

Research on purely azimuthal passive positioning of unmanned aerial vehicles based on geometric analysis and three-point intersection circle positioning

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Abstract. With the development of UAV technology, this paper discusses the problem of the positioning model of UAVs passively receiving radio signals in UAV clusters. For the case of a circular formation, two possible point distributions are analysed, and the positioning model of UAVs passively receiving signals is established by geometric analysis. In the case where the positions of UAVs numbered FY00 and FY01 are known, the problem of determining how many UAVs need to transmit signals in order to effectively locate a certain UAV with a slightly deviated position is addressed. The paper goes on to prove that only one UAV of unknown number is required to transmit signals for effective localization by analyzing the structure of a positive nine-sided shape on a circle. The article establishes a model of the problem through geometric analysis and proposes a corresponding solution, which provides some theoretical support for the formation flight of UAV clusters.

Keywords: Geometric analysis, passive positioning, three-point intersection circle positioning.

1. Introduction

With the development of the times, drones have come into people's view, and nowadays drones provide a different experience for our life. UAVs can solve the problem of high-altitude remote shooting for us, and UAV cluster performance enriches our life; at the same time, UAVs are also widely used in the military field, and have shown obvious advantages and outstanding performance in this field [1-2].

In order to avoid external interference and detection, UAV clusters should maintain radio silence and send out fewer radio signals as far as possible during formation flight [3-4]. In order to maintain the formation formation, it is proposed to use the pure orientation passive positioning method to adjust the position of UAVs, i.e., a few UAVs in the formation transmit signals, and the rest of the UAVs receive the signals passively, from which the orientation information is extracted for positioning to adjust the position of the UAVs. Each UAV in the formation has a fixed number, and its relative position relationship with other UAVs in the formation remains unchanged. The direction information received by a UAV that receives signals is agreed to be the angle between the UAV and the line connecting any two UAVs transmitting signals [5-6]. In order to model the positional adjustment of passive signal-receiving UAVs [7], and at the same time to determine the minimum number of signal-transmitting UAVs, this paper takes a circular formation as an example to build a passive positioning model [8-10]. Some symbols in the modelling process are described in Table 1 below:

Table 1. Description of symbols

notation	Meaning
α_i	Magnitude of directional signals for drones that passively receive radio signals
j_i	The i th drone that deviates from its position
δ	Maximum threshold for total drone deviation

2. Passive positioning modelling and solving

The UAVs are all kept flying at the same altitude based on their own perceived altitude information. Therefore, only this plane of UAV flight altitude needs to be considered.

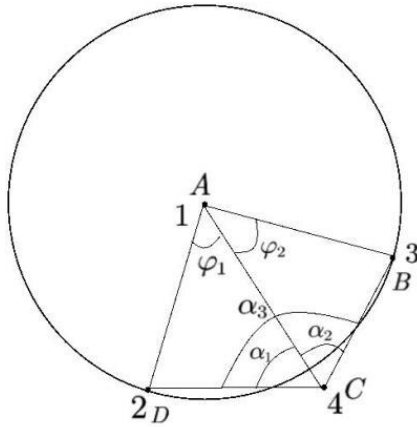


Figure 1. Schematic diagram of the relative positions in case 1

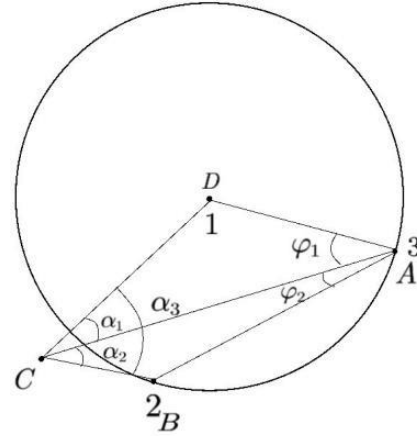


Figure 2. Schematic diagram of the relative positions in case 2

There are two scenarios for the location of UAVs that passively receive radio signals, as shown in Figures 1 and 2. To order the magnitude of the received directional signals: $\alpha_3 \geq \alpha_2 \geq \alpha_1$, then by the angular relationship from the figure: $\alpha_3 = \alpha_2 + \alpha_1$.

(1) For the situation in Figure 1:

It is clear from the question that the position of the UAV transmitting the radio signal is accurate without deviation and the number is known. Therefore, the size of $\angle BAD$ is known, so that $\angle BAD = \varphi$, $\angle DAC = \varphi_1$, $\angle CAB = \varphi_2$, then $\varphi_1 + \varphi_2 = \varphi$ is a fixed value, so that the radius of the circle r , $AC = \rho$.

Inside $\triangle ACD$, by the sine theorem, we get

$$\frac{AC}{\sin(\pi - \varphi_1 - \alpha_1)} = \frac{AD}{\sin \alpha_1} \Rightarrow \frac{\rho}{\sin(\varphi_1 + \alpha_1)} = \frac{r}{\sin \alpha_1} \quad (1)$$

Inside $\triangle ABC$, by the sine theorem:

$$\frac{AC}{\sin(\pi - \varphi_2 - \alpha_2)} = \frac{AB}{\sin \alpha_2} \Rightarrow \frac{\rho}{\sin(\varphi_2 + \alpha_2)} = \frac{r}{\sin \alpha_2} \quad (2)$$

From equations (1) and (2) we have:

$$\frac{\sin(\varphi_2 + \alpha_2)}{\sin(\varphi_1 + \varphi_2)} = \frac{\sin \alpha_2}{\sin \alpha_1} \quad (3)$$

$$\alpha + \beta = \varphi_2 + \alpha_2 \quad (4)$$

$$\alpha - \beta = \varphi_1 + \alpha_1 \quad (5)$$

$$\alpha = \frac{(\varphi_1 + \varphi_2) + (\alpha_1 + \alpha_2)}{2} = \frac{\varphi + \alpha_3}{2} \quad (6)$$

Since both φ and α_3 are known, α is a known constant value.

From equations (4) and (5) it follows that:

$$\beta = \frac{(\alpha_2 - \alpha_1) + (\varphi_2 - \varphi_1)}{2} \quad (7)$$

Where α_1 and α_2 are known, so the inverse solution $\varphi_2 - \varphi_1 = 2\beta + (\alpha_1 + \alpha_2)$, then the value of $\varphi_2 - \varphi_1$ can be obtained by determining β .

Substituting equations (4) and (5) into equation (3) yields

$$\frac{\sin(\alpha + \beta)}{\sin(\alpha - \beta)} = \frac{\sin\alpha_1}{\sin\alpha_2} \quad (8)$$

Since α_1 and α_2 are known, $\frac{\sin\alpha_1}{\sin\alpha_2}$ is a known value. Let $\frac{\sin\alpha_1}{\sin\alpha_2} = k$, then we have

$$\frac{\sin(\alpha + \beta)}{\sin(\alpha - \beta)} = k \Rightarrow \frac{\sin\alpha\cos\beta + \sin\beta\cos\alpha}{\sin\alpha\cos\beta - \sin\beta\cos\alpha} = k \quad (9)$$

The above equation is simultaneously divided by $\cos\alpha\cos\beta$ to obtain:

$$\frac{\tan\alpha + \tan\beta}{\tan\alpha - \tan\beta} = k \quad (10)$$

The solution is therefore:

$$\beta = \arctan\left(\frac{k-1}{1+k}\tan\alpha\right) \quad (11)$$

By α and k are known, then β is a known value.

From the inverse solution equation $\varphi_2 - \varphi_1 = 2\beta + (\alpha_1 + \alpha_2)$ and $\varphi_1 + \varphi_2 = \varphi$ associatively, we get:

$$\begin{cases} \varphi_1 = \frac{\varphi + (\alpha_2 - \alpha_1)}{2} - \arctan\left(\frac{\sin\alpha_1 - \sin\alpha_2}{\sin\alpha_1 + \sin\alpha_2} \tan\frac{\varphi + \alpha_3}{2}\right) \\ \varphi_2 = \frac{\varphi + (\alpha_2 - \alpha_1)}{2} + \arctan\left(\frac{\sin\alpha_1 - \sin\alpha_2}{\sin\alpha_1 + \sin\alpha_2} \tan\frac{\varphi + \alpha_3}{2}\right) \end{cases} \quad (12)$$

Through the formula (1) we can get: $\rho = \frac{r}{\sin\alpha_1} \sin(\varphi_1 + \alpha_1)$, so with the position of FY00 as the origin and AD as the polar axis we can determine the positioning of the UAV that receives the radio signal passively as

$$\left(\frac{r}{\sin\alpha_1} \sin(\varphi_1 + \alpha_1), \frac{\varphi + (\alpha_2 - \alpha_1)}{2} - \arctan\left(\frac{\sin\alpha_1 - \sin\alpha_2}{\sin\alpha_1 + \sin\alpha_2} \tan\frac{\varphi + \alpha_3}{2}\right)\right) \quad (13)$$

(2) For the case of Fig. 2:

Taking A as the origin and AB as the polar axis, the same can be obtained as the coordinates of the drone positioning as:

$$\left(\frac{r}{\sin\alpha_2} \sin(\varphi_2 + \alpha_2), \frac{\varphi + (\alpha_2 - \alpha_1)}{2} + \arctan\left(\frac{\sin\alpha_1 - \sin\alpha_2}{\sin\alpha_1 + \sin\alpha_2} \tan\frac{\varphi + \alpha_3}{2}\right)\right) \quad (14)$$

3. Solving for the minimum number of signal-transmitting drones

If the position deviation UAV's position is shown in Fig.1. Let the radio signals transmitted by UAVs FY00 and FY01 so that the slightly deviated UAVs receive the signal at an angle of α_1 , as shown in Fig. 3 below

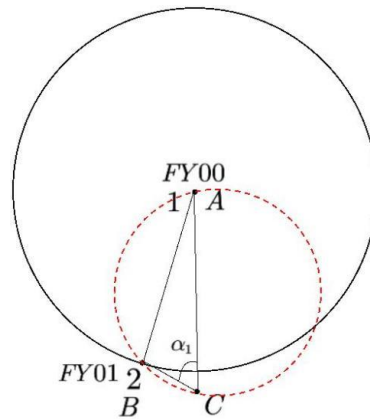


Figure 3. Schematic of Points

Then the radius $2R_1 = \frac{r}{\sin \alpha_1}$ corresponding to the circumferential angle α_1 with $AB=r$ as the chord, then the circle is drawn with R_1 as the radius, and the red dashed line superior arc part is the position of the C possible deviation.

If another direction deviation angle α_2 is obtained from the signal of another unknown UAV, the position UAV is D (unknown), at this time, C that can satisfy the group conditions is shown in blue in Fig. 4 below

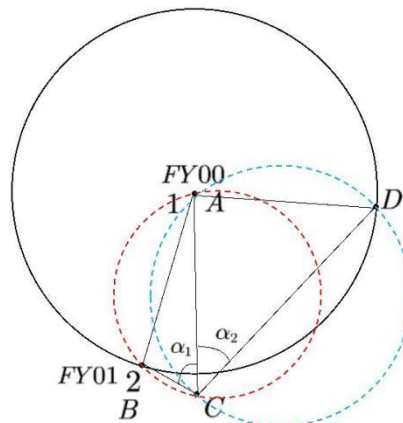


Figure 4. Schematic of Points

Thus, the unique determination of point C can be given by the focus of the red and blue trajectories. However, the blue trajectory is a dynamic graph that can be rotated around node A (radius must be R_2), while the unknown numbered UAV transmitting the radio signal is given by the intersection of the black and blue circles

Next this paper needs to determine how many UAVs are needed to transmit signals for effective positioning.

1) If the slight deviation of the positioning UAV is the error of the polar angle within "0~" " π " / "9", then at this time only 1 UAV with unknown number is needed to transmit the radio signal for effective positioning.

Proof: since the UAV formation is uniformly distributed on the circumference, the neighbouring positions of the 9 UAVs on the circumference

Two by two, they form a square nine-sided shape, with each side corresponding to a central angle of $\frac{2\pi}{9} = 40^\circ$, Let the drone that deviates from the unknown be j_i , then the known drone that sends out the signal is numbered j, and the unknown is k.

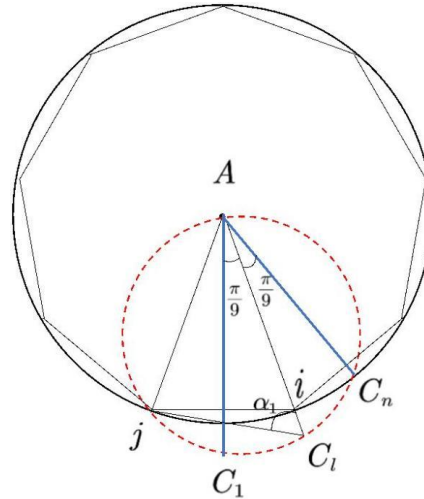


Figure 5. Schematic representation of the possible location of point C.

Thus, as shown in Figure 5, the location of point C may be $C_1 \dots C_i \dots C_n$, then the corresponding chord on the corresponding dynamic map of the declination signal α_2 emitted by the UAV with unknown number. Since the radius of the dynamic map R_2 is known, the arc of the area that AC can sweep through is $2\pi/9$, i.e., the possible positions of point D at this time are $D_1 \dots D_p \dots D_n$, as shown in the figure below.

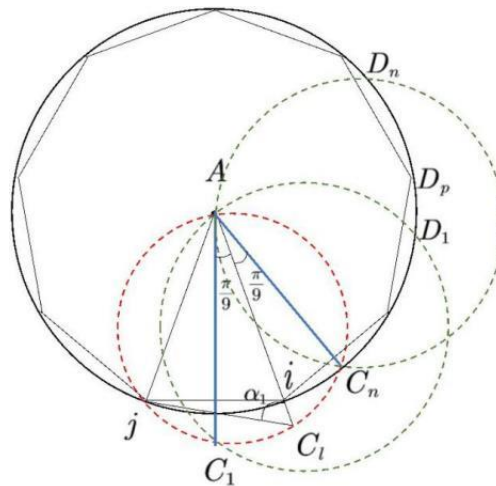


Figure 6. Schematic representation of the possible location of point D.

From Figure 6 above, it can be seen that in the $2\pi/9$ circumcentre angle corresponding to the region swept by D, there is and only possibility of a

a vertex of a square nine-sided shape, at which point it can be determined that the unknown numbered UAV is the UAV with this unknown vertex.

4. Conclusion

In this paper, by establishing a mathematical model and carrying out derivation and analysis, the UAV position location positioning problem is successfully solved and the number of UAV transmitting signals required to solve the problem is proposed. Firstly, the positional localisation formulas of the UAV in two cases are derived by sorting the height information perceived by the UAV and the magnitude of the direction signals received. The sine theorem and the angular relationship are applied to derive the positioning coordinate formula for the other case. Then, considering the case of position deviation, using the nature of circle and the error range of declination angle, it is determined that only one UAV with unknown number is needed to transmit signals to locate effectively when the

error of polar angle is within " $0 \sim \pi/9$ ". And the method of determining the UAV with unknown number is further deduced.

Through the above analysis, it can be concluded that this paper has achieved important results in solving the problem of UAV location positioning, which provides a theoretical basis for the application of UAV positioning technology. In addition, in the practical application, the required number of UAV transmitting signals can be flexibly adjusted according to the specific requirements of the problem and the requirements of positioning accuracy. Overall, the research in this paper is of great significance to the development of UAV-related fields and helps to improve the positioning accuracy and application range of UAVs

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