Sex Ratio of Seven Gill Eels Based on System Dynamics and Dynamic Generalisation Models

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Abstract. This paper provides an in-depth study of the ecological impacts of changing sex ratios in the lampreys, an ancient species known as a living fossil, through mathematical modeling. First, a dynamic model was constructed to simulate the response of the sex ratio of the lampreys to food availability and to predict subsequent changes in predator populations within the ecosystem. The model also considered reproductive success to assess broader ecological impacts. The results suggest that changes in lampreys sex ratio can significantly affect ecosystems in many ways. However, risks such as overfishing, environmental stress and predation emerged as clear disadvantages. Further exploration through more advanced dynamic modeling confirmed the stabilizing effect of higher male ratios on the ecosystem. The study also ventured into competition modeling, revealing that lamprey populations may inadvertently favor other species, such as parasites. Taken together, the mathematical modeling in this study provides insights into the ecological role of lampreys and is intended to inform conservation efforts and optimize the ecological use of lampreys.

Keywords: Dynamic models, Dynamic generalization models, Competitive models.

1. Introduction

The phenomenon of sex-ratio adjustment in lampreys, which are uniquely capable of altering their sex distribution in response to environmental resources, provides a compelling subject for ecological and evolutionary studies. This adaptive trait allows for more females in nutrient-rich environments and more males in resource-scarce environments and plays a critical role in the survival and reproductive success of lampreys [1]. Our study was devoted to constructing and analyzing a model reflecting the mechanisms regulating the sex ratio of the lampreys and assessing its ecological impacts and adaptations in different environmental contexts. First, we created a dynamic model to simulate how lampreys sex ratio changes in response to food. Other factors, such as reproductive success, were also considered to synthesize effects on various parts of the ecosystem. Second, we examined a more complex dynamic generalization model based on the dynamic model and found that a higher proportion of males in the lampreys helps maintain ecosystem stability. Finally, a competition model was constructed, in which we found that lampreys populations can create more favorable conditions for other species (e.g., parasitoids) to thrive. The experimental results show that the model proposed in this paper improves the accuracy of the assessment and helps us understand the significance of sex ratio adjustment in animal species.

2. Dynamic Models

2.1. Model Establishing

In this study, a discrete dynamic model is developed to elucidate the interplay between food availability and the sex ratio of lampreys across generations. The model employs a recursive formula to simulate a 1% change in the sex ratio per generation [2], influenced by the food resource levels. This approach underscores the adaptive response of lampreys to environmental resource fluctuations, where a scarcity of food leads to an increased sex ratio, while abundance results in its decrease. The model further explores the relationship between food availability and sex ratio changes through a linear interpolation method, highlighting how the sex ratio's evolution is contingent upon the initial conditions, aiming for optimal reproductive efficiency.

We develop a simple discrete dynamic model for simulating changes in the sex ratio of lamprey as a function of food availability and generation.

In this paper, we define sex ratio as the ratio of the number of males to the total number of males, taking values from 0 to 1. Consequently, we used a linear interpolation formula to calculate the sex ratio for each generation of lamprey:

$$r_n = r_0 + \frac{n-1}{N_g-1} \cdot \left(r_f - r_0\right) \tag{1}$$

Where r_n denotes the sex ratio of the nth generation of lamprey. r_0 denotes the initial sex ratio, r_f denotes the final sex ratio, and N_g denotes the total number of generations.

As shown in Fig. 1(a). From the figure, it can be seen that when the initial sex ratio is low, the sex ratio of the lampreys gradually rises close to 1. When the initial sex ratio is high, the sex ratio of the lampreys gradually decreases close to 0. This suggests that the change in the sex ratio of the lampreys is correlated with the initial ratio, probably to achieve optimal reproductive efficiency.

Additionally, we used a simplified relational equation to calculate the average food resources lived by each lamprey individual of each generation:

$$fn = \frac{f_0}{r_n} \tag{2}$$

Where f_n denotes the food supply of the nth generation of lamprey and f_0 denotes the initial amount of food resources.

It is evident from Fig. 1(a) that the lamprey's food availability increases with decreasing sex ratio and decreases with increasing sex ratio. This suggests that there is an inverse relationship between the sex ratio and the food availability of lampreys.

Next, we used a continuous dynamic model to simulate the process of changing sex ratios and population sizes of lampreys with food availability and time.

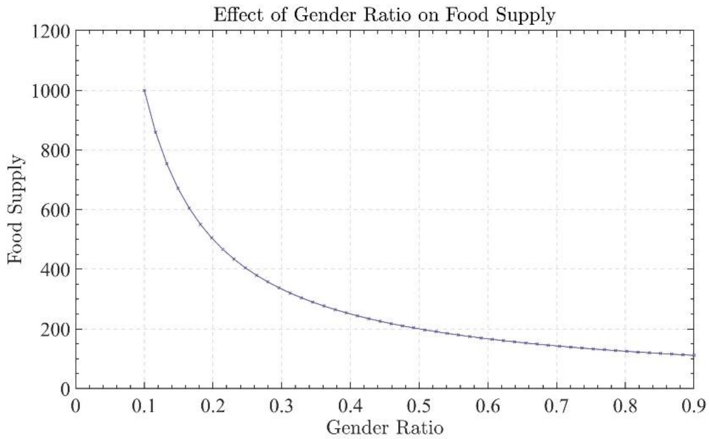
$$\frac{dM}{dt} = bMA - dM - pMP \tag{3}$$

Sex ratio and population size can be calculated from the number of males and females. Sex ratio is defined as the ratio of the number of males to the total number of males and can range from 0 to 1. Population size is defined as the sum of the number of males and females. The formula is given below:

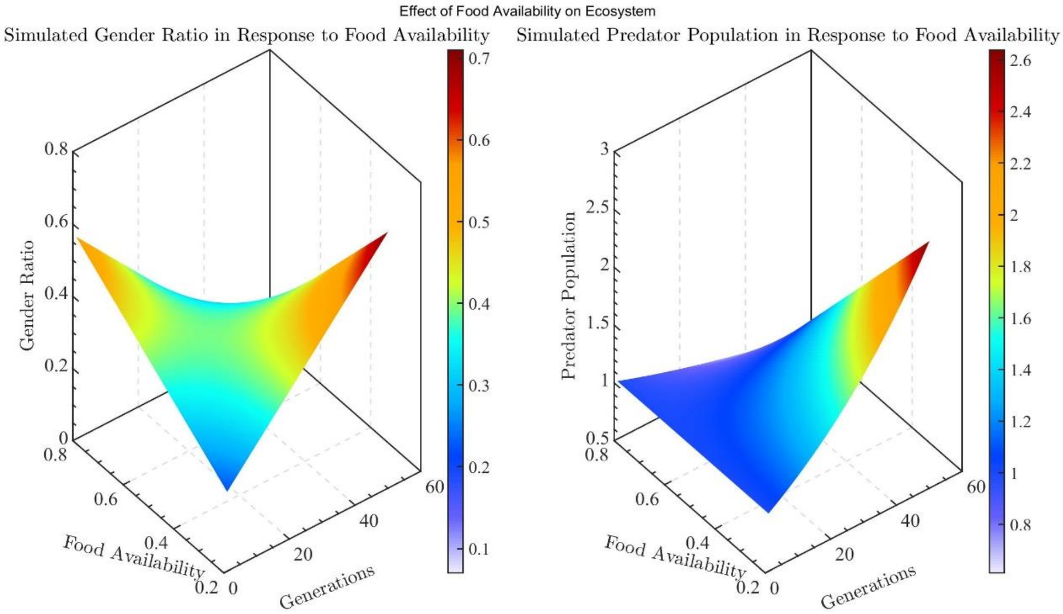
$$r = \frac{M}{M+F} \tag{4}$$

$$N = M + F \tag{5}$$

As shown in Fig. 1(b), the sex ratio of lamprey increases under conditions of low food availability. Predator abundance increases under conditions of low food availability. And the number of predators decreases under conditions of high food availability.



(a) Effect of gender ratio on food supply



(b) Effect of food availability on ecosystem

Figure 1. Food source effect

We developed a simple model to simulate variation in reproductive success of lampreys. At each time step, reproductive success was calculated based on resource availability and individual attributes (sex and size). Below is the mathematical formula expressed in symbols:

Reproductive success was calculated:

$$RS = r * I \tag{6}$$

Individual size updates:

$$I' = I + r \tag{7}$$

Finally, reproductive success over time was visualized by plotting a curve of reproductive success over time and marking the location of the highest reproductive rate, as in Fig. 2.

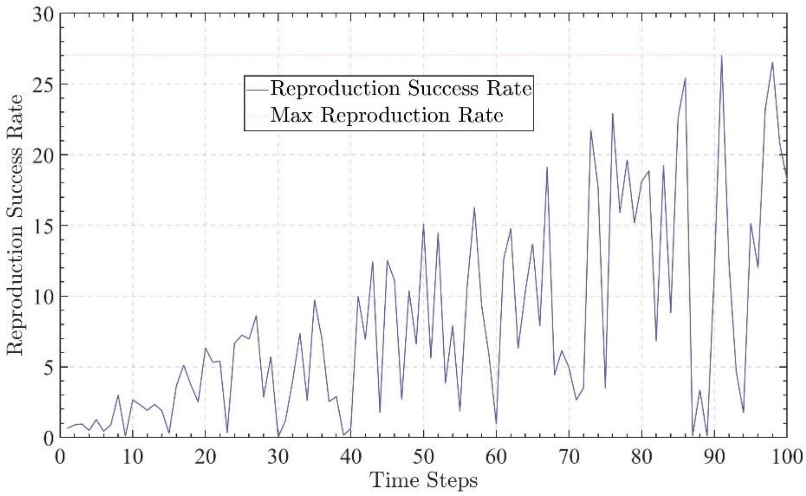


Figure 2. The reproduction success rate changes with time steps

We use the following formula to represent habitat:

Sex ratio column:

$$r = r + r_a \times 0.2 - 0.1 \quad (8)$$

Habitat occupancy:

$$o = c \times r \quad (9)$$

As shown in Fig. 3(a), the percentage of habitat tends to increase with the constant change in sex and the number of iterations.

Use the following equation to represent breeding areas:

Sex ratio:

$$r = r + r_a \times 0.2 - 0.1 \quad (10)$$

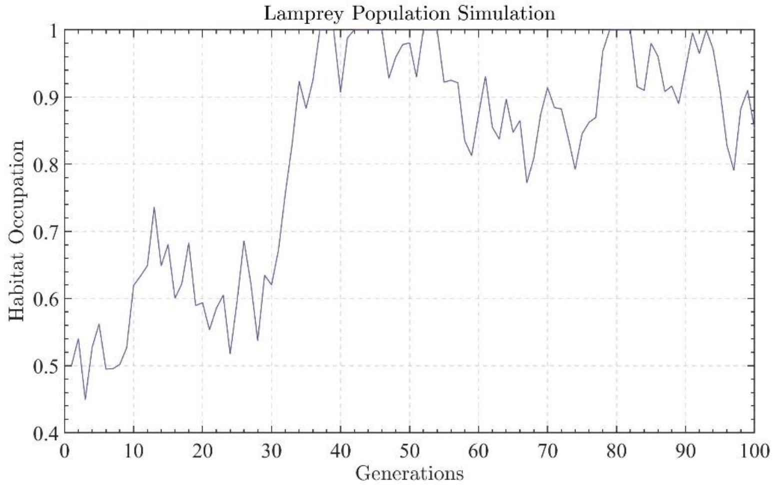
Resource area occupancy:

$$a = a + r_a \times 0.4 - 0.2 \quad (11)$$

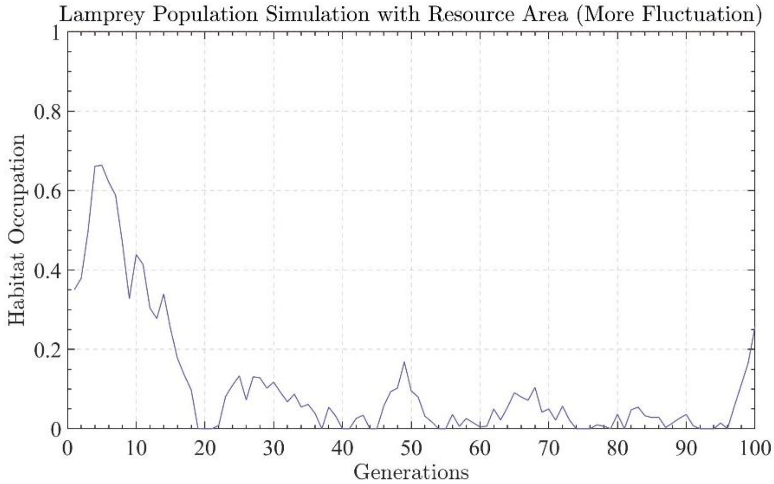
Habitat occupancy:

$$o = c \times f \times a \quad (12)$$

Habitat occupancy within 100 generations and the maximum habitat capacity are shown in Fig. 3 (b). Habitat occupancy in the breeding area showed a decreasing trend with the continuous change in sex and the increase in the number of iterations.



(a) Simulation of lampreys populations



(b) Simulation of lampreys populations in the resource area

Figure 3. Lampreys populations

2.2. Result

As shown in Fig. 4 with increasing number of iterations, when food resources are scarce, the proportion of males increases, predator population size decreases, reproductive success decreases, and food resources decrease; when food resources are moderate, the proportion of males remains constant, predator population size decreases, reproductive success decreases slowly, and food resources remain almost constant; when food resources are abundant, the proportion of males decreases, predator population size decreases, reproductive success decreases rapidly, and food resources increase; and when food resources are abundant, the proportion of males decreases, predator population size decreases, and food resources increase. When food resources are abundant, the proportion of males' declines, predator populations decline, reproductive success declines rapidly, and food resources increase.

The weights of the varied factors were summed to obtain a composite indicator, the larger the composite indicator, the greater the impact on the larger ecosystem, and finally, the curve was smoothed using a normal distribution and the smoothed composite indicator was returned. As shown in Fig. 4 and Fig. 5, when the number of iterations is small, the impact on the larger ecosystem under each female-male ratio rises rapidly; as the number of iterations increases, the growth rate of composite indexes under the female-dominated condition slows down at first, but the growth rate becomes significantly larger after about 40 iterations; the male-dominated and the equal-sex ratio, the growth rate of the male-dominated and the same-sex ratio, after about 20 iterations, produces a large difference in growth rate, and the curve of the equal-sex ratio is approaching to that of the female-dominated curve, and the male-dominated curve is approaching, and the male dominated curve is approaching to that of the female-dominated curve. The male dominance and equal sex ratio, after about twenty iterations, have different growth rates, with the equal sex ratio curve approaching the female dominance curve, and the male dominance curve always maintaining a higher growth rate.

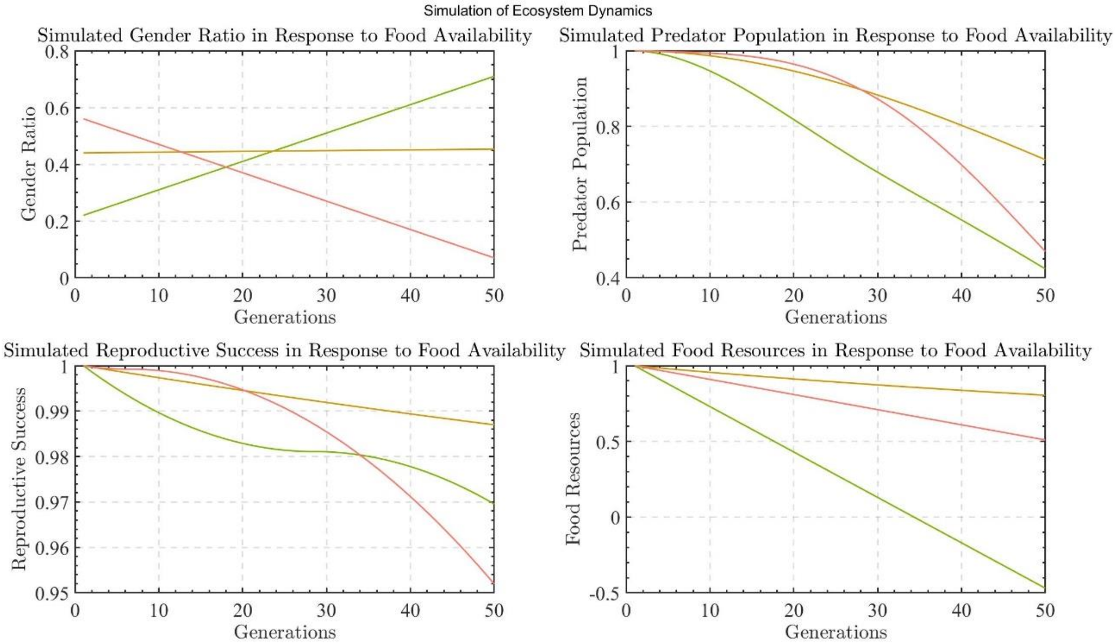


Figure 4. Simulation of ecosystem dynamics

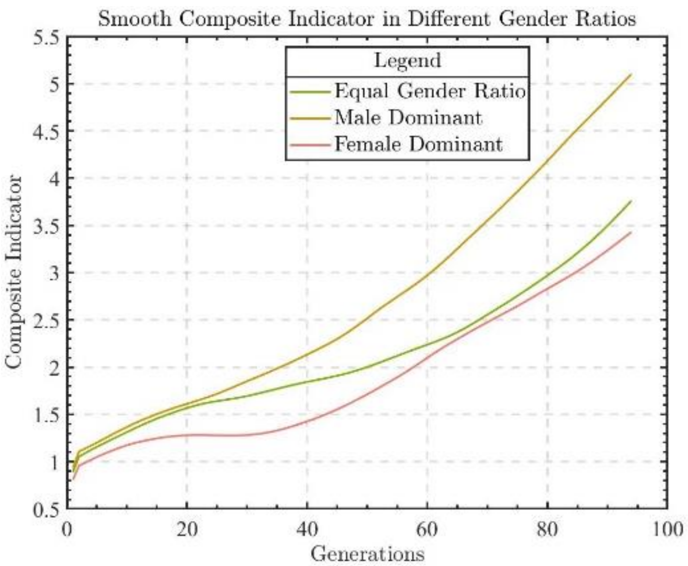


Figure 5. Smooth composite indicator in different gender ratio

3. Dynamic System Dynamics Modeling

3.1. Model Establishing

We can use a decreasing sigmoid function to model the relationship between the sex ratio and the food supply [3], where the sex ratio decreases with increasing food supply. Curve function affecting the sex ratio is:

$$P = \frac{1}{1+e^{-k\cdot(F-F_0)}} \tag{13}$$

To conduct a system dynamic modeling analysis of the population dynamics of the lamprey [4,5], we will consider a combination of a multi-species competition model, a logistic growth model and previously established models. By exploring the competition between the lamprey population and other species, the food chain relationship, and the effect of sex ratio changes on the whole ecosystem, we build on the original model to establish a system dynamics model. The lamprey population dynamics equation is:

$$\frac{dN}{dt} = r_N \cdot N \cdot \left(1 - \frac{N}{K_N}\right) - \alpha_{NO} \cdot (N - O) \tag{14}$$

$$\alpha_{NO}(N, O) = 0.005 \cdot \frac{N}{O} \tag{15}$$

The same reasoning can be used to obtain dynamic equations for the female lamprey.

$$\frac{dF}{dt} = r_F \cdot F \cdot \left(1 - \frac{F+M}{K_F}\right) + \alpha_{FN} \cdot (N - F) \tag{16}$$

Male sea lamprey dynamic equation:

$$\frac{dM}{dt} = r_M \cdot M \cdot \left(1 - \frac{F+M}{K_M}\right) + \alpha_{MN} \cdot (N - M) \tag{17}$$

Other species have dynamic equations:

$$\frac{dO}{dt} = r_O \cdot O \cdot \left(1 - \frac{O}{K_O}\right) - \beta_{ON} \cdot (O - N) \tag{18}$$

We established the relationship between S and N, F, M, O to determine changes in lamprey sex ratio, effects on ecosystem stability.

The calculation formula is follows:

$$\frac{dS}{dt} = \gamma \cdot \left(\frac{dN}{dt} + \frac{dF}{dt} + \frac{dM}{dt} + \frac{dO}{dt}\right) \tag{19}$$

Finally, we conducted the sensitivity analysis of the model, and the specific process is as follows: the calculation of the elasticity coefficient:

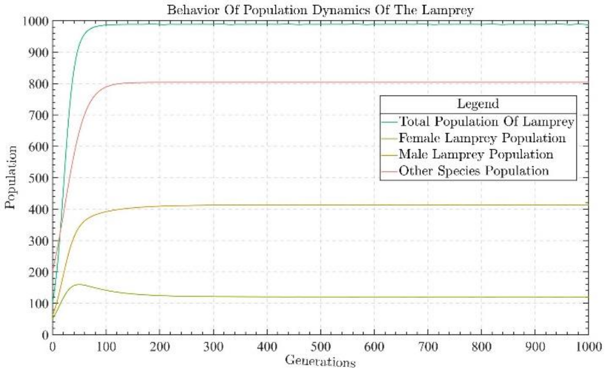
$$\varepsilon(i) = \frac{dN}{dp} \cdot \frac{p(i)}{N} \tag{20}$$

The calculation of the sensitivity of the sex ratio to the food supply is:

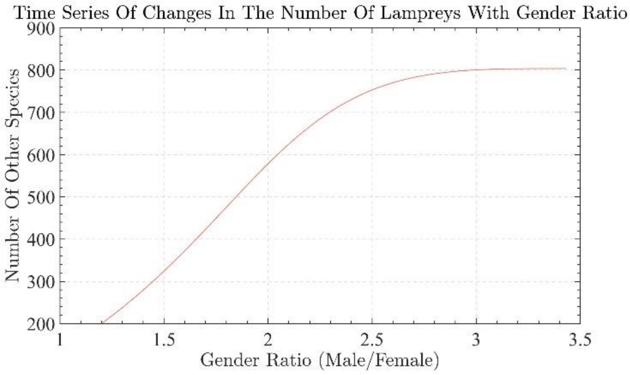
$$\eta = k' \cdot exp\left(-k' \cdot (f_0 - f_0')\right) \cdot \left(1 + exp\left(-k' \cdot (f_0 - f_0')\right)\right)^{-2} \tag{21}$$

3.2. Result

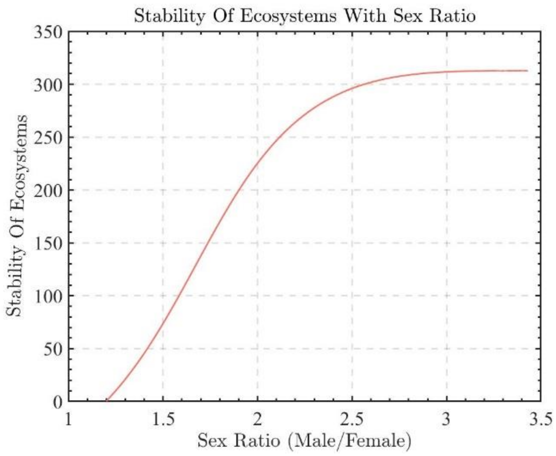
After simulation to obtain Fig. 6, from Fig. 6(a), with the increase in the number of iterations, the number of various populations first increased rapidly and then stabilized; from Fig. 6(b), with the increase in the sex ratio (male/female), the number of the other populations first increased and then stabilized; from Fig. 6(c), with the increase in the sex ratio (male/female), the ecosystem stability first increases and then tends to stabilize; Fig. 6(d) is a three-dimensional graph, as the number of lamprey populations increase, the number of female lampreys increases linearly, and the number of male lamprey decreases first and then increases; With the increase of k, the total number of lampreys and the number of female lampreys are both increased substantially, the number of male lampreys decreases, and the number of other populations almost unchanged, and these four values change independently of F_0; As k increases, the proportion of male lampreys first decreases rapidly and then stabilizes, and the values change independently of F_0.



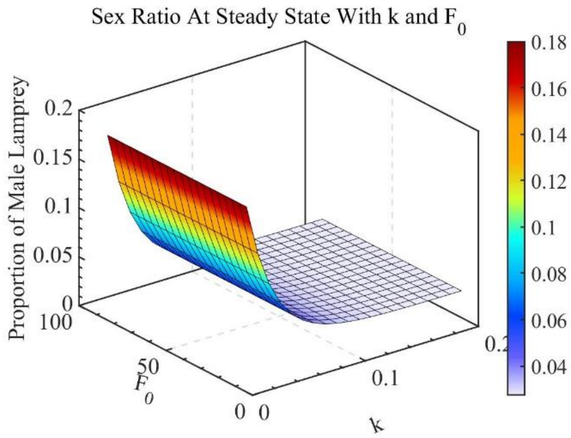
(a) Behavior of population dynamics of the lamprey



(b) Time series of changes in lamprey abundance with sex ratio



(c) Stability of ecosystems with sex ratio



(e) The relationship

Figure 6. Simulation result

4. Competition Model

4.1. Model Establishing

We constructed the following environmental, food and predation pressure functions to model the population dynamics of lamprey and parasites as well as changes in food resources.

A temperature function that varies with the number of days:

$$T(t) = 10 + 5sin\left(\frac{2\pi t}{365}\right) \tag{22}$$

The effect of temperature on birth rate and mortality rate:

$$f_T(\tau) = 1 + 0.01(\tau - 10) \tag{23}$$

A regenerative capacity function of food resources:

$$f(t) = 0.5sin\,(0.1t) + 0.5 \tag{24}$$

A predation pressure function of lamprey:

$$g(t) = 0.4sin\,(0.2t) + 0.6 \tag{25}$$

A food availability function:

$$F(t) = F_{min} + (F_{max} - F_{min})\frac{f(t)-g(t)+1}{2} \tag{26}$$

Next, to investigate the effects of changing sex ratios in lamprey populations on other species in the ecosystem (such as parasites) [6], we considered the effects of changes in the sex ratio of lamprey populations on other species in the ecosystem. Lampreys' parameters such as birth rate, death rate, and growth rate, and introduced competitive relationships to model competition between different parasite populations and their interactions with the lamprey. Consequently, we set out modeling equations.

Growth rate of lamprey populations (male and female) is:

$$\begin{aligned} \frac{dF}{dt} &= \left(r_F F\left(1-\frac{F}{K_F}\right) - S_f F\right)(1-r_F) \\ \frac{dM}{dt} &= \left(r_M M\left(1-\frac{M}{K_M}\right) - S_m M\right)(1-r_M) \end{aligned} \tag{27}$$

The growth rate of the three kinds of parasites population is.

$$\begin{aligned} \frac{dP_1}{dt} &= \left(r_{p1} P_1\left(1-\frac{P_1}{K_{p1}}\right) - S_{p1} P_1\right)(1-E_1) \\ \frac{dP_3}{dt} &= \left(r_{p3} P_3\left(1-\frac{P_3}{K_{p3}}\right) - S_{p3} P_3\right)(1-E_3) \end{aligned} \tag{28}$$

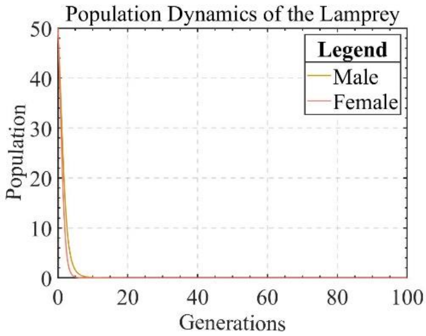
Predation rate of sea lamprey on parasites is:

$$\begin{aligned} E_1 &= k_1 F P_1 \\ E_3 &= k_3 F P_3 \end{aligned} \tag{29}$$

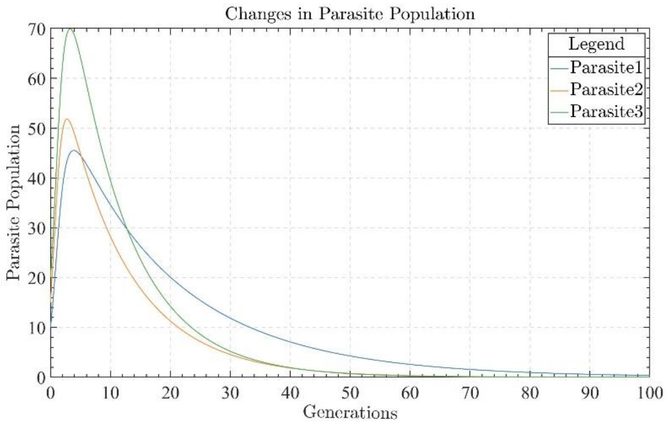
4.2. Result

By simulation analysis, we get three figures. Fig. 7(a) shows that as the number of iterations increased, both male and female lamprey numbers decreased, and females declined faster than males. Fig. 7(b) shows that the population of the three parasites first increased and then decreased, and finally stabilized. The population with parasite three was the largest, parasite twice, and parasite one was the smallest. Fig. 7(c) shows that food availability was on the rise.

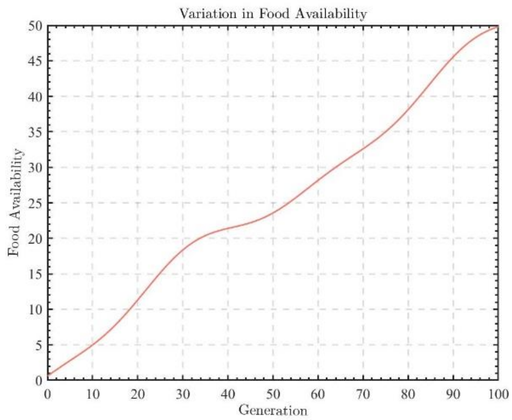
In conclusion, due to the competitive relationship between the parasite and the lamprey, it increases significantly first when the lamprey population decreases sharply, and due to the predation relationship between the parasite and the lamprey, parasite numbers subsequently decreased significantly and below the initial numbers, indicating that loss of lamprey is detrimental to parasite survival. According to the previous model, the sex ratio change in lamprey is beneficial to their survival, so the ecosystem with the sex ratio change in lamprey population can provide advantages for other species (such as parasites) in the ecosystem.



(a) Population dynamics of the lamprey



(b) Changes in parasite population



(c) Variation in food availability

Figure 7. Simulation analysis

5. Summary

The purpose of this paper is to develop a series of mathematical models for analyzing the strengths and weaknesses of lamprey's populations in ecosystems and their interactions with other components of the ecosystem as the sex ratio of lamprey's populations changes. First, this paper creates a dynamic model to simulate how lampreys sex ratios change in response to food. Synthesizing the effects on various parts of the ecosystem, we find that changes in lampreys sex ratios can fluctuate and affect the larger ecosystem in a variety of ways. Second, we examined more complex dynamic generalization models and found that lampreys populations are more susceptible to predation rates and that the sensitivity of sex ratios to food availability peaks under specific circumstances. Finally, the construction of a competition model reveals that lampreys' populations can create more favorable conditions for other species (e.g., parasitoids) to thrive. The methodology presented in this paper contributes to our understanding of the significance of sex ratio adjustment in animal species and provides practical guidance for biodiversity management and conservation.

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