

Research on a Simulation and Prediction Method of Traffic Flow Information Based on PINN

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Abstract. Traffic flow prediction can help intelligent transportation effectively be carried out. This paper employs the Physics-Informed Neural Network (PINN) as a deep learning method and partial differential equation (PDE) multivariable methods to approximate solutions for the Lighthill-Whitham-Richards (LWR) traffic flow equation system. The relationship between traffic flow density function and time/displacement is obtained by setting different function conditions and initial boundary constraints. Furthermore, a series of three-dimensional graphics are generated during the research process through dynamic comparisons and evaluations of independent variables to intuitively display and analyze traffic flow trends and gradients. Additionally, before the commencement of the study, the authors propose specific initial condition combinations corresponding to real-world application scenarios to enhance the practicality of the research. Moreover, through the interpretation and reflection on the obtained images, the study identifies inconsistencies between actual situations and predictions, leading to necessary adjustments. Furthermore, the research explores the compatibility of the $u(x)$ function and hard constraint conditions in different scenarios, aiming to accurately predict traffic information and ultimately enhance the utilization of traffic resources. By leveraging PINN and PDE technologies, this study contributes to advancing traffic flow analysis and optimization, potentially influencing urban planning and traffic management strategies positively.

Keywords: Deep learning; Physics-Informed Neural Network; LWR; Traffic flow prediction.

1. Introduction

In recent years, traffic congestion has become an increasingly serious issue in both developed and developing countries. The rapid increase in the number of motor vehicles on roads has put immense pressure on existing transportation systems, resulting in chronic traffic congestion and causing significant but unavoidable economic losses [1]. Since the Second Industrial Revolution, internal combustion engine vehicles have played an increasingly important role in human history, leading to a growing demand for road and traffic planning. To address this phenomenon, the field of "traffic flow theory" has emerged, aiming to understand, model, and optimize vehicle movement on road networks. This field has played a crucial role in providing traffic patterns and congestion dynamics [2].

With the development of the times, various solutions have been derived for traffic flow problems according to different requirements. Despite a macroscopic phenomenon appearing by the general public, traffic flows can be approximated in certain contexts as "a fluid in which numerous tiny particles are directed to move in a certain direction." Therefore, fluid-related analysis methods or mechanical wave-related formulas and methods can sometimes be applied to help solve [3]. In the early days, several vehicles did not display an obvious burden to the traffic. Additionally, with the inferior equipment, cars can hardly switch their route quickly. So, the statistic theory could satisfy the demand for arrangement. With the development of hardware and the number of vehicles, people need more accurate mathematic tools to solve further questions. Micro calculation became a great solution, which can combine the velocity of flow and cars, density of cars, find out and add up the derivative solution of tiny time periods [4]. After the virtual of Artificial Intelligence (AI) tech, a ton of research fields benefited from it, as the traffic flow. Machine learning can approximate the trend of the future density of cars according to the past database or based on existing algorithms. This essay is going to focus on this aspect, using an algorithm to construct a trend of a given time period.

The main objective of this research is to predict the traffic flow density solution within a given time period by introducing a Physical Informed Neural Network (PINN). PINN combines physical information with the Divine Network Learning method, which integrates physical equations and neural networks to provide more accurate and reliable predictions. This means that PINN can adhere to the laws of physics and incorporate them into its training process to enhance prediction accuracy. Specifically, first, PINN and Partial Differential Equation (PDE) are utilized to establish the fundamental network structure for approximating the true density value through computational processing of provided data and adjustment of neural unit weights. Second, Lighthill-Whitham-Richards (LWR) function sets and models are employed to reduce conditions, enabling the simulation of various real-world scenarios to vary solution conditions [4]. Thirdly, different models' prediction performance is analyzed and compared. Experimental results demonstrate that the proposed model effectively predicts and evaluates traffic flow, with quantitative results presented in tabular form in Section 4. This study aims to optimize traffic arrangements more efficiently and facilitate routing for achieving higher vehicle capacity.

2. Methodology

The fundamental tools used in this project include PDE solving, LWR traffic flow model, and PINN network architecture.

In this study, PINN is introduced to predict the density of traffic flow in a given time period to propose a solution for effective traffic organization. The PINN is used to improve the optimization of the model for infinite approximation by combining physical information and good network learning. Specifically, first, the PINN and PDE are utilized to provide the basic network structure that computationally processes the given data and varies the neural unit weights to approximate the true value of the density. Second, the LWR function set and model reduction conditions are utilized, which helps to vary the solution conditions to simulate various real-world situations. Third, the prediction performance of different models is analyzed and compared. The overall process is shown in Fig. 1.

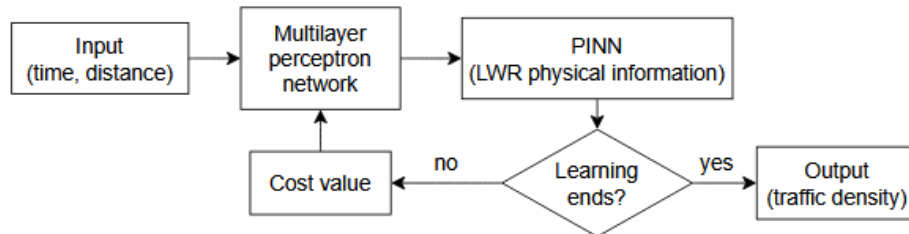


Fig. 1 Overview processing graph of this essay

2.1. Partial Derivative Equation (PDE)

In solving the PDE for traffic flow density, a series of initial conditions were established, which includes different $u(x)$ as mentioned in the previous equation, as well as a hard limitation for another independent variable (manifested as traffic density at the starting distance in this paper). The ability to set the value of the independent variable is due to the construction of the traffic flow model being based on solving two independent variables - space and time. When solving for one variable, known initial conditions can be set for another. In this experiment, by setting different boundary conditions, approximate values of traffic flow density within a specified range and predictions of its changing trend were provided.

The author found the proper initial parameters and function models (a, b, c) from [5].

Greenshields' parabolic curve (1): Traffic density relationship forms a parabolic curve.

$$q(\rho) = \frac{4q_{max}}{k_{max}} \left(k - \frac{k^2}{k_{jam}} \right) \quad (1)$$

Piecewise linear traffic density relationship (2): To solve for the $u(x)$ function the change in traffic flow with density varies linearly within different density ranges.

$$q(p) = \min\{\rho \cdot u_{max}, q_{max}, \omega(\rho_{max} - \rho)\} \quad (2)$$

Nonlinear traffic density relationship (3): In nonlinear relationship cases, the traffic density relationship may be nonlinear, such as in congested traffic flow.

$$q(p) = -\lambda \log \left(\exp \left(-\frac{\rho \cdot u_{max}}{\lambda} \right) + \exp \left(-\frac{q_{max}}{\lambda} \right) + \exp \left(-\frac{\omega(\rho_{max} - \rho)}{\lambda} \right) \right) \quad (3)$$

2.2. Physical Informed Neural Network (PINN)

A physics-informed Neural Network PINN combines neural networks with physical information, allowing for the integration of model parameters (Fig.2). (σ = weight, u = output, l , ∂_t , ∂_x , ∂_{xx} ..., = after several times derivative calculation. The graph did not show the whole structure, R = cost value, L = sum of loss value from initial, boundary, PDE function).

The underlying principle of PINN capitalizes on the strengths of neural networks and the constraints imposed by physical equations. With the parameters mentioned in the first paragraph confirmed, the operation can be carried out. Initially, a neural network is employed to represent unknown physical quantities, such as temperature, speed, or displacement. Subsequently, the network parameters are optimized by minimizing the loss function, ensuring that the neural network's output satisfies both the physical equations and boundary conditions. The loss function comprises two components: the residual term of the equation and the boundary condition. The equation's residual term quantifies the disparity between the physical equation and the neural network's output, while the boundary conditions are applied to the network's output to guarantee accuracy at the boundaries.

Although scholars briefly researched it in the last century, the effective utilization of this architecture was hindered due to limited resources and demand. However, with the advancement of science and technology, scholars revitalized this technology in 2017 and promoted its application [6].

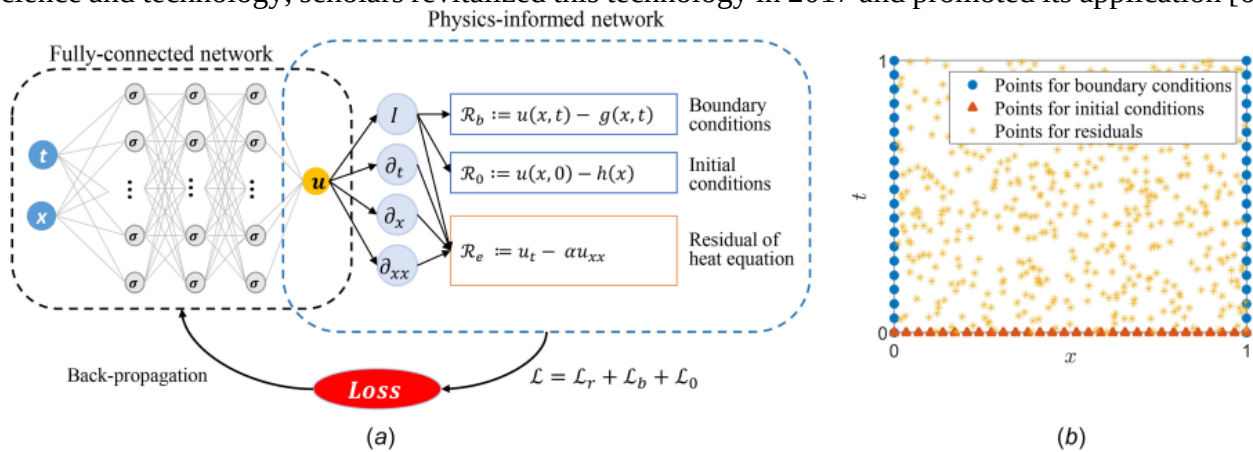


Fig. 2 Physical informed neural network [7]

2.3. Lighthill-Whitham-Richards (LWR) function group

The LWR equation group integrates the basic logic of these three components, elucidating and establishing the relationship between traffic flow speed, traffic flow density, and vehicle speed. Initially, LWR consisted of a system of three equations.

$$\begin{cases} \frac{\partial \rho}{\partial t} + \frac{\partial q}{\partial x} = 0 \end{cases} \quad (4)$$

$$u = u(\rho) \quad (5)$$

$$q = \rho \cdot u \quad (6)$$

However, researchers in practical situations typically transform it from a group form into a single like (7):

$$\frac{\partial \rho(x,t)}{\partial t} + \frac{\partial q(\rho(x,t))}{\partial x} = 0 \quad (7)$$

This has been extensively researched and rigorous derivations have been proposed [8]. In simple terms, when considering traffic congestion scenarios in the LWR model, traffic density increases with time, while traffic flow speed decreases with increasing positive displacement (reflecting an increase or decrease in positive or negative forms). The vehicle speed is represented by constructing a function based on traffic density. In this paper, quadratic functions, piecewise functions, and high-order complex functions from Felix Rempe are used.

After introducing the LWR equation, researchers found limitations in its ability to describe real-world traffic flow situations, particularly in dealing with non-equilibrium states. Several higher-order continuous equations were proposed as improvements [9] [10]. However, these higher-order equations have the complexity of parameter acquisition and expensive computational time costs, which may not necessarily make them more practical than the LWR model. Therefore, this paper focuses on solving the relevant problems based on the LWR equation.

2.4. Applying in Experiments

In terms of neural network structure, this passage uses a multi-layer perceptron (MLP) as the basic structure of PINN. After the testing, the author decided the network in Fig.3 as the final structure. If the layers are too much, the hardware requirement and time cost will increase explosively. If vice, the accuracy will decline which will hardly approximate a satisfying model. Usually, the network preserves thick at the middle and thin at the beginning as well as the end, while the parameter in perceptron neural network will correct the parameter stable effectively according to the initial designing prototype, i.e. the neural system like mammal brain.

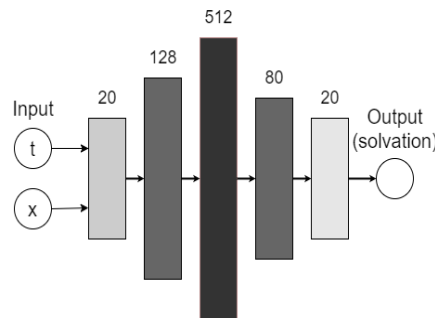


Fig. 3 Structure applied to this study

2.5. Loss Function

This passage uses a loss function to measure the difference between the model output and the actual values and trains the model by optimizing the loss function, such as adjusting the learning rate. Common loss functions include Mean Squared Error (MSE), which measures the average squared difference between the output and actual values, as well as other loss functions suitable for specific problems. The author uses MSE (8) to solve the problem.

$$MSE = \frac{1}{n} \sum_{i=1}^n (Y_i - \hat{Y}_i)^2 \quad (8)$$

3. Experiment and Analysis

This chapter first introduces the operational principles of the experiment. Second, the author shows and discusses the experimental results.

The goal of the study is to predict traffic flow density by providing necessary initial constants, such as maximum traffic flow velocity and maximum traffic flow density, and solving boundary relationships with partial differential equations, which are multivariable equations. Approximate predictions of traffic flow density at specific times and distances can be made. When applying hard

constraints to the same function, setting different initial conditions wisely is crucial. Different $u(x)$ functions can lead to different fitting accuracies and reflect different scenarios. The following content demonstrates the different fitting results when the hard constraint conditions interact with the $u(x)$ function and provide corresponding dynamic analyses, giving possible scenarios that may apply to different initial conditions.

3.1. Periodic Boundary Condition (PBC)

The sine function corresponds to periodic traffic congestion: When the boundary condition is a sine function, this passage can simulate traffic congestion with PBC. In this case, the density and velocity of traffic flow will vary periodically, leading to periodic traffic congestion.

For the expected results of the sine periodic function, a slanted pattern with time delay or stable periodic variation is expected. This means that under the change of time t , different positions related to distance x can be observed for the conduction of traffic flow density, or it can stabilize within a range of fluctuations. To explain the phenomenon of time "delay," it can be understood as the peak of traffic flow density spreading towards the direction closer to the origin with the increase of time.

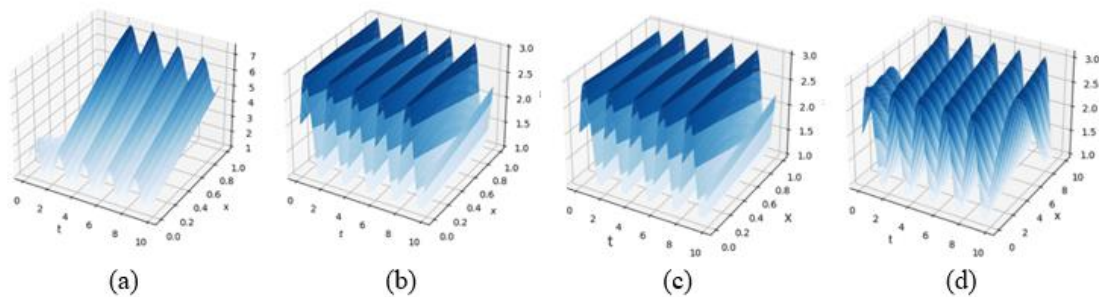


Fig. 4 hard limitation: sin function; $u(x)$: quadratic(a), piecewise (b)

piecewise (smaller range in large limitation) (c); piecewise (bigger range in large limitation) (d)

In Fig.4(a), the quadratic function exhibits a relatively regular increasing trend. As time changes, the global traffic flow density shows periodic fluctuations. The traffic flow density at different positions gradually stabilizes within a fixed range of fluctuations. When the input values are low, the global value is also low, and vice versa. The quadratic function describes the PBC state of traffic flow, which is more in line with the actual situation.

In Fig.4(b), only a slight slope can be observed at the peak. Therefore, it may be that the limited numerical range in the function cannot provide enough information to reflect the overall characteristics. To address this issue, the training range is expanded, and the function is retrained, resulting in the above charts:

It can be observed that the piecewise function in Fig.4(d), after expanding the training range, shows a reverse relationship between traffic flow density and time, similar to the expected time "delay." The purpose of Fig.4(c) is to compare the behavior of solutions at small distances under different large intervals. The results show almost no difference in trend compared to before the interval is expanded, indicating that the difference in solution behavior is caused by the interval size.

3.2. Constant Boundary Condition (CBC)

A constant function corresponds to an unobstructed road condition: when the boundary condition is a constant function, it indicates an unobstructed road. In this case, the density and velocity of traffic flow remain constant, without congestion.

However, constant function boundary conditions often contradict common sense. Even in an ideal system where traffic flow density is the same at all positions and vehicle spacing is equal, different output patterns can be obtained based on the specific dynamics of the system. The expected behavior of a quadratic function is a stable surface, with fluctuations within a constant range being acceptable.

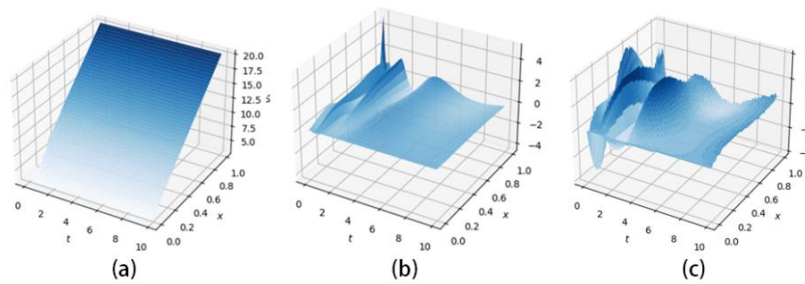


Fig. 5 hard limitation: constant function; $u(x)$: quadratic(a), piecewise (b), exponential (c)

In Fig.5(a), when traffic flow density tends towards infinity, the results of the quadratic function model are unsatisfactory. Therefore, it is decided to introduce some random fluctuations into the function, resulting in the following output:

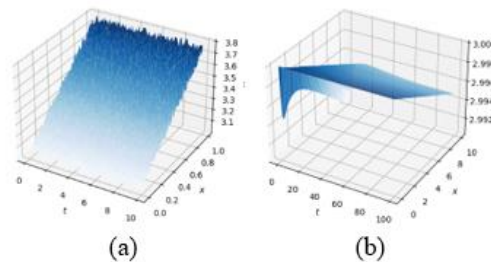


Fig. 6 hard limitation: constant function; $u(x)$: quadratic with noise(a) piecewise, large limitation(b)

Fig.6(a) shows some random oscillations, with a limited numerical range, but still no significant trend changes. Therefore, it can be concluded that the quadratic function itself may not be suitable for analyzing the behavior of constant functions. It only considers the relatively average traffic flow density and provides some reference values when considering traffic flow as particles. However, it is not in line with reality and has limited reference value.

In Fig.5(b) and Fig.5(c), piecewise and exponential functions exhibit acceptable fluctuations over time. Both functions, after experiencing an initial chaotic period, show a gradual trend towards smoothness, and weak periodic characteristics can be observed, which is consistent with expected behavior. Therefore, after increasing the time interval constraint, the functions are retrained. However, the training results show that traffic flow density repeatedly converges abnormally near zero. Finally, after expanding the time and distance intervals, the piecewise function obtained a relatively normal graph (the exponential function still could not be plotted).

In the dynamic system of Fig.6(b), for the piecewise function, it is assumed that if traffic flows uniformly in a certain interval, due to the adaptive phenomenon, each vehicle will undergo collective acceleration, thereby increasing the average vehicle speed and linearly decreasing the traffic flow density. This output to a certain extent can reflect objective reality and has reference value.

3.3. Normal Distribution Boundary Condition (NDBC)

Normal distribution function corresponds to traffic congestion: when the boundary condition is a normal distribution function, this passage can simulate traffic congestion. In the NDBC case, the density and velocity of traffic flow will exhibit a normal distribution, with the peak indicating a region of dense congested vehicles. Regarding the prediction of this graph, it roughly matches periodic functions, both exhibiting some delay over time. The periodic function itself is similar to the accumulation of multiple traffic congestions, while the normal distribution function is more akin to a single traffic congestion process, showing some similarities between the two. Unfortunately, after the double examination, the three functions are considered as unable to produce the expected outputs.

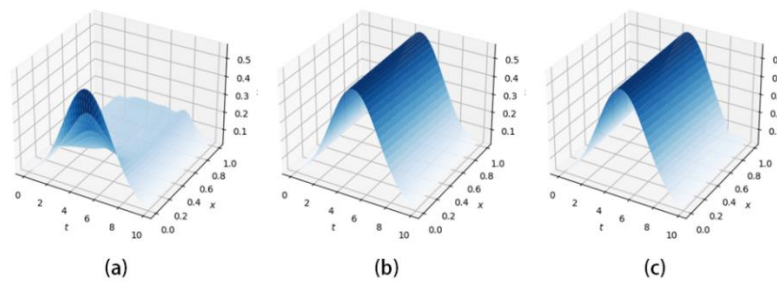


Fig. 7 hard limitation: normal distribution function; $u(x)$: quadratic(a), piecewise (b), exponential(c)

In Fig.7, the quadratic function graph shows a slight variation in traffic flow density at $x = 0$. However, this density change quickly dissipates in the subsequent traffic flow and eventually disappears. It is speculated that this may be due to the insufficient size of the solving range to reflect the influence of traffic flow density at different positions. Alternatively, this may be a limitation of the function itself, as the limited oscillations are not sufficient to have a significant impact on the entire system. Therefore, the author modifies the graph to a half-period sine function while increasing the numerical range, as shown in Fig.8. In addition, there are a few changes, that and quite different from the quadratic function apart from the quadratic function. Therefore, it can be inferred that a single oscillation is not sufficient to affect the overall density change of traffic flow.

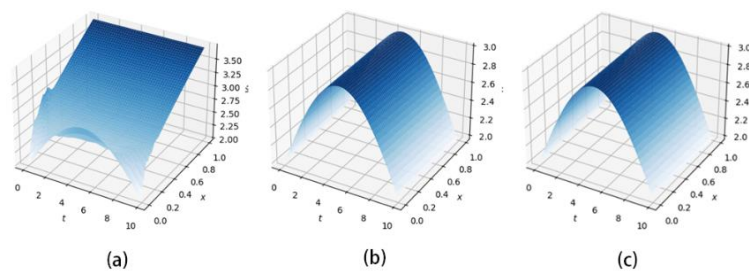


Fig. 8 hard limitation: half sin function; $u(x)$: quadratic(a), piecewise (b), exponential (c)

Finally, a comparison of the cost values of each function is presented in Table.1 (the error values generated by the interference-free function correspond to the normal state, i.e., $x \in (0, 1)$ and $t \in (0, 10)$). These cost values indicate the fitting accuracy of each function. Meanwhile, the basic findings are summarized in Table 2.

Table 1. The cost value of different function conditions (Keep two decimal places)

	PBC	CBC	NDBC	Half-sin
Quadratic	3.60e-04	2.35e-07	1.57e-03	2.57e-06
Piecewise	2.28e-05	8.79e-10	1.57e-05	5.72e-08
Exponential	nan	5.86e-10	2.26e-04	9.89e-08

Table 2. Predicting results of different original conditions

	PBC	CBC	NDBC
Quadratic	Excessive and timely periodic congestion	Unlimited linear congestion	
Piecewise	Periodic congestion with time delay	From minimum density to uniform flow velocity	Lack of reference value, as confirmed
Exponential	nan		

4. Conclusion

This paper constructs a traffic density prediction model based on the PINN by imposing hard constraints on initial conditions with realistic characteristic functions and selected PDEs (LWR equation system). The traffic flow density prediction and approximate solutions are conducted by referencing the research on PDE solutions by scholars such as Rempe and simulating various scenarios to constrain the initial conditions. Furthermore, when simulating traffic conditions using periodic functions and constant functions, the results show fitting predictions that closely match real-world scenarios, representing periodic awareness and the transition from minimum density to uniform flow velocity, respectively. However, this approach also has some limitations. For instance, in the process of solving the quadratic function for a constant function, this paper has not explicitly identified the detailed reasons for the unbounded increase in traffic flow density, as well as the anomalous results under normal distribution function and single-periodic function conditions. Further research may consider altering initial constant values, such as maximum traffic flow density and maximum flow, to observe if the situation improves. Moreover, the study focuses on the basic cases only, implying that more diverse $u(x)$ functions could be considered. The analytical model proposed in this study contributes to addressing traffic issues, enhancing better route planning, and preparing for the upcoming era of autonomous driving technology in the smart transportation landscape.

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