Research on Multibeam Bathymetric System Based on Greedy Algorithm and Variable Step Size Traversal Algorithm

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Abstract. Multi-beam bathymetry system is an advanced technology to measure the seabed based on single-beam bathymetry. In this paper, the greedy algorithm and variable step traversal method will be used to establish the mathematical models of coverage width and overlapping rate of adjacent strips of multi-beam sounding, and the best scheme of survey line layout will be designed by establishing the optimization model. According to the established mathematical model of overlap rate, the mathematical model of coverage width of multi-beam sounding in three-dimensional rectangular waters is optimized. Finally, according to the real life, with the overlapping rate of adjacent strips within 10% to 20% as the limiting condition and the minimum total length of survey lines as the objective function, a single-objective optimization model is established. Then, Python is used to solve the problem through greedy algorithm and variable step-size traversal algorithm, and it is found that the included angle of survey line direction in the optimal scheme is 90, and the minimum total length of survey line is 68 nautical miles. Stability analysis is carried out by changing the constraint range of overlap rate.

Keywords: Multibeam system; Submarine topography detection; Greedy algorithm; Variable step traversal.

1. Introduction

With the continuous improvement of seabed sounding technology, single-beam sounding has developed into multi-beam sounding. Single-beam bathymetry can only obtain continuous bathymetric data on the route, while multi-beam bathymetry can measure a wider range of seabed topography by transmitting multiple beams at the same time, which overcomes the shortcoming of limited distribution of single-beam bathymetry data.

I In multibeam bathymetry, the overlap coverage between neighboring lines is related to many factors, including the opening angle of the transducer, the slope of the seafloor, the spacing of the lines, and the water depth, etc. In order to optimize the measurement design of the multibeam system, it is necessary to establish a more reasonable mathematical model to calculate the coverage width, adjacent overlap rate and other indicators, and take into account the specific topographic and geomorphological characteristics of the sea area, to design the optimal number of lines, the shortest total length, and to meet the coverage requirements and overlap rate requirements of the best line program [1].

In this paper, a mathematical model of the coverage width of multibeam bathymetry and the overlap rate between adjacent strips is firstly established, and the width coverage model is optimized for the relative angles between different survey lines and seabed slopes in the three-dimensional space, and then, according to the actual situation, a set of survey lines with the shortest measurement lengths is designed in a rectangular sea area, which covers the whole sea area and the overlap rate between adjacent strips is between 10% and 20%. Then, a single-objective optimization model is established with the objective function of minimizing the total length of the survey lines. The model is then solved by the greedy algorithm and the variable step size traversal algorithm using Python.

2. Model establishment and solution

It should be noted that this paper puts forward the following reasonable assumptions: (1) suppose that the survey ship can be regarded as a particle; The direction of the survey line is parallel to the horizontal plane, and the survey ship sails in a straight line on the sea level and the survey process will not be disturbed by other factors; The submarine slope is approximately a smooth plane.

2.1. Covering width and overlapping rate between adjacent strips

Firstly, the mathematical models of coverage width and overlap rate between adjacent strips of multi-beam sounding are established. Let the horizontal distance between the i-th survey line and the i-1-th survey line be d_i, the water depth be D_i, the left coverage width of the i-th multi-beam sounding strip on the seabed slope be W_l, and the right coverage width be W_r. When calculating the overlap ratio, the coverage width between adjacent strips can not be expressed by the actual coverage width of the i-th survey line (W_i) [2], but by the sum of the horizontal projection of the left coverage width of the i-th strip and the horizontal projection of the right coverage width of the i-1-th strip ($W_i^{'}$), that is

$$W_i^{'} = W_{i-1}^{r}\cos\alpha + W_i^{l}\cos\alpha \tag{1}$$

The overlap rate between adjacent strips can be expressed as

$$\eta_i = 1 - \frac{d_i}{W_i^{'}} \tag{2}$$

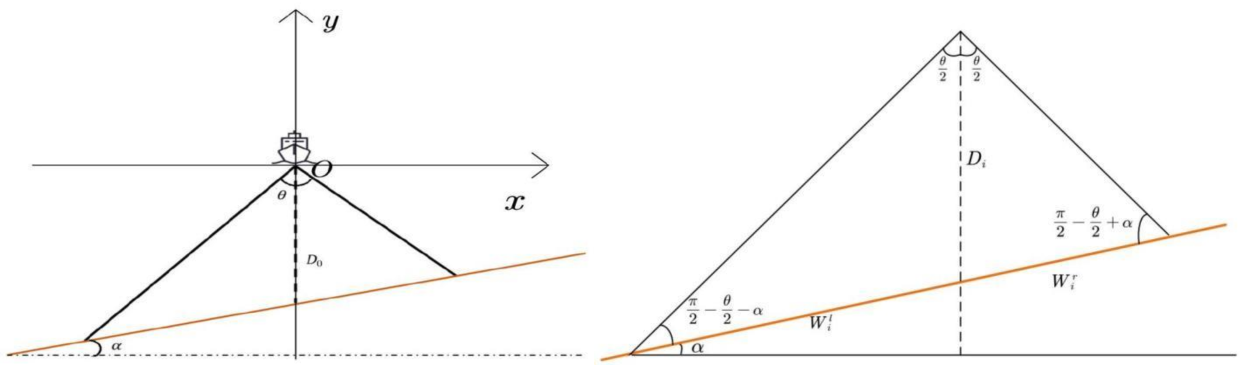

Fig. 1 (a) establishes a coordinate system with the center point where the probe ship is located as the origin; (b) Schematic of the geometric relationship between $\mathrm{W_i^l}$ and $\mathrm{W_i^r}$

Combined with Fig. 1(a), it is easy to know that the water depth D_i at each survey line is

$$D_i = |tan\ \alpha \cdot x_i - D_0| \tag{3}$$

As can be seen from Fig. 1(b), the coverage width W_i of multi-beam sounding is

$$W_i = \frac{sin\frac{\theta}{2}}{cos\left(\frac{\theta}{2}+\alpha\right)}|tan\alpha \cdot x_i - D_0|cos\alpha + \frac{sin\frac{\theta}{2}}{cos\left(\frac{\theta}{2}-\alpha\right)}|tan\alpha \cdot x_i - D_0|cos\alpha \tag{4}$$

The overlap rate η_i between adjacent strips is

$$\eta_i = 1 - \frac{d_i}{cos\alpha\left(\frac{sin\frac{\theta}{2}}{cos\left(\frac{\theta}{2}+\alpha\right)}|tan\alpha \cdot x_i - D_0| + \frac{sin\frac{\theta}{2}}{cos\left(\frac{\theta}{2}-\alpha\right)}|tan\alpha \cdot x_{i-1} - D_0|\right)} \tag{5}$$

2.2. Coverage width of multi-beam sounding in rectangular sea area in 3D space

Establish a spatial rectangular coordinate system as shown in Fig. 2, in which the starting position of the ship is the origin, the xOy plane is parallel to the horizontal plane, and the Z axis is vertical to the horizontal plane. In this paper, the mathematical model of the coverage width of multi-beam sounding at each position when $\pi < \beta < \pi$ will be given, and the modeling process in other cases is similar, so this paper will not discuss it too much.

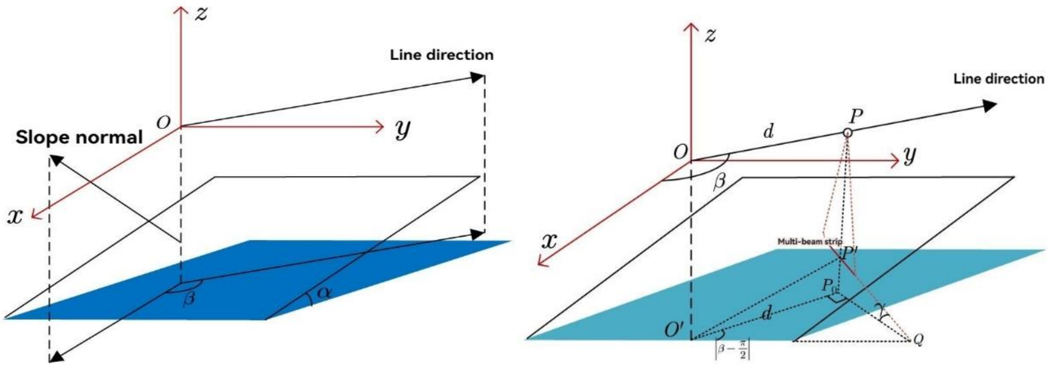

Fig. 2 (a) a schematic diagram of spatial rectangular coordinate system; (b) After the horizontal plane is moved up.

Firstly, the X- axis is projected to the seabed slope along the opposite direction of the Z axis, and the projection $O'x'$ is obtained. Suppose $\overrightarrow{O'x'} = (1, y_1, z_1)$, then $tan\alpha = \frac{-z_1}{1}$, so $z = -tan\alpha, y_1 = 0$, The first vector in the submarine slope is $\overrightarrow{O'x'} = (1,0,-tan\alpha)$. Similarly, $\overrightarrow{O'y'} = (0,1,0)$. According to the vector outer product formula

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & -tan\ \alpha \\ 0 & 1 & 0 \end{vmatrix} = \begin{vmatrix} 0 & -tan\ \alpha \\ 1 & 0 \end{vmatrix} \vec{i} - \begin{vmatrix} 1 & k \\ 0 & 0 \end{vmatrix} \vec{j} + \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \vec{k} \tag{6}$$

You can get the normal vector of the seabed plane $\vec{n} = (tan\alpha, 0,1)$. Therefore, the plane equation of submarine slope is.

$$tan\alpha \cdot x + z + D_0 = 0 \tag{7}$$

For the convenience of calculation, the horizontal plane is translated upward along the Z axis until it passes through the O' point, as shown in Fig. 2(b). When the ship drives to the P point along the survey line, let $OP = d$, the projections of P on the slope and the horizontal plane are P' and P_0 respectively, and the intersection of the extension line of the strip and the lower boundary of the horizontal plane in the figure is Q. Easy to know the new slope γ is.

$$\gamma = arctan\frac{-tan\alpha \cdot cos\beta}{tan\left|\beta-\frac{\pi}{2}\right|} \tag{8}$$

Finally, under general conditions, the mathematical model of coverage width of multi-beam sounding at various positions is as follows:

$$\begin{cases} W_i = \frac{sin\frac{\theta}{2}cos\gamma}{cos\left(\frac{\theta}{2}+\gamma\right)}\left|tan\gamma \cdot x_i - D_0\right| + \frac{sin\frac{\theta}{2}cos\gamma}{cos\left(\frac{\theta}{2}-\gamma\right)}\left|tan\gamma \cdot x_i - D_0\right|, \\ \gamma = arctan\frac{-tan\alpha \cdot dcos\ \beta}{tan\left|\beta-\frac{\pi}{2}\right| \cdot d}. \end{cases} \tag{9}$$

2.3. The shortest survey line length covering the whole rectangular sea area

According to the actual situation, the overlap rate between adjacent strips is between 10% and 20%. Because the submarine slope is rectangular, only consider the case where β is $(90°, 180°)$. Because the submarine slope passes through its center $(x_0, y_0, -D_0)$, D_0 indicates its depth from the sea level. The equation of submarine slope can be obtained as follows.

$$tan\alpha \cdot (x - x_0) + z + D_0 = 0 \tag{10}$$

In order to facilitate the layout of survey lines, the wiring will start from the nearest place to the origin [3]. For the first survey line, the sounding strip should include the origin, which can be obtained.

$$d_{0max}tan\alpha + \frac{d_{0max}}{tan\frac{\theta}{2}} = (d_{0max} - x_0) + D_0 \tag{11}$$

Where d_{0max} is the maximum distance between the first survey line and the origin, θ is the opening angle of the transducer, and α is the slope of the seabed slope.

The intersection point between the i-th survey line and the sea plane is $P_i^{(1)}$ and $P_i^{(2)}$, and the distance between the i-th survey line and the i+1-th survey line is d_i. Because the seabed slope is uniform, the minimum coverage width W of sounding strips on the i-th survey line is point $P_i^{(1)}$, that is, the position with the minimum repetition rate between adjacent strips, while the position with the maximum repetition rate between adjacent strips is point $P_i^{(2)}$. For the depth of point $P_i^{(2)}$

$$D_i^{(1)} = tan\alpha \cdot \left(x_i^{(1)} - x_0\right) + D_0 \tag{12}$$

Where $x_i^{(1)}$ is the abscissa of $P_i^{(1)}$. Then the overlap rate between the sounding strip and the next strip on the i-th measuring line at point $P_i^{(1)}$ is expressed as

$$\eta_i^{(1)} = 1 - \frac{d_i}{cos\gamma sin\frac{\theta}{2}\left(\frac{D_i^{(1)}+d_i tan\,\gamma}{cos\left(\frac{\theta}{2}-\gamma\right)} + \frac{D_i^{(1)}}{cos\left(\frac{\theta}{2}+\gamma\right)}\right)} \geqslant 10\%, \tag{13}$$

After simplification,

$$d_i \leqslant 0.9cos\gamma sin\frac{\theta}{2}D_i^{(1)}\left(\frac{1}{cos\left(\frac{\theta}{2}-\gamma\right)} + \frac{1}{cos\left(\frac{\theta}{2}+\gamma\right)}\right) \Big/ \left(1 - 0.9\frac{sin\gamma sin\frac{\theta}{2}}{cos\left(\frac{\theta}{2}-\gamma\right)}\right) \tag{14}$$

In the same way,

$$d_i \geqslant 0.8cos\gamma sin\frac{\theta}{2}D_i^{(2)}\left(\frac{1}{cos\left(\frac{\theta}{2}-\gamma\right)} + \frac{1}{cos\left(\frac{\theta}{2}+\gamma\right)}\right) \Big/ \left(1 - 0.8\frac{sin\gamma sin\frac{\theta}{2}}{cos\left(\frac{\theta}{2}-\gamma\right)}\right) \tag{15}$$

Because the survey line is laid from the origin, the position of the last survey line can be determined only by considering whether the point farthest from the origin can be included in the last survey line. If the minimum number of survey lines according to the above survey line layout scheme is n, the maximum distance of the survey line farthest from the origin is.

$$d_{max} = \frac{|y^* - x^* tan\ \beta|}{\sqrt{1+tan^2\beta}} = |y^* cos\beta - x^* sin\beta| \tag{16}$$

Where x^*、y^* are the abscissa and ordinate of the farthest point from the origin in the sea area. Then the total length of the survey line can be expressed as.

$$\sum_{i=1}^{n} \sqrt{\left(x_i^{(1)} - x_i^{(2)}\right)^2 + \left(y_i^{(1)} - y_i^{(2)}\right)^2} \tag{17}$$

Based on the above analysis, the following constraints are set, and the minimum overlap rate of adjacent strips is not less than 10%; The maximum overlap rate of adjacent strips shall not exceed 20%; The sounding strip of the farthest survey line shall contain the point farthest from the origin. Finally, the following single-objective optimization model is established:

$$\min \sum_{i=1}^{n} \sqrt{\left(x_i^{(1)} - x_i^{(2)}\right)^2 + \left(y_i^{(1)} - y_i^{(2)}\right)^2} \tag{18}$$

For each direction included angle β, a set of survey line layout scheme can be determined to make the total length of lateral line reach the optimum under this direction included angle, so greedy algorithm should be used to solve it. Greedy algorithm is an important algorithm design idea, and its basic principle is to choose the solution that looks optimal in the current situation in every step of selection. Greedy algorithm usually gets a local optimal solution in a certain sense, but not necessarily a global optimal solution. The key lies in choosing the appropriate greedy criteria to get a global optimal solution. Therefore, when using greedy algorithm, we need to carefully analyze the characteristics and requirements of the problem and determine the selection strategy of the optimal solution [4].

For each β, the first survey line is the farthest from the origin on the premise of satisfying the constraint conditions. Because the overlap rate of sounding strips corresponding to the shallowest point is the lowest, in order to meet the conditions, it is only necessary to make the overlap rate of sounding strips corresponding to the shallowest point be 10%, so that the number of sounding lines can be minimized under the condition that the maximum overlap rate is less than or equal to 20%. This scheme can be regarded as the optimal survey line layout corresponding to this β, and then the β can be traversed with variable step size. By solving the optimization model of the total length of the survey line with Python [5], the local optimal solution corresponding to the included angle β in each direction can be obtained, as shown in Fig. 3.

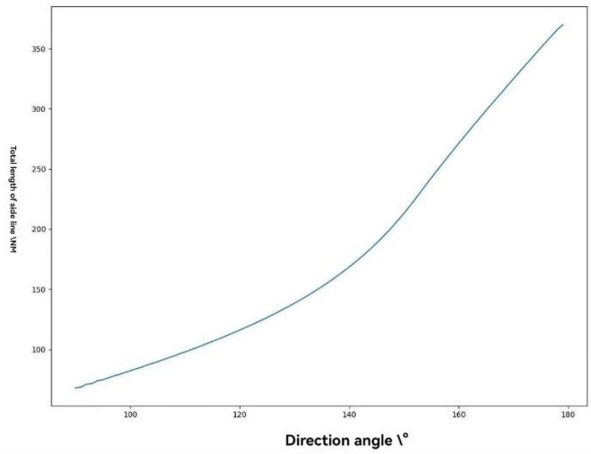

Fig. 3 Relationship between total length of survey line and direction included angle

When β changes in [90°, 180°], the minimum value of the total length of the survey line also increases with the increase of the included angle in the direction. Therefore, when $\beta = 90°$, the total length of the survey line reaches the minimum, which is the global optimal solution. At this time, 34 survey lines need to be laid, and the minimum total length of survey lines is 68 nautical miles.

With the direction included angle β gradually increasing on [90°, 180°], the total length of the survey line shows an increasing trend. Let's change the limit range of the overlap rate of the strip coverage width and observe the change of the total length of the survey line, as shown in Fig. 4.

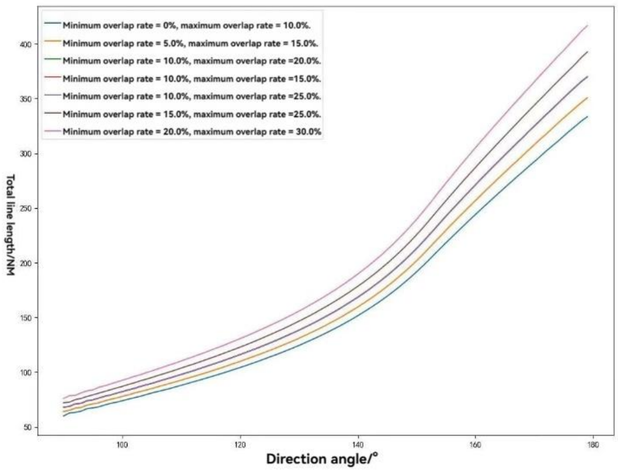

Fig. 4 The change of total length of survey line when the limit range of overlap rate is changed.

Under the condition that the difference between the upper and lower limits of the overlap rate of strip width is unchanged, the change trend of the total length of survey lines with different overlap rate ranges is basically the same, and they all show positive correlation. When the optimal scheme with the smallest total length of the survey line is selected, that is, when β takes the optimal solution of 90°, the change range of the total length of the survey line is small and the stability is strong; However, with the increase of the direction angle, the change amplitude of the total length of the survey line increases gradually, and the stability is also decreasing gradually. Based on this, we can draw the following conclusions: the optimal scheme has strong stability.

3. Summary

In this work, the strips on the seafloor slope are projected to the horizontal plane when calculating the coverage width and overlap rate, which is more in line with the factual situation. A mathematical model of the coverage width and the overlap rate between adjacent strips of multibeam bathymetry is established, taking into account the joint influence of two adjacent strips on the overlap rate. The width coverage model is optimized for the case of different survey line relative angles to the seafloor slope in three-dimensional space, and the greedy algorithm is used to determine a set of survey line placement schemes to make the total length of the sidetrack reach the optimum under this directional angle through the calculation. The slope angle γ in the vertical direction is converted into a coverage width model to establish, then according to the actual situation, a set of survey lines with the shortest measurement length in a rectangular sea area is designed, and the iterative relationship between each line and the next line is calculated from the starting point, so that the bathymetric strips can minimize the total length of the survey lines while the overlap rate is kept in the constraint range, which can be generalized to a more general and general wiring model.

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