

Research On Numerical Wave Tank Based on SPH Method

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Abstract. Numerical wave-current flume is one of the effective methods to study waves and their effects on structures. In this paper, the Smoothed Particle Hydrodynamics (SPH) method is used to simulate and analyze the numerical wave flume. Based on the SPH method, an efficient and flexible numerical wave tank model is constructed to simulate complex water wave dynamics. Firstly, the basic principle of SPH method is summarized, and then the establishment process of numerical wave tank model is described in detail, including particle initialization, boundary condition setting and so on. Through the flume simulation under different wave conditions, the accuracy and stability of the model are verified. Finally, the interaction between wave and structure is studied. It provides a new and effective tool for the numerical simulation of wave flume, and also provides a useful reference for the related research in the fields of marine engineering and coastal protection.

Keywords: Smooth particle hydrodynamics; Numerical wave tank; Water wave dynamics.

1. Introduction

With the continuous growth of global energy demand, the development and utilization of renewable energy has attracted more and more attention. As a clean, renewable and widely distributed energy form, ocean wave energy has great potential for development. As one of the most important factors in coastal and marine engineering, wave propagation, deformation and interaction with structures are all important contents of research. The wave flume is the most basic equipment for studying similar problems. However, the physical model experiment is limited by factors such as cost, complex modeling, environmental simulation and scale effect. Therefore, the use of numerical simulation technology has become an effective means. The numerical wave flow tank provides great convenience for studying similar problems. The experiment is fast, easy to compare multiple schemes, and there is no scale problem. In the traditional method of establishing numerical wave flume based on grid method, the grid is relatively well arranged, but when dealing with large deformation and complex boundary problems such as waves, it requires a high computational domain and is not easy to implement.

The Smoothed Particle Hydrodynamics (SPH) method proposed by Lucy [1-2] et al. is a meshless method, which can effectively avoid the above problems and is suitable for dealing with large deformation and complex single-phase and multi-phase fluid dynamics problems. It has unique advantages in simulating complex fluid motion and can deal with complex phenomena such as large deformation, fracture and merging.

Liang [3] et al. used ISPH method (Incompressible SPH) to study the motion of seawall constrained by springs under the impact of solitary waves. Using the WCSPH method (Weakly compressible SPH), Li [4] et al. used WCSPH method (Weakly compressible SPH) to study the interaction of solitary waves with gentle slopes and abrupt steps. Wang [5] et al. considered the $k-\varepsilon$ turbulence closure equation and used the adaptive time algorithm to calculate the non-breaking propagation of solitary waves on the obstacles installed at the bottom and the breaking propagation on the 1:50 slopes. Monaghan [6-10] and other scholars have also studied the SPH method.

2. Theoretical basis of SPH method

2.1. SPH method

SPH numerical approximation usually has two key steps. The second step is to accumulate and sum the values of the nearest adjacent particles to approximate the integral expression of the function and obtain the approximate expression of the particles.

2.1.1 Kernel function approximation

Any function (density, mass, temperature, etc.) in a continuous variable field can be written as an integral of the product of the function and an impulse function:

$$f(r) = \int_{\Omega} f(r') \delta(r - r') dr' \quad (1)$$

Where f is the function of coordinate vector r ; Ω is the integral domain of r , and δ is the Dirac function. $W(x - x', h)$ is called a kernel function in many literatures related to SPH. The δ function is replaced by the smooth function W :

$$\langle f(r) \rangle = \int_{\Omega} f(r') W(r - r', h) dr' \quad (2)$$

h is the smooth length of the influence region of the smooth function W , and the above is the initial form of the kernel approximation.

2.1.2 Particle approximation

The particle approximation method is a discretization process corresponding to the superposition of particles. The solution of particle i is related to the smooth function. It is solved by the particle expression in its support domain. The function particle approximation at particle i can be expressed by the following formula:

$$\langle f(r_i) \rangle = \sum_{j=1}^N \frac{m_j}{\rho_j} f(r_j) \cdot W_{ij} \quad (3)$$

N is the total number of particles in the search domain, m_j and ρ_j are the mass and density of particle j acting on particle i , respectively, and W_{ij} is a smooth function of the influence of particle j on particle i .

Approximation of its derivative:

$$\langle \nabla \cdot f(r_i) \rangle = \sum_{j=1}^N \frac{m_j}{\rho_j} f(r_j) \cdot \nabla_i W_{ij} \quad (4)$$

Where:

$$\nabla_i W_{ij} = \frac{x_i - x_j}{r_{ij}} \frac{\partial W_{ij}}{\partial r_{ij}} = \frac{x_{ij}}{r_{ij}} \frac{\partial W_{ij}}{\partial r_{ij}} \quad (5)$$

Where: r_{ij} is the distance between particle i and particle j .

This process is particle approximation, which is essentially the interpolation and weighted summation of N particle information around particle i . The weight W_{ij} is a function of the distance between particles i and j .

Using the above formulas, the governing equations and boundary conditions can be rewritten to solve the partial differential equations in an explicit way, which is the basic idea of the SPH method.

2.2. Governing equation

According to the three basic laws of physical conservation, the governing equations of fluid dynamics are obtained, which are the law of conservation of mass, the law of conservation of momentum and the law of conservation of energy. The N-S equations in Lagrangian form are as follows:

2.2.1 Equation of mass conservation

The expression of the mass conservation equation (also known as the continuity equation) is:

$$\frac{d\rho}{dt} = -\rho \nabla \cdot \vec{v} \quad (6)$$

Where: ρ is the density of water, t is the time, and v is the velocity vector.

Expanded by density summation method, the modified equation is as follows:

$$\rho_i = \frac{\sum_{j=1}^N m_j W_{ij}}{\sum_{j=1}^N \left(\frac{m_j}{\rho_j} \right) W_{ij}} \quad (7)$$

2.2.2 Momentum equation

In the SPH method, the velocity change of each particle is calculated by the momentum conservation equation. The Lagrangian Navier-Stokes momentum equation is as follows:

$$\frac{dv}{dt} = \frac{1}{\rho} \frac{\partial \sigma}{\partial r} + g \quad (8)$$

Where: v is the velocity vector; g is the acceleration of gravity; σ is the total stress tensor, including pressure p and viscous stress τ .

By derivation and transformation of SPH formula, the SPH approximation of particle i can be obtained as follows :

$$\frac{dv_i}{dt} = -\sum_{j=1}^N m_j \frac{P_i + P_j}{\rho_i \rho_j} \cdot \frac{\partial W_{ij}}{\partial r_i} + \sum_{j=1}^N m_j \frac{\sigma_i + \sigma_j}{\rho_i \rho_j} \cdot \frac{\partial W_{ij}}{\partial r_i} + g \quad (9)$$

The first item on the right side of the equal sign is the pressure approximation, and the second item is the viscous force. According to different calculation conditions, different viscous force calculation formulas will be selected to meet the accuracy requirements of the calculation results.

2.2.3 Energy equation

The Lagrangian Navier-Stokes energy equation can be written as follows :

$$\frac{de}{dt} = -\frac{p}{\rho} \frac{\partial v}{\partial r} + \frac{\mu}{2\rho} \varepsilon_1 \varepsilon_2 \quad (10)$$

Where ε is the shear strain rate ; μ is the ratio of shear stress to shear strain rate.

2.3. Boundary condition

2.3.1 Solid boundary

Many scholars have done a lot of research on the treatment of boundary problems. Monaghan [11] arranges a set of virtual particles on the solid boundary to repel the particles near the boundary, thus preventing these particles from penetrating the boundary. Liu [12] et al. used virtual particles to deal with fixed boundary conditions. They proposed two types of virtual particles : type I virtual particles

and type II virtual particles. Type I particles are distributed on a fixed boundary, and type II particles are distributed in the boundary domain. This study uses type I virtual particles.

The repulsive force of virtual particles near the boundary of real particles is calculated by the following formula:

$$PB_{ij} = \begin{cases} D \left[\left(\frac{r_0}{r_{ij}} \right)^{n_1} - \left(\frac{r_0}{r_{ij}} \right)^{n_2} \right] \frac{x_{ij}}{r_{ij}^2}, \frac{r_0}{r_{ij}} \leq 1 \\ 0, \frac{r_0}{r_{ij}} > 1 \end{cases} \quad (11)$$

In the formula, n_1 and n_2 are the empirical coefficients, usually taking 12 and 4, and D is the order of magnitude that the parameter is generally equal to the square of the maximum velocity.

2.3.2 Free surface boundary

There are no fluid particles outside the free surface. The SPH method calculates the particle density of the free surface by summing the density values of the surrounding particles. Koshizuka's [13] algorithm is usually used to realize the Dirichet condition on the free surface. If the particle density satisfies the condition of the following formula 12, it is considered as a free surface particle, and its density is forced to be equal to the actual density of the free surface fluid, thus ensuring stability.

$$\rho < \beta \rho_0 \quad (12)$$

In the formula, ρ_0 is the calculated density of each particle, and β is a parameter less than 1, which is generally between 0.8 and 0.98.

3. Wave-making and wave-eliminating methods

Numerical wave making technology is a computer application method based on the field of fluid mechanics. It can simulate the physical wave maker in the numerical flume and produce various waveforms and waves needed. In this paper, the numerical simulation adopts the push plate wave maker.

3.1. Wave-making module

3.1.1 Regular wave making wave.

In this paper, the propagation of second-order Stokes waves in still water is simulated to simulate waves.

The analytical solutions of the velocity potential and the free surface position of the second-order Stokes wave propagating along the x direction are:

$$\Phi = \frac{gH}{2\omega} \frac{\cosh(k(h+z))}{\cosh(kh)} \sin(kx - \omega t) + \frac{3\omega H^2}{32} \frac{\cosh(2k(h+z))}{\sinh^4(kh)} \sin 2(kx - \omega t) \quad (13)$$

$$\eta = \frac{H}{2} \cos(kx - \omega t) + \frac{H^2 k}{16} \frac{\cosh(kh)}{\sinh^3(kh)} (2 + \cosh(2kh)) \cos 2(kx - \omega t) \quad (14)$$

Where, H , ω , k and h are wave height, circular frequency, wave number and water depth, respectively.

The velocity U of the wave plate can be expressed by the following formula:

$$U = \frac{\omega}{W^\#} \eta \quad (15)$$

In the formula, $W^\#$ is the hydraulic transfer function, which represents the ratio of the amplitude of the wave surface to the amplitude of the wave plate. According to the following formula :

$$W^\# = \frac{4\sinh^2 kh}{2kh + \sinh 2kh} \quad (16)$$

The horizontal displacement of a time step of the wave plate is $\Delta x = U\Delta t$.

3.1.2 Irregular wave generation

The irregular wave used in this paper uses the JONSWAP spectrum as the target spectrum. An irregular wave is composed of N linear regular waves, and its free surface can be expressed as:

$$\eta(t) = \sum_{i=1}^N A_i \cos(\omega_i t + \delta_i) \quad (17)$$

$$A_i = \sqrt{2S_\eta(f_i)\Delta f} \quad (18)$$

In the formula, $S_\eta(f_i)$ is the spectral density of JONWASP, N is the total number of waves, ω_i is the angular frequency, A_i is the amplitude of each wave component, δ_i is the random phase angle between 0 and 2π , $f_i = (f - i\Delta f - 0.5\Delta f)$ is the wave frequency.

3.2. Wave elimination module

The structures in the flume will reflect the wave back to the wave maker, and encounter the wave pusher plate to form a secondary reflection. The wave is superimposed many times in the flume and the energy is continuously accumulated, which will cause the actual wave height to be significantly greater than the target wave height, which will have a serious impact on the reliability of the experimental results.

3.2.1 Active wave absorption

The active absorption wave-making system is Active Wave Absorption System (AWAS). In this paper, the AWAS module is used to eliminate the wave. The basic idea is to monitor the elevation of the front wave surface of the wave plate in real time through the sensor, use the target wave surface elevation to subtract the measured wave surface elevation to obtain the reflected wave surface elevation, and obtain the corrected flow velocity through the reflected wave surface elevation, and then correct the motion of the wave plate in real time to realize the active absorption wave generation. The specific implementation steps are as follows:

1) Real-time monitoring of the wave elevation in front of the wave plate, the measured wave elevation η_{SPH} , and then use the target incident wave elevation η_I minus the measured wave elevation, calculate the reflected wave free surface elevation η_R , as shown in the following formula.

$$\eta_R(t) = \eta_I(t) - \eta_{SPH}(t) \quad (19)$$

2) According to the free surface elevation η_R of the reflected wave, the velocity U_R corresponding to the reflected wave is obtained. The velocity correction U_R expression of the reflected wave is shown as follows:

$$U_R(t) = \eta_R(t)\sqrt{g/d} \quad (20)$$

Where, η_R is the reflected wave height at time t , and d is the water depth.

3) The velocity of the wave plate is corrected to offset the velocity generated by the reflected wave. The theoretical velocity of the wave plate is U_I , the corrected velocity of the wave plate is U_C , and the velocity of the reflected wave is U_R . The post-wave plate velocity $U_C(t+dt)$ at time $t+dt$ is

the sum of the theoretical wave plate velocity $U_I(t)$ and the reflected wave velocity $U_R(t)$ at time t , as shown in Formula 21. The theoretical velocities $U_I(t)$, $e(t)$ and $U_I(t)$ of the wave plate are obtained by deriving the displacement $e(t)$ of the wave plate from time, see formulas 22 and 23.

$$U_c(t+dt) = U_I(t) + U_R(t) \quad (21)$$

$$e(t) = \frac{S_0}{2} \sin(\omega t + \delta) \quad (22)$$

$$U_I(t) = \omega \frac{S_0}{2} \cos(\omega t + \delta) \quad (23)$$

4. Physical flume experiment and verification

4.1. Experimental installation

The physical experiment flume is 30m long, 0.6m wide, 0.8m high, and the test water depth is 0.7m. One end of the flume is equipped with a hydraulic push plate wave maker, and the other end is equipped with an energy dissipation facility. The wave-making period range is 0.5s ~ 3.0s, and the maximum wave-making capacity is 0.15m ~ 0.20m when the static water depth is 0.7m, as shown in Fig.1.

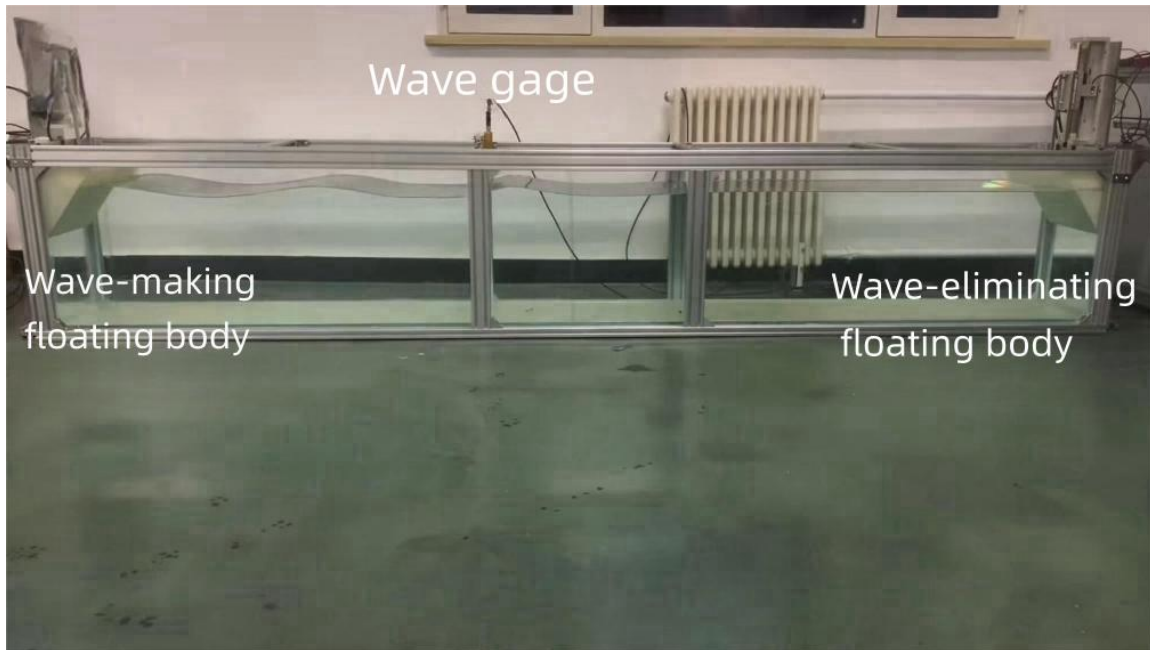


Fig. 1 Experimental tank layout schematic diagram

4.2. Regular wave experiment

4.2.1 With wave elimination module

Build a 30.0 m long tank. The push plate is used to make waves, and the initial position of the push plate is set as the origin of coordinates. There is a straight wall at the right end of the flume, the bottom plate of the flume is horizontal, the straight wall and the bottom plate are composed of solid boundary particles, the water depth is 0.5m, and the wave height meter is set at $X = 2.0m, 12m$ and $25m$ respectively. The numerical model is shown in Fig.2. Make regular waves, wave height $H = 0.15m$, wave period $T = 2s$, simulated 20s.

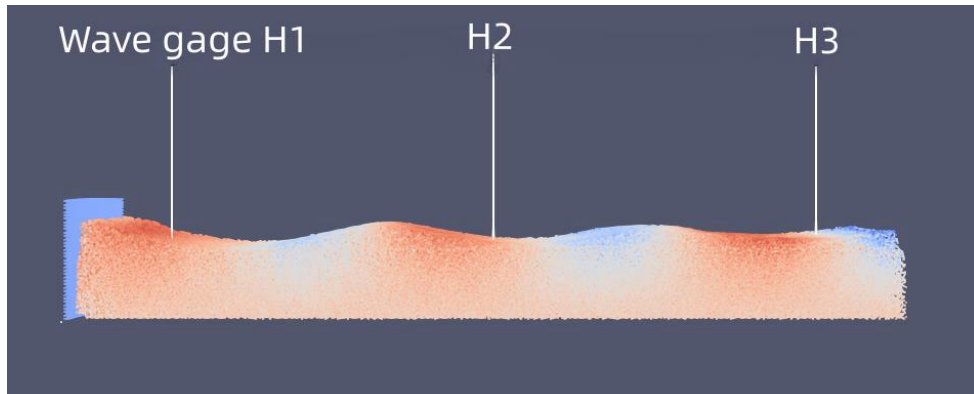


Fig. 2 Experimental tank layout schematic diagram

In 20 s, the wave peak has not yet reached the straight wall to produce the reflected wave, and will not be affected by the reflected wave. At the same time increase the wave elimination module.

The wave height at $X = 2.0\text{m}$ is extracted, and the free surface time series curve is drawn, as shown in Fig.3.

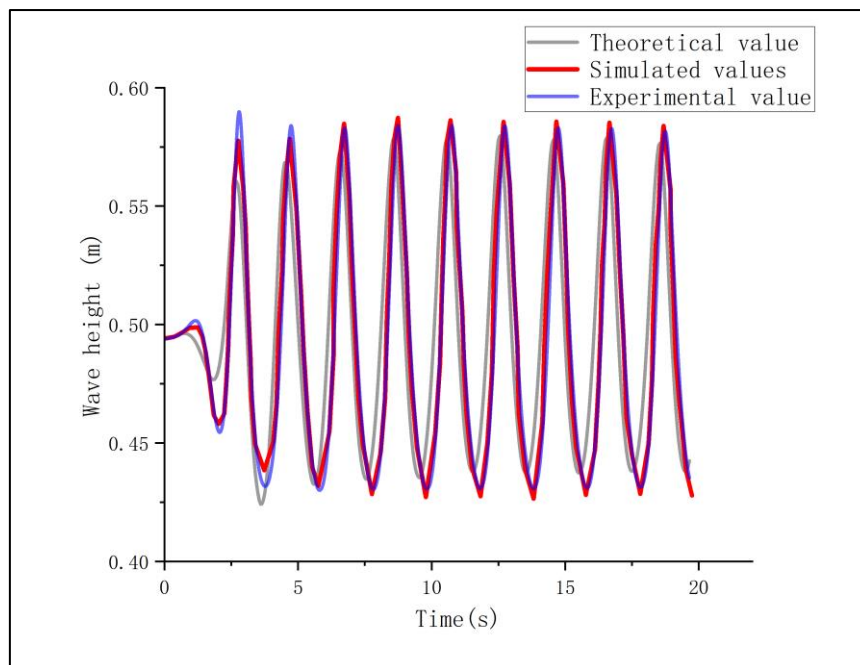


Fig. 3 Comparison of free surface time series curves at $x = 2\text{m}$

It can be seen from the figure that the wave surface motion is unstable in the first few motion cycles, and the amplitude of the wave-making plate is small in the first cycle, and then it can basically reach stability. A stable and uniform traveling wave column is formed in the tank. It can be seen that the second-order Stokes wave simulated by the numerical wave flume established in this paper is basically consistent with the theoretical value. The error may be caused by the influence of viscous parameters and water oscillation, and the error is within 5%.

4.2.2 Without wave elimination module

A 7.0m long flume was established to make regular waves with the same parameters as the above experiments, and the simulation was carried out for 20s. The wave height at $X = 2.0\text{ m}$ was extracted, and the free surface time series curve was drawn as Fig.4.

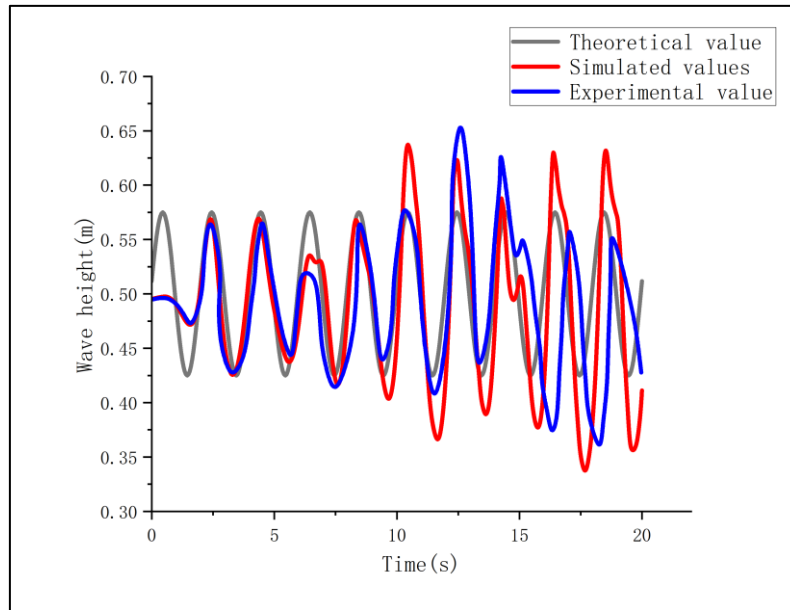


Fig. 4 Comparison of free surface time series curves at $x = 2\text{m}$

It can be seen from the figure that the numerical simulation results are basically consistent with the theoretical solution in the first 5 seconds. Since then, the deviation between the simulation results and the theoretical values has become larger and larger. The same as the experimental phenomenon, this is because the wave reflection, energy can not be dissipated, resulting in no regularity.

The above experiments show that the wave flume constructed in this paper can simulate regular waves well, and AWAS has a good wave dissipation effect.

4.3. Irregular wave experiment

Build a 30m tank. The water depth is 0.5m. Make irregular waves, using JONSWAP spectrum. The wave height meter is set up at $x = 2$, and the wave height is recorded as follows Fig.5.

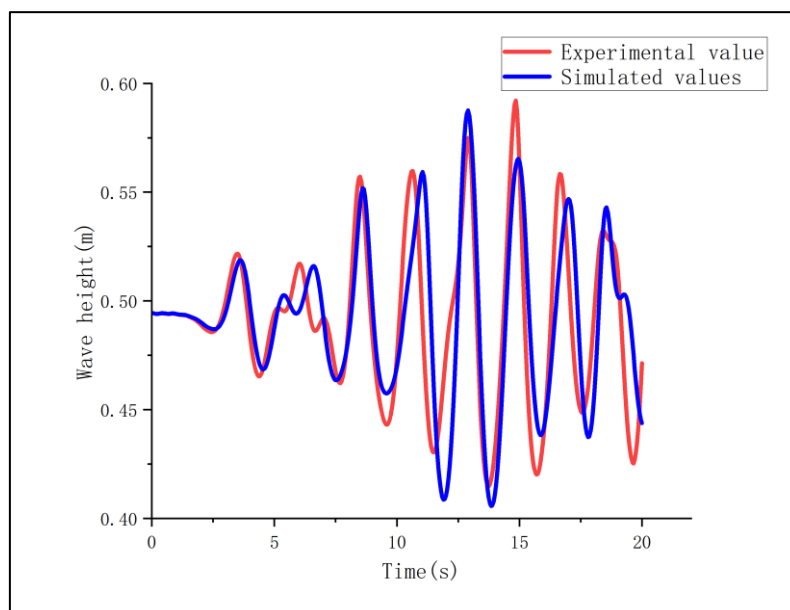


Fig. 5 Comparison of free surface time series curves at $x = 2\text{m}$

It can be seen from the figure that the elevation change of irregular wave surface is irregular. The simulated wave height is basically consistent with the experimental waveform, indicating that the spectral density and spectral peak period of the two are basically the same, and the error is within 10%. It can be concluded that the numerical flume model constructed in this paper can correctly generate irregular waves.

4.4. Interaction with structure

The numerical simulation of the oscillating water column wave energy converter (OWC) chamber is carried out by using the numerical wave tank constructed above. In this paper, the physical model test data of OWC gas chamber refer to the experimental data of Dr.Liu Zhen[14-15].

The regular wave is used as the incident wave in the experiment, and the wave height in the tank and the surface heave change in the air chamber are measured by the wave height meter. The wave height meters are arranged at 10m, 20m and 25m away from the wave plate to monitor the wave height changes along the way, and the wave height meters in the air chamber are in the central position to monitor the water surface heave in the air chamber. The experimental device and schematic diagram are shown in Fig.6.

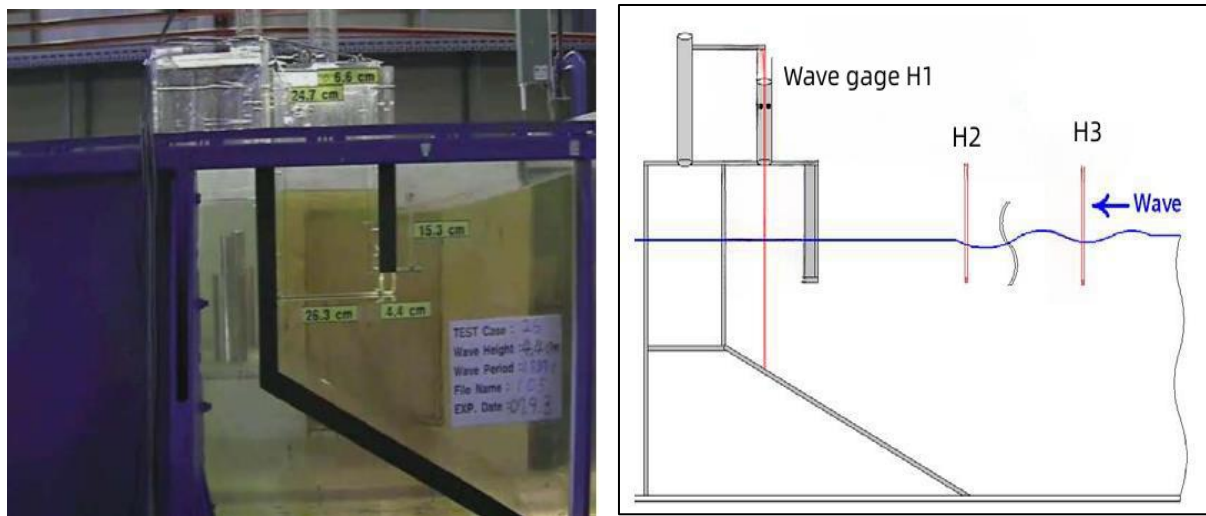


Fig. 6 OWC gas chamber experimental device and schematic diagram

The simulated regular wave elements are: incident wave period $T=4s$, wave height $H=0.5m$, and the wave height changes inside and outside the gas chamber are studied. The numerical simulation process is shown in Fig.7.

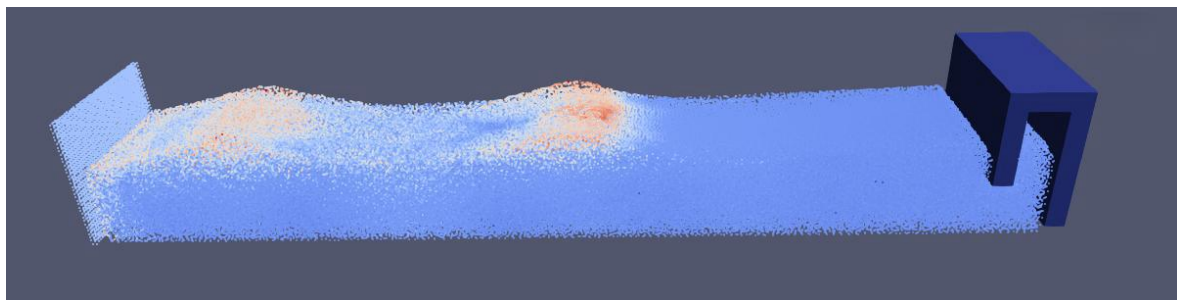


Fig. 7 Numerical model simulation process

Fig.8 and Fig.9 show the time-history curves of the oscillating water column wave surface displacement and the incident wave height at the center of the air chamber under the test conditions and compare them with the simulated values.

It can be seen from the diagram that the water surface heave in the air chamber changes quasi-sinusoidally, the motion period is basically the same as the incident wave, but the oscillation amplitude is not equal to the incident wave. The simulation results are in good agreement with the experimental results, and the error is within 5%.

In summary, the numerical wave flume can not only simulate the heave change of the water surface inside and outside the air chamber, but also the simulation and experimental results of the liquid level change of the oscillating water column in the air chamber are in good agreement. Therefore, the numerical wave tank constructed in this paper can be used to study the interaction between waves and structures.

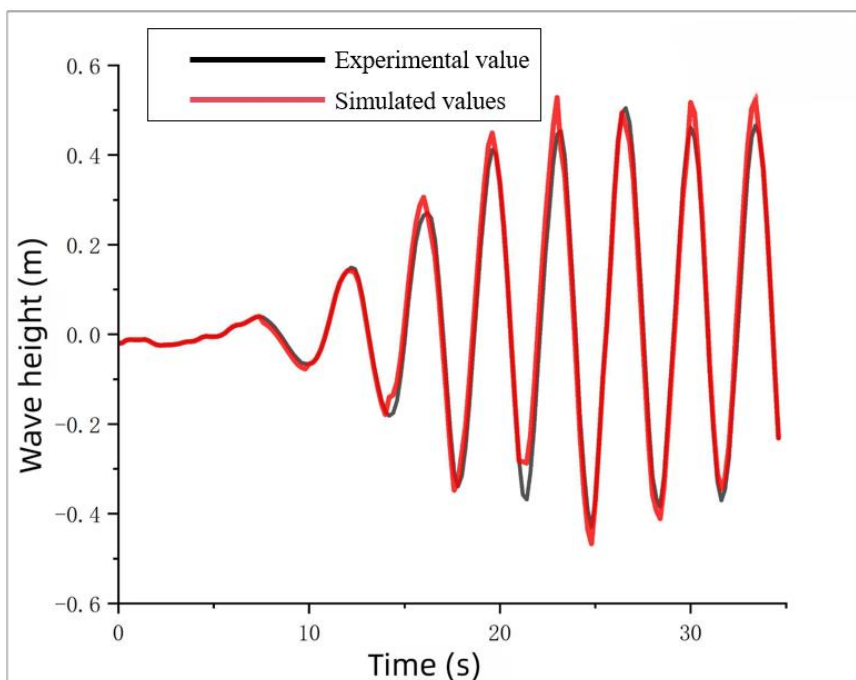


Fig. 8 Wave height curve of incident wave

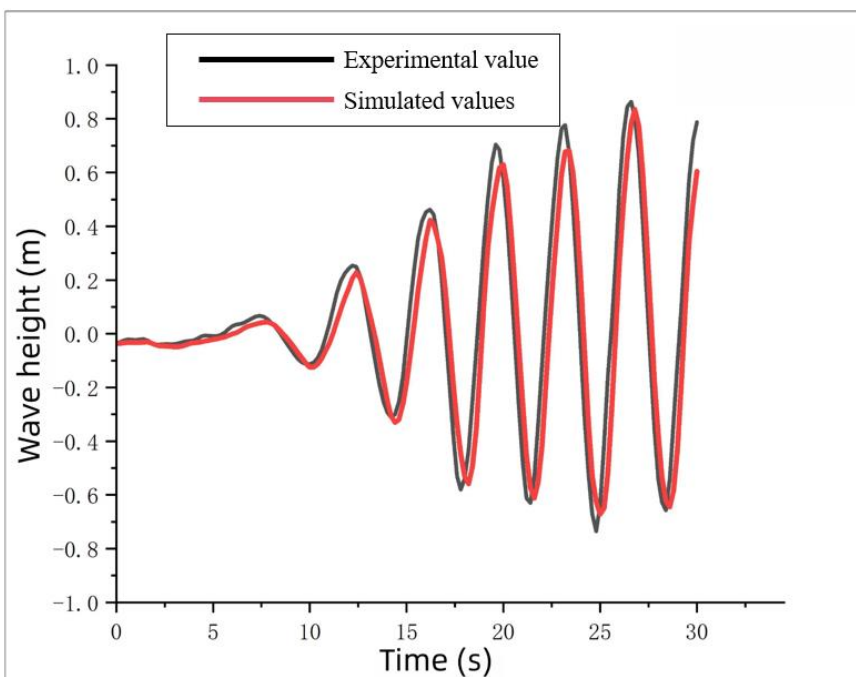


Fig. 9 Wave surface change curve of water column in air chamber

5. Summary

In this paper, a numerical wave tank model based on SPH method is constructed, and its accuracy and applicability are verified by a series of numerical experiments. Through comparative analysis, it is found that the numerical wave flume based on SPH method can effectively simulate different types of wave motion, and in most cases, it is in good agreement with the analytical solution or experimental data. At the same time, when simulating the interaction between waves and structures, it can also reasonably reflect the main characteristics of wave propagation, such as wave height.

The model provides an effective analysis tool for wave simulation in the fields of ocean engineering and hydraulic engineering, which is helpful to deepen the understanding of wave motion law and improve the level of related design. With the continuous improvement of computer

performance and numerical methods, the numerical wave tank based on SPH method is expected to play a greater role in higher precision, larger scale and more complex scenarios.

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