(m + t\cdot\frac{r}{r+s})$; $(n-2t)(m + t\cdot\frac{s}{r+s}) + (m + t)(m + t\cdot\frac{s}{r+s} + n - t)$	$m + t + (n-2t)(m + t + n - 2t) + (m + t)(m + t)$; $(n-2t)(m + t) + (m + t)(m + t + n - 2t)$

Situation 9. If A and C have the same score, A is 2 points ahead of B, and 1 point ahead of D, i.e., $a = c, a - b = 2, a - d = 1$. This situation is the same as case 1.

Situation 10. If A, B, C, and D have all the same score, i.e., $a = b = c = d$. In this case, the tiebreak points are known, with A > C > D, and A > B. If BD do not win, their rankings will be lower than AC, so they must adopt an aggressive strategy. For C, regardless of whether B's tiebreak points are higher or lower, if C does not win, and if AB have a result, the winner's score is destined to be higher than C's. If AB draw, A, with the advantage of tiebreak points, will be ranked higher than C. Therefore, C must strive for a win and adopts an aggressive strategy. A's conditions for winning the championship are either A wins or A draws and CD draw. If A chooses a conservative strategy, the probability is $m + t\cdot\frac{s}{r+s} + (n-t)(n-2t)$. If A chooses an aggressive strategy, the probability is $m + t + (n-2t)(n-2t)$. If $n > t\cdot(\frac{r}{r+s} + 2)$, A should choose a conservative strategy; otherwise, A should choose an aggressive strategy.

3. Real Data Analysis

The above is a theoretical analysis. Now, let's combine it with real-world data to interpret the results. According to the data recorded by Dongping Chess Network (http://www.dpxq.com)as of November 27, 2023, there are a total of 126,738 chess games. Among them, the Red won 47,551 games, there were 35,473 draws, and Black won 43,714 games, with win rates of 37.5%, draw rate of 28.0%, and loss rate of 34.5%. In the model, we disregarded the impact of color and only distinguished between wins, draws, and losses. Therefore, we set the win probability m=36%, draw probability n=28%. As $0 < t < 0.5n$, we chose t to be 5%, and let p represent $\frac{r}{r+s}$, then $1 - p$ represents $\frac{s}{r+s}$, and we set p = 0.4. We denote the probability of A winning the championship as p_1, and C winning the championship as p_2, with the difference Δ.

3.1. Results of Different Cases

If A has no opponents with the same score. The results are consistent regardless of the values of m, n, and t, indicating independence from these parameters.

If A has opponents with the same score. If A and B have the same score which is higher than C's, i.e., $a = b, a - c > 0$. The results are consistent regardless of the values of m, n, and t, indicating independence from these parameters. If A, B, and C have the same score which is higher than D's, i.e., $a = b = c > d$. As $n < 2t + \frac{r}{r+s}$, A chooses an aggressive strategy.

If A and C have the same score and are higher than D, and B is trailing AC by 1 point, i.e., $a = c > d, a - b = 1$. If B's cumulative tiebreak points is lower than C's. According to Table 3 and superiority in game theory, due to -0.0572<0.1954, 0.071734<0.2005, A should choose an aggressive strategy. Since 0.1954<0.2005, C chooses a conservative strategy.

Table 3. The probabilities of A and C winning the champion of Situation 8

A, C	conservative	aggressive
conservative	Δ=0.3992-0.4564=-0.0572	Δ=0.3658-0.294066=0.071734
aggressive	Δ=0.5198-0.3244=0.1954	Δ=0.5162-0.3157=0.2005

If B's cumulative tiebreak points is lower than C's. As $m+n>t\cdot(\frac{r}{r+s}+1)$, A chooses an aggressive strategy. If A and C have the same score, A is 2 points ahead of B, and at least 2 points ahead of D, i.e., $a=c, a-b=2, a-d>1$.

Table 4. The probabilities of A and C winning the champion of Situation 9

	conservative	aggressive
conservative	Δ=0.473-0.291214=0.181786	Δ=0.5216-0.3299=0.1917
aggressive	Δ=0.5039-0.3244=0.1795	Δ=0.5162-0.3157=0.2005

According to Table 4 and superiority in game theory, due to 0.181786<0.1917 and 0.1795<0.2005, C should choose an aggressive strategy. Since 0.1795<0.181786, A chooses a conservative strategy.

If A and C have the same score, A is 2 points ahead of B, and 1 point ahead of D, i.e., $a=c, a-b=2, a-d=1$. If A, B, C, and D have all the same score, i.e., $a=b=c=d$. Since$n>t\cdot(\frac{r}{r+s}+2)$, A chooses a conservative strategy. In fact, the direction of this inequality is highly influenced by the value of t. If t is slightly larger, it might lead to the inequality reversing.

3.2. Discussion

In summary, when A has no opponents with the same score, A adopts a conservative strategy, while the other three players adopt an aggressive strategy.

When A has opponents with the same score: If all players have the same score, A adopts a conservative strategy, and the other three players adopt an aggressive strategy. If only A and B have the same score, A adopts a conservative strategy, and the other three players adopt an aggressive strategy.

If only A and C have the same score: If A is one point ahead of B: If B's cumulative tiebreak points are lower than C's, ABD adopt an aggressive strategy, and C adopts a conservative strategy. If B's cumulative tiebreak points are higher than C's, all four players adopt an aggressive strategy. If A is two points ahead of B, A adopts a conservative strategy, while the other three players adopt an aggressive strategy.

4. Conclusion

Through theoretical analysis of different strategies in Swiss-system chess tournaments and validation with real-world data, the author has gained important insights into the dynamics of player decision-making. However, this is only an initial exploration of this complex topic, and this paper can further delve into research and discussion in the following areas.

Firstly, the paper can consider more variables in the games, such as the impact of playing first or second and the varying strengths of different players. Secondly, the paper can expand the research framework to apply it to larger-scale tournaments, including scenarios where more than the top two players have a chance to win the championship.

Additionally, the paper may introduce concepts from other decision theories and game theories into the research. For example, if the paper makes slight adjustments to variables like m, n, and t, there may exist Nash equilibria worth exploring. Furthermore, if each player's goal is not necessarily to become the champion but rather to maximize their ranking, the complexity and practical significance of the study will be further enhanced.

In the future, the paper hopes to reveal more about the essence of player decision-making in Swiss-system chess tournaments through in-depth research. This will provide players with more strategic guidance and contribute to the enrichment of the applications of game theory and decision theory.

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