Research on the Vector Space and Its Linearity

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Abstract. A vector space is an abelian group over a field. From the viewpoint of group theory, the tangent space is a theoretical concept. Tangent space is similar to a vector space, particularly in terms of linearity. From a field's perspective, a subfield is a subset of the original field that shares the same properties as the original field. Subfield is logically similar to a subspace of a vector space. A vector is an element of a vector space. Weight vectors can be used to describe motion, which can be applied in pattern recognition. A linear combination of weight vectors can describe a sequence of motions. Kernel graph theory treats a graph as an element of its theoretical structure. Kernel graph theory can also be applied in pattern recognition. So, basically, this article presents a perspective on applying vector space in pattern recognition. The inherent logic of vector space can be applied to both motion synthesis and graph pattern recognition. The linearity of the vector space is the main focus of this article.

Keywords: Vector space; linearity; tangent space; kernel graph theory; pattern recognition.

1. Introduction

This article aims to provide a brief review of vector space. A vector space is an additive abelian group based on a field. The operations of a vector space are addition and multiplication. The elements of a vector space are called vectors or points. Vector has an inverse and is closed. So does the point. Vector spaces form a group. There are two types of fields: the real field and the complex field. This article only discusses the real field. A vector space is a type of linear space. This article also discusses examples of linear space because vector space and linear space examples share similarities.

After completing the review, this article demonstrates the application of vector space in various contexts. Since a vector space is a fundamental concept in linear algebra. This concept can be applied in both mathematics and computer science. This article discusses some fundamental concepts associated with vector spaces. Tangent space is a concept based on the tangent line, tangent surface, and vector space. Tangent space can be described as a linear space of differentials. The tangent space can be used in the study of differential geometry [1]. Based on the structure of the tangent space, additional concepts in differential geometry can be introduced. Tensor is also a linear space. This article will introduce two concepts.

Since a vector space is a group over a field. One basic concept, called a subfield, can be introduced. Basically, a subfield can be considered as a part of the field and also shares similarities with the original field [2]. This concept can be applied in field theory. Field theory is a branch of Galois Theory. The mathematics section of this article is now complete. This article demonstrates the application of vector space in computer science. In motion synthesis, vectors can be used to describe motions. Linear combination is a concept based on addition and multiplication. In motion synthesis, it can be used to describe a variety of different motions. Linear combination is effective in describing sequences of motions. This concept can be used to describe human movements [3]. And when people made 3D geometric models of faces. Some motion data can be described by a shape vector and a timing vector. Shape-vector shows the variation in shape. The timing vector shows the variation of time. And these two types of vectors adhere to the principles of the vector space. Two vectors can essentially form a variety of shapes depending on time. Since motion synthesis can describe the variation of motion, especially human movements [4]. This technique can be used in the entertainment industry. Such as animation, film, games, and other visual works. Another application is related to kernel theory. Kernel theory is a concept about clustering. The graph kernel is a bilinear space. The element of a graph kernel is a graph. Kernel theory can be applied in pattern recognition. It is used in the transformation from one graph to another graph without altering each individual feature vector. The focus of graph kernel objects here is feature vector spaces. The graph lacks mathematical structure [5]. Because of the kernel theory. Studies of the graph can provide a fresh perspective on pattern recognition. Pattern recognition can be used in various fields. Since pattern recognition is a part of clustering. Pattern recognition can be applied in various fields such as finance, artificial intelligence, data science, and other data-related work [6].

This article divides the concepts of vector space into several sections. Each part presents specific cases in this article. A vector space is an additive abelian group over a field. Focusing on linear space, this article discusses tangent space, tensors, and kernel theory. This article focuses on the elements of a vector space and demonstrates linear combinations. This article focuses on field theory and introduces some fundamental concepts related to it. This article explores the mathematical structure and significance of vector spaces. Based on the mathematical concept of vector space, this article demonstrates the application of several related concepts. This article can inspire ideas for applying vector spaces. This concept can be transformed into various forms because it is timeless and uncomplicated.

2. Methods

A vector space is a fundamental concept in linear algebra, which is a basic course in mathematics. A vector space is essentially an additive abelian group over a field.

2.1. Vector Space Properties

Commutativity: $\forall a, b \in V: a + b = b + a$; Associativity:$\forall a, b, c \in V: a + b + c = a + (b + c)$

Additive identity:$\forall a \in V, \exists 0 \in V: a + 0 = 0 + a = a$; Additive inverse:$\forall a \in V, \exists(-a) \in V: a + (-a) = (-a) + a = 0$ Multiplicative identity: $\forall a \in V, \exists 1 \in F: 1 \cdot a = a \cdot 1 = a$ Distributive properties:$\forall a, b \in V, \forall \alpha, \beta \in F: \alpha(a + b) = \alpha a + \alpha b; \forall a, b \in V, \forall \alpha, \beta \in F: (\alpha + \beta)a = \alpha a + \beta a$

This concept is widely used in solving various mathematical problems. Such as differential geometry and Galois Theory, among others. This concept is widely used in solving various questions related to computer science. Such as motion synthesis and pattern recognition, among others [7]. Vector spaces can be applied in differential geometry.

2.2. Definition of Tangent Space and Cotangent Space

Any point on a regular surface or curve has a tangent plane or tangent line at that location. It makes sense to build a differentiable structure on a topological manifold and to use a linear space (or occasionally a vector space) to approximate the neighborhood of any point. Thus, it is possible to introduce the ideas of tangent space and cotangent space [8].

2.3. Definition of F-valued Function and Dual Space

Concepts in differential geometry can be introduced. Suppose f: V→F is an F-valued function on V. If for any $v_1, v_2 \in V$and $\alpha^1, \alpha^2 \in F$, $f(\alpha^1 v_1 + \alpha^2 v_2) = \alpha^1 f(v_1) + \alpha^2 f(v_2)$.

Then f is called an F-valued linear function on V. Obviously, if f, g are F-valued linear functions on V and $\alpha \in F$, then f+g and αf are also F-valued linear functions on V. Thus, the set of all F-valued linear functions on V forms a vector space over F, called the dual space of V, denoted by V^*.So,the concepts of linear function and dual space can be introduced [9].

2.4. Definition of Tensor

Suppose V is an n-dimensional vector space over F with dual space V^*.The elements in the tensor product: $V_r^s = V \otimes \ldots \otimes V \otimes V^* \otimes \ldots V^*$. r times V and s times V^* are called (r, s)-type tensors, where r is the contravariant order and s is the covariant order [9]. Vector space can be applied in Field theory.

3. Results and Discussion

If F is a field and E is a subset of F, then F as a whole becomes a field upon addition and multiplication. In other words, we will refer to F as an extension of E if E is a subfield of F. Thus, the idea of extension can be discussed [10]. Vector spaces can be applied in motion synthesis.

The idea of a linear combination is founded on vector space features. Motion synthesis involves the linear combination of stationary objects. Its foundation is a data set of stationary items belonging to the same class. It is expected that all of these exemplar items correspond fully, which can be accomplished by employing methods based on optic flow algorithms. A linear combination of a set of m exemplar objects in full correspondence, each of which is characterized by a feature vector $X_1, \ldots\ldots, X_m$(such as a pixel for a 2D image or a vertice for a 3D geometries), yields a new item in the same class: $X = \sum_{i=1}^{m} w_i X_i$

Unless the 2D images align pixel-to-pixel, for instance, a simple linear combination of them would appear like a translucent superposition of various images rather than a new image in the same class. Hence, this linear combination is meaningful or legitimate because all the samples are in full correspondence. A linear vector space's foundation is made up of the vectors. The procedure parametrizes a continuous class of objects, and each object in this class is compactly characterized by the weight vector $\vec{w} = (w_1, \ldots\ldots, w_m)$ [4].

Pattern recognition can make use of vector space: One common task in pattern recognition is clustering. Kernel process:$R^n \times R^n \to R$ One can profit from the large library of algorithms found in vector spaces as well as the high representational power of graphs from the definition of the kernel function [5].

3.1. Theorem of Partial Derivative of Tensor

Suppose $f^1, \ldots, f^s \in C_p^\infty$ and F $(y^1 \ldots y^s)$ is a smooth function in a neighborhood of $(f^1(\text{p}), \ldots, f^s(\text{p}))$ is a smooth function in a neighborhood of $(f^1, \ldots, f^s) \in R^s$. Then $\text{f} = \text{F}(f^1, \ldots, f^s) \in C_p^\infty$ and $(df)_p = \sum_{k=1}^{s}[(\frac{\delta F}{\delta f^k}(f^1(\text{p}), \ldots, f^s(\text{p})) * (df^k)_p]$.

Proof: Suppose the domain of f^k containing p is U_k. Then f is defined in $\bigcap_{k=1}^{s} U_k$, for q$\in \bigcap_{k=1}^{s} U_k$, f (q) =F $(f^1(\text{q}) \ldots\ f^s(\text{q}))$. Since F is smooth function, f$\in C_p^\infty$.

Let $a_k = \frac{\partial F}{\partial f^k}(f^1(\text{p}) \ldots\ f^s(\text{p}))$.Then for any $\gamma \in \Gamma_p, \ll \gamma, [\text{f}] \gg = \frac{d}{dt}|_{t=0}(\text{f} \circ \gamma) = \frac{d}{dt}|_{t=0}\text{F}(f^1 \circ \gamma(\text{t}), \ldots, f^s \circ \gamma(\text{t})) = \sum_{k=1}^{s} a_k \frac{d}{dt}|_{t=0}(f^k \circ \gamma(\text{t})) = \ll \gamma, \sum_{k=1}^{s} a_k [f^k] \gg$.

Thus, $[\text{f}] - \sum_{k=1}^{s} a_k [f_k] \in \text{H}_p$, i.e., $(df)_p = \sum_{k=1}^{s} a_k (df^k)_p$[2]

3.2. Theorem of Division between Fields

If F, B, E are three fields such that $\text{F} \subset \text{B} \subset \text{E}$, then $(\text{E/F}) = (\text{B/F})(\text{E/B})$. The elements of E are$A_1$,$A_2 \ldots$, A_r.The elements of B are C_1,$C_2 \ldots$,C_s.

Assume that elements in E that are linearly independent of B and elements in B that are independent of F. Next, the products $C_i A_j$.where the elements of E that are independent of F are i=1, 2…s and j=1, 2…r.As a result, we get $\sum_i a_{ij} C_i$=0 for each j. If $\sum_{i,j} a_{i,j} C_i A_j$=0, then$\sum_j(\sum_i a_{ij} C_i)A_j$ is a linear combination of the with coefficients in B. The A_j were independent of B. Then, for the C_i to be independent of F, each a_{ij} must equal 0. Given that there are r·s elements, we have demonstrated that the degree $(\text{E/F}) \geq \text{r} \cdot \text{s}$ for any $\text{r} \leq (\text{E/B})$ and $\text{s} \leq (\text{B/F})$. Consequently, $(\text{E/F}) \geq (\text{B/F})(\text{E/B})$. The theorem is proved if one of the latter integers is infinite. We can assume that the A_j and the C_i are generating systems of E and B, respectively, if both (E/B) and (B/F) are finite, say r and s, respectively. We then demonstrate that the set of products $C_i A_j$ is a generating system of E over F. You can express each A $\in$ E linearly in terms of the coefficients in B. So, $\text{A} = \sum B_j A_j$ Furthermore, each element of B may be written linearly in terms of the C_i using coefficients in F, i.e., $B_j = \sum a_{ij} C_i$ j=1, 2… r. As a result, $A = \sum a_{ij} C_i A_j$ and the $C_i A_j$ create a separate generating system [3].

3.3. Motion Attributes

We compute the weighted sums $\Delta S = \sum_{i=1}^{m} \mu_i (S_i - S), \Delta T = \sum_{i=1}^{m} \mu_i (T_i - T)$. based on a collection of motions (S_i, T_i) with labels μ_i indicating the markedness of the attribute. Now, each single motion produced by the motion model can have multiples of $(\Delta S, \Delta T)$ added to or subtracted from it. This will modify a particular attribute while maintaining the stability of all other attributes. Caricature in motion is also an option. By making a motion further far from the usual motion, it might be caricatured for E over F [4].

3.4. Definition of Graph Kernel and Definition of Graph Embedding

Definition 1 (Graph Kernel). Let G be a finite or infinite set of graphs, $g_1, g_2 \in G$ and $\varphi : G \to R^n$ a function with $n \in N$. A graph kernel function is a mapping $\kappa : G \times G \to R$ such that $\kappa(g_1, g_2) = <\varphi(g_1), \varphi(g_2)>$.

Definition 2 (Graph Embedding). If $G = \{g_1, \dots, g_n\}$ is a set of graphs and $P = \{p_1, \dots, p_n\} \subseteq G$ is a set of prototype graphs, the mapping $\varphi_n^P : G \to R^n$ is defined as the function $\varphi_n^P(g) \to (d(g, p_1), \dots, d(g, p_n))$ where $d(g, p_i)$ is the graph edit distance between graph g and the i-th prototype [5].

4. Conclusion

This article introduces some fundamental concepts about vector spaces. Vector spaces can be used in various mathematical and computational applications. Vector spaces can be used to introduce some basic definitions. Some basic theories can also be introduced. Since a vector space is a classic example of linearity. This article focuses on demonstrating the linearity of concepts. This article introduces basic concepts in differential geometry, field theory, pattern recognition, and kernel graph theory. Differential geometry and field theory are mathematical theories. Pattern recognition and kernel graph theory are computational theories. Since the definition of a vector space involves three concepts: group, field, and vector. This article explores the concept of tangent space and linear space within a group. This article explores a specific aspect of the field by introducing the concept of subfields. Weight vector uses a vector to represent the properties of motion. A vector is an element of a vector space. This article discusses kernel graph theory, which is a theory that treats graphs as elements. From the perspective of the definition of a vector space, this article can help readers expand their understanding of vector spaces, particularly the concept of linearity within a vector space. This article shows that linearity is a crucial property of vector spaces. The main focus of this article is linearity.

Both linear operations and bilinear operations are widely used in various works in the fields of mathematics and computer science. All concepts in this article are based on these two principles. The definition of a vector space is described by linear operations involving vectors and numbers. Similarly, linear operations and bilinear operations encompass various theoretical structures related to linearity.

Based on the concept of vector space. This article can also help readers understand the structure of space. Kernel graph theory shows that the elements of space are not necessarily numbers or vectors. The definition of the tangent space shows that the structure of space can be a topological manifold. This article demonstrates the inherent logical relationship between vector spaces and other fundamental concepts. This article presents both theoretical and practical concepts.

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