

Using Matrix Analysis to Assist in Completing Investment Decisions

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Abstract. Economics is an essential type of science currently, and with the development of linear algebra becoming increasingly rich, it is widely used in problem-solving in economics, especially in the matrix analysis part. When making investment decisions, the help of matrices is indispensable, therefore, extensive learning is needed to make good use of mathematics to solve practical problems. Many models can be applied to matrix analysis. This article utilizes the famous Markowitz combination optimization model, utilizes the covariance matrix, and analyzes how to invest in four stocks based on the idea of minimizing investment risk while obtaining expected returns, making investment decisions very reasonable. In many similar situations, Markowitz combination optimization models can be used to assist investment decision-making, often resulting in better results. Finally, the Markowitz optimization portfolio can be used to solve many investment portfolio points on the effective frontier, and then fitted to these points to obtain the effective frontier. Based on the effective frontier and combined with the Sharpe ratio, the best investment portfolio can be selected from the effective frontier. This article can solve many investment decision-making problems in economics.

Keywords: Linear algebra; matrix; investment decision; Markowitz combinatorial optimization.

1. Introduction

Linear algebra is widely used in economics, so how is it used? This is where this paper comes in. Linear algebra is a branch of mathematics that is of great significance and its application in economics is receiving more and more attention. In some complex economic systems, linear algebra provides such a powerful tool for analyzing various economic problems, from market supply and demand to risk management, from optimization to investment decisions. This article explores the application of linear algebra in the field of economics and its potential for solving and predicting economic problems, especially in investment decisions.

Linear algebra was born from the study of determinants in 1693. Between 1800 and 1900, there were rapid and significant developments in the context of linear algebra. Some theorems for determinants, the concept of eigenvalues, diagonalization of a matrix, and similar matrix concepts were added to linear algebra by Cauchy [1]. Through the efforts of these mathematicians, the embryonic of linear algebra has been enriched.

Another mathematician Cayley was the first to study the matrix as an independent object. In 1858 he published an extremely important article in the field of matrices, "A Memoir on the Theory of Matrices", which defined the representation of determinants, matrix addition, zero matrices, inverse matrices, and so on. Covariance is a measure of the degree and direction of interdependence between two random variables. Later, in 1993, the definition of covariance matrix emerged. When there are more than two variables, the matrix formed by the covariance between the two variables is called the covariance matrix. Whether in econometrics or financial engineering, covariance matrices are important tools that can be used. In the following text, the covariance matrix will also be used.

In terms of economics, David Hume was known as a philosopher during the Scottish Enlightenment who studied history, philosophy, religion, economics, and more extensively. At the same time, he was also the earliest and most systematic expositor of monetary quantity theory, influencing the entire classical era of monetary theory.

Hume used the theory of quantity of money to oppose the mercantilist concept of equating money with wealth, believing that the amount of money does not have any impact on the country and is not directly related to the amount of wealth [2]. The founder of classical economics, Adam Smith, is known as the father of economics. He wrote the book "The Wealth of Nations". He said that guided by the "invisible hand" of price mechanisms, people not only maximize their interests but also promote public interests [3]. In investment decision-making, minimizing risk and maximizing benefits are also the goals pursued.

Besides, Acemoglu, Johnson, et al. developed a research agenda that went in quest of a better understanding of the historical origins of recent institutions and their meanings for long-run economic development [4]. Game theory is the mathematical study of the possible choices that players can make to win in games. Therefore, game theory can be used to improve the correctness of investment decisions. This project will show how to use linear algebra matrix computations as a powerful tool to solve game theory problems [5]. To make a better investment decision, the investor ought to fully and correctly understand the possible opportunities he may face, and these decisions should not be made in haste. A wrong investment decision can even lead companies to bankruptcy [6].

The history of the development of economics is a rich and colorful process, and the rise and development of different schools and theories reflect different understandings and interpretations of economic phenomena in different periods and economic conditions. With the help of linear algebra, economic problems can be easily solved, and at the same time, new problems will arise. To achieve the goal of this paper, reading some classic literature on economics and some classic literature on linear algebra is significant.

Through a comprehensive literature review and case analysis, this article aims to provide a comprehensive perspective for economic researchers and policymakers. Prove the importance of linear algebra in making investment decisions and predicting future trends. Especially, matrix will help a lot in investment decisions. Believe that linear algebra will continue to play a crucial role in the field of economics, helping to better understand and respond to ever-changing economic challenges.

2. Methods

2.1. Markowitz Combinatorial Optimization

2.1.1. Brief introduction

In the process of investment decisions, people often hear terms such as "investment portfolio" and "asset allocation", which mean reducing the risks in the investment process through diversified investment. Therefore, the question arises, how to define investment risk and how to diversify investment to achieve the optimal ratio of returns and risks are our assignments.

Harry Markowitz came out with an article titled "Securities Portfolio Selection" in 1952, marking the start of modern securities portfolio theory. In a single period, of investment, investors purchase and hold a portfolio of securities at a predetermined ratio of funds. At the end of the maturity period, the Markowitz model intends to optimize the Portfolio to obtain the optimal point of risk and return.

In the opinion of Markowitz's portfolio theory, the risk of investing in a unique stock can be grouped into systemic and non-systemic these two risks. Systemic risks are generally referred to as market risks, such as macroeconomic or policy changes. These risks have particularly significant impacts on individual stocks. However, if ways can be found to increase the number of stocks in the portfolio, this type of risk can be dispersed, which is also commonly known as "don't put chicken eggs in the same basket".

2.1.2. The presumption of the Markowitz model

Assumption 1: Investors are rational and have the same investment goals and time perspective.

Assumption 2: Investors' investment decisions are based on expected returns and risks, ignoring other factors.

- Assumption 3: The expected returns and risks of assets can be measured using statistical methods.
 Assumption 4: Investors can borrow or invest any amount of funds without costs or restrictions.
 Assumption 5: Investors can trade at any time with zero transaction costs.
 Assumption 6: The securities market is completely efficient.

2.2. Definitions

Definition 1. Expected return on investment portfolio:

$$\mu_p = w^T \mu \quad (1)$$

Among them, (μ_p) is the expected return on the investment portfolio, and (w) is the weight vector of the investment portfolio, (μ) is the expected return vector of an asset.

Definition 2. Variance of expected investment portfolio:

$$\sigma_p^2 = w^T \Sigma w \quad (2)$$

Among them, (σ_p^2) is the variance of the investment portfolio, (Σ) is the covariance matrix of asset returns.

Definition 3. Sharpe ratio:

$$\text{Sharpe Ratio} = \frac{\mu_p - r_f}{\sigma_p} \quad (3)$$

Among them, (r_f) is a risk-free interest rate.

Definition 4. Optimal Portfolio Weight:

$$w^* = \frac{\Sigma^{-1}(\mu - r_f \mathbf{1})}{\mathbf{1}^T \Sigma^{-1}(\mu - r_f \mathbf{1})} \quad (4)$$

$$\Sigma w^* = (\mu - r_f \mathbf{1}) \quad (5)$$

Among them, $(\mathbf{1})$ is a column vector that is all 1. These formulas are the foundation of Markowitz's combinatorial optimization. In investment decision-making, especially when using matrix analysis, some concepts and formulas of linear algebra are very useful. The following are some formulas related to linear algebra that may involve the construction, optimization, and analysis of investment portfolios.

Definition 5. Singular value decomposition, SVD.

In some cases, singular value decomposition can be used to analyze the asset return matrix. Assuming the asset return matrix is (X) , and its SVD is $(X=U \Sigma V^T)$, where (U) and (V) are orthogonal matrices, (Σ) is a diagonal matrix. Such decomposition may help identify the main market factors or specific patterns in investment portfolios.

The Markowitz problem has three forms at least. The first one is to minimize the risk and limit the return at the same time, the second is to maximize the return while keeping the risk under control, and the third is a multi-objective problem that minimizes the mixture of risk and return after parameterizing their ratio. We can choose the most suitable way to complete the decision based on the actual needs of the problem.

3. Results and Discussion

The stock market has always been a hot spot for investment and understanding the price fluctuations of stocks is crucial for investment decision-makers. Here, we will construct an investment portfolio using four stocks: Apple (AAPL), Tesla (TSLA), Microsoft (MFST), and Amazon (AMZN) as examples.

Firstly, we obtained the daily closing price data of the four stocks mentioned above in the second half of 2022 and the first half of 2023. Through Python and Pandas data reader and matplotlib in Python to plot the stock price data into curves. Furthermore, the yield series of four stocks can be calculated. Then calculate the average daily returns of four stocks separately (Table 1).

Table 1. Average daily return.

Stock Name	Average daily return
AAPL	a%
TSLA	b%
MFST	c%
AMZN	d%

And the next step is to calculate the covariance matrix of stock returns.

Table 2. The covariance matrix of stock returns.

	AAPL	TSLA	MFST	AMZN
AAPL	a_{11}	$a_{12}= a_{21}$	$a_{13}= a_{31}$	$a_{14}= a_{41}$
TSLA	a_{21}	a_{22}	$a_{23}= a_{32}$	$a_{24}= a_{42}$
MFST	a_{31}	a_{32}	a_{33}	$a_{34}= a_{43}$
AMZN	a_{41}	a_{42}	a_{43}	a_{44}

It should be noted that the covariance matrix is symmetric, with variance on the diagonal and covariance on the non-diagonal (Table 2). Based on being able to define investment risk and return, it is worth returning to the original intention of the mean-variance theory, which is to obtain a set of weights to achieve the optimal ratio of risk and return for investment portfolios configured with this weight. Without considering how to find the optimal weights for this group, if one randomly generates many weight combinations and allocates the four stocks mentioned above according to these weight combinations, one can obtain the corresponding risks and returns for each combination. If one plots these combination points on the horizontal axis of risk and the vertical axis of returns, there could be a chart.

For a given four stocks, constructing all possible combination configurations can miraculously discover that these combination points converge into a bullet-like shape. From the analysis in the figure, it can be concluded that the lower half of the bullet is invalid because the lower combination has higher returns than it under the same risk, the lower combination could be taken. Using the same analytical framework, one can assume that there is an optimal investment portfolio at the boundary above the bullet, which is also known as the Markowitz efficient frontier. Of course, since the bullet boundary in the figure above was obtained by simulating a finite number of times, the actual effective front is different from the boundary shown in the figure.

Finally, solving the problem of how to diversify risks. And at the same time, achieving the optimal ratio of returns and risks is needed. The core of the Markowitz model is to minimize risk based on a given rate of return, that is, to minimize non-systemic risks as much as possible by diversifying investments [7].

Therefore, Markowitz's Optimal Investment Portfolio and Analyzing Portfolio Risk could be constructed like this [8]. $Min \sigma^2(r_p)=\sum \sum x_i x_j Cov(R_i, R_j), R_p = \sum x_i R_i$, and $\sum x_i = 1, x_i \geq 0$.

Among them, (R_p) is the expected return on the investment portfolio, (x_i) is the proportion of stock i in the investment portfolio, $[\sigma^2(r_p)]$ is the risk faced by the investment portfolio. Based on the model, many investment portfolio problems on the effective frontier could be solved and fit these points to obtain the effective frontier. Further, the optimal investment portfolio can be selected from the effective frontier based on the risk-adjusted return index (Sharpe Ratio $= \frac{\mu_p - r_f}{\sigma_p}$).

Table 3. Effective frontier.

	weight	Portfolio income	Risk of portfolio	Sharpe ratio
AAPL	A			
TSLA	B			
MFST	C	E	F	G
AMZN	D			

Assuming an initial investment amount of \$10000, the optimal portfolio asset value change curve can be plotted, and the asset portfolio value change curve with equal weight allocation can be used as the benchmark. It is easy to find that the optimal investment portfolio can generally outperform the benchmark investment portfolio.

SVD can be used to help identify major market factors or specific patterns in investment portfolios. Because the Markowitz portfolio investment model is based on a series of strict assumptions, its actual effectiveness and application scope are limited to some extent [9]. What are the shortcomings of the Markowitz model? The first point is that Markowitz's model did not consider the issue of transaction costs. Secondly, many stock markets are not the fully efficient markets mentioned by Markowitz in his hypothesis. The third point is that the returns of individual stocks relying on are a series of returns from the past, while stocks are predictions of future profitability, and the future is unknown. History may have similarities. However, it will not be the same or repeated [10]. Everything has both positive and negative sides. Some means can be used to minimize the negative side. Relaxing the assumptions of the model, expanding it, and establishing corresponding models are examples.

4. Conclusion

This article constructs a Markowitz optimization combination model, which combines covariance matrices in linear algebra, and calculates the corresponding investment weights of four stocks, resulting in a relatively good investment decision-making method. In addition to calculating the minimum investment risk based on obtaining the expected return, it is also possible to achieve the maximum investment return at an acceptable level of investment risk. Although Markowitz's model provides us with a new perspective on thinking problems, however, extreme values can easily occur through the Markowitz model.

Therefore, when using the Markowitz portfolio optimization model, one can compare and choose the appropriate securities portfolio. If necessary, the Markowitz portfolio optimization model can be expanded to consider more practical securities market conditions, so that such a portfolio optimization model can be used in more situations.

From this perspective, securities portfolio investment models should be considered with transaction costs, minimum transaction units, time-varying securities portfolio investment models, Merton continuous time stochastic models, or mean semi-variance securities portfolio investment models to improve the Markowitz portfolio model.

The Markowitz combination optimization model can solve many investment decision-making problems, proving that eggs should not be put in one basket. At the same time, some shortcomings and improvement methods of this model were also discovered to better serve economics, which is the purpose of this article's writing.

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