

Modeling and analysis of ecosystem dynamics based on the Lotka-Volterra model

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Abstract. In this paper, the ecological dynamics of sea lamprey populations were explored using the Lotka-Volterra model. Its ecosystem dynamics were also modeled. The analysis showed that changes in the sex ratio of sea lampreys can lead to a decrease in the abundance of other species within a certain range. Thus, changes in sex ratio affect the abundance of other species in the larger ecosystem. In addition, the study assessed the strengths and weaknesses associated with sea lamprey populations through iterative modeling. Six advantages (growth rate, environmental adaptation, importance in the food chain, ecosystem contribution, predation control, and reproductive efficiency) and four disadvantages (environmental stress, risk of overfishing, threats to reproduction, and threats to predation) were assessed in terms of population stability, reproductive fitness, and ecological impact. The results showed that when sex ratios were equal, the greatest advantage was predation control and the greatest disadvantage was overfishing risk. When males were dominant, the greatest advantage was adaptability, while the greatest disadvantage was predation threat.

Keywords: Lotka-Volterra model; Clustering; Ecosystem dynamics; Iterative model.

1. Introduction

In various environmental conditions, species can exhibit changes in sex ratios, such as lamprey. When food is scarce in the environment, the proportion of male lamprey significantly increases, where as in environments with abundant food, the proportion of males decreases markedly. This variation in sex ratios not only reflects individual survival strategies of lamprey in response to resource scarcity and abundance but also unveils intricate interactions among components within the ecosystem. Ecosystems comprise a network of interconnected species, so the changes in sex ratios of lamprey may trigger a cascade of reactions.

The study of system dynamics in ecosystems is crucial for ecological conservation. At present, some scholars have already studied the change problem. Li et al [1] analyzed the global PES study in 1919 by combining natural language processing methods. PES themes, spatial and temporal distributions, and hotspot PES patterns in 114 countries around the world were obtained. A PES framework that combines socio-economic and ecosystem dynamics and is oriented to sustainable development is proposed to make integrated management decisions. Based on this, a consistent PES solution can be provided for future conservation theory research and implementation strategies. Kishor Aryal et al [2] utilized the Driver-Pressure-State-Impact-Response (DPSIR) framework, which assessed 38 drivers of sustainable development on a decadal basis and analyzed more than 30 policy instruments as well as 15 formulated periodic development plans (viz. five-year and three-year plans). In addition, the paper conducted a 300-household survey to understand the needs of local residents for ecosystem management. Hampton Randall and Peter Brewitt [3] Using the Advocacy Coalition Framework (a proven model for analyzing policy structure and change), the paper conducted a series of semi-structured interviews with the partner organization to analyze the organization's functioning and the framework through which stakeholders viewed the coalition, their roles, and mission. The paper found that it had succeeded in making itself a policy subsystem for the region and a venue that encompassed all relevant stakeholders. Aparna Howlader [4] explored the

determinants of large-scale farmland afforestation projects and their consequences, using the example of the Great Plains Protected Forestry Project in the late 1930s. The paper used instrumental variables approach to study the long term sustained effects of the planting project and showed that the adoption of shelterbelts reduced wind erosion in the 1930s and even in the early 21st century and that this effect was mainly concentrated in the rangeland areas. Jaakko Paasi et al [5] studied the dynamics of innovative ecosystems using a system dynamics approach. The aim of this study was to support the creation of innovative ecosystems and the harmonization of existing ecosystems so that they can be more successful. The key factors that generate dynamics in innovation ecosystems were identified. They constitute the elements of a system dynamics model of innovation ecosystems.

The Lotka-Volterra model has been instrumental in understanding the dynamics of ecological systems, particularly in examining predator-prey interactions. In this study, we employ the Lotka-Volterra model to delve into the intricate dynamics of the sea lamprey population, focusing on the influence of gender ratio variations on the broader ecological dynamics. Through our analysis, we aim to shed light on the interconnectedness of species within ecosystems and how seemingly isolated changes, such as gender ratio shifts, can reverberate through the entire ecological web. Additionally, our investigation expands to assess the advantages and drawbacks associated with the sea lamprey population, providing valuable insights into its ecological role and potential risks.

2. Ecological Modeling of the Lamprey Population Ecosystem

2.1. The model built by the Lotka-Volterra model

The quantity of food available for the lamprey can alter its sex ratio, and these changes in sex ratios can have repercussions on a broader ecosystem. To analyze the impact of sex ratio changes in lamprey on the larger ecosystem, and considering that lamprey primarily function as predators within the ecosystem, we have constructed a dynamic population ecosystem model based on the Lotka-Volterra model.

The Lotka-Volterra model is as follows:

Setting:

N_1 and N_2 represent the population sizes of two species.

K_1 and K_2 represent the carrying capacities of the two species.

R_1 and R_2 represent the population growth rates of the two species.

When the two species compete for or utilize the same spatial resources:

$$\frac{dN_1}{dt} = R_1 \times N_1 \times \left(1 - \frac{N_1}{K_1} - \alpha \times \frac{N_2}{K_2}\right) \quad (1)$$

At this point, we establish:

α as the competition coefficient of Species 1 against Species 2.

$$\frac{dN_2}{dt} = R_2 \times N_2 \times \left(1 - \frac{N_2}{K_2} - \beta \times \frac{N_1}{K_1}\right) \quad (2)$$

β as the competition coefficient of Species 2 against Species 1.

Based on the relationship between the lamprey and food availability, we construct a dynamic population ecosystem model for the lamprey based on the Lotka-Volterra model.

Variables:

N represents the total population of the lamprey species.

F represents the number of females in the lamprey population.

M represents the number of males in the lamprey population.

O represents the number of other species.

The dynamic equations for the lamprey population are as follows:

$$\frac{dN}{dt} = R_N \times N \times \left(1 - \frac{N}{K_N}\right) - \alpha_{NO} \times (N - O) \quad (3)$$

Where R_N represents the growth rate of the lamprey population, K_N represents the carrying capacity of the lamprey population, α_{NO} represents the competition coefficient between the lamprey and other species:

$$\frac{dN}{dt} = R_F \times N \times \left(1 - \frac{N}{K_F}\right) - \alpha_{FN} \times (N - O) \quad (4)$$

Where R_F represents the growth rate of the female lamprey population, K_F represents the carrying capacity of the female lamprey population, α_{FN} represents the competition coefficient between female lamprey and the total lamprey population.

$$\frac{dM}{dt} = R_M \times N \times \left(1 - \frac{N}{K_M}\right) + \alpha_{MN} \times (N - M) \quad (5)$$

Where R_M represents the growth rate of the male lamprey population, K_M represents the carrying capacity of the male lamprey population, α_{MN} represents the competition coefficient between male lamprey and the total lamprey population.

The dynamic equations for other species populations are as follows:

$$\frac{dO}{dt} = R_O \times O \times \left(1 - \frac{O}{K_O}\right) - \beta_{ON} \times (O - N) \quad (6)$$

Where R_O represents the growth rate of other species populations, K_O represents the carrying capacity of those other species, β_{ON} represents the competition coefficient between other species and the lamprey population.

2.2. Parameter Settings

We employed MATLAB to conduct an analysis of the Lamprey population ecosystem dynamics model based on the Lotka-Volterra framework. The specific parameters of the model come from the Great Lakes Fishery Commission, and part of the data was roughly assessed by us through consulting population materials. As shown in Table 1.

Table 1. notations used in this section

Symbol	Description	data
R_N	the growth rate of the lamprey population	0.1
K_N	the carrying capacity of the lamprey population	800
α_{NO}	competition coefficient between the lamprey and other species	0.01
R_F	the growth rate of the lamprey population	0.08
K_F	the carrying capacity of the female lamprey population	500
α_{FN}	competition coefficient between female lamprey and the total	0.005
R_M	the growth rate of the female lamprey population	0.14
K_M	the carrying capacity of the male lamprey population	500
α_{MN}	competition coefficient between male lamprey and the total	0.005
R_O	the growth rate of other species populations	0.12
K_O	the carrying capacity of those other species	600
β_{ON}	competition coefficient between other species and the lamprey	0.002

2.3. Model Results Analysis

The result are shown in Figure 1 and Figure 2.

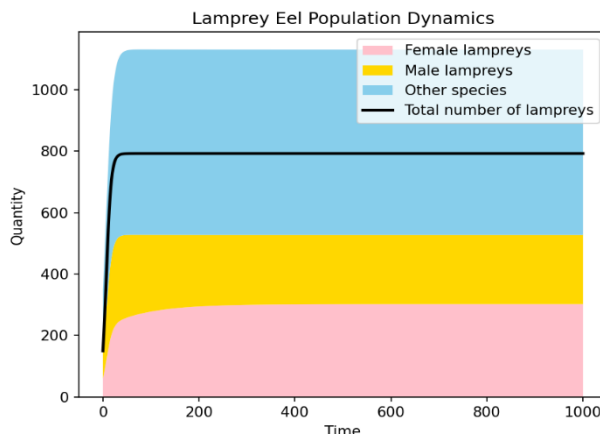


Fig. 1 The Relationship Between Lamprey Population Size and Time

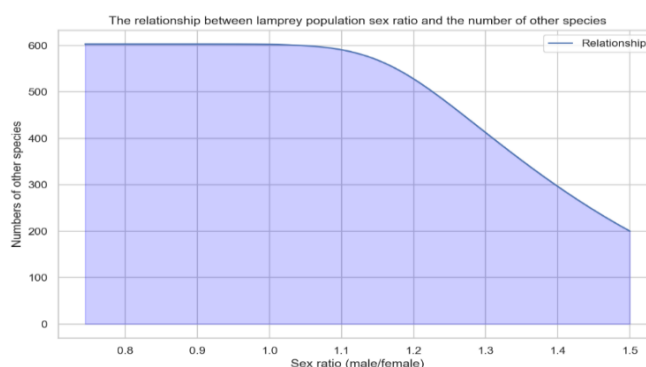


Fig. 2 The Relationship Between Sex Ratios in the lamprey Population and the Population Size of Other Species

According to the model analysis, it was found that changes in the sex ratio of the lamprey will cause a reduction in the number of other populations within a certain range. Therefore, when the sex ratio changes, it will have an impact on the number of other species in the larger ecosystem.

3. Evaluation Model For Strength & Weakness Based On Iteration

To study the dominance and inferiority of the lamprey population, we need to assess the stability of the lamprey population itself, the adaptability of its reproductive rate, and its impact on the food web and overall ecosystem services within the ecosystem.

We have gathered information from relevant sources and identified potential strengths and weaknesses of the lamprey population as follows:

Strengths: Individual growth rate of lamprey, adaptability of the population to environmental changes, significant role in the food chain, crucial contribution to ecosystem services, substantial impact on predator control, and reproductive success, among others.

Weaknesses: Environmental stressors, risks posed by overfishing, threats to reproduction, and predation risks.

3.1. Establishment of the Iteration Model

Establishment of the Iteration Model To evaluate both the strengths and weaknesses, we have developed an iteration model.

We have set the following parameters:

$\rho, \Phi, \sigma, \theta$ represent the parameters to be considered for each factor, with the calculation formula as follows:

$$C = \alpha \times \rho + \beta \times \Phi + \gamma \times \sigma + \delta \times \theta \quad (7)$$

$\alpha, \beta, \gamma, \delta$ denote the weights of each parameter.

3.2. Establishment of Models for Various Dominant Factors

In the study of dominant influencing factors, we established models to investigate the individual growth rate of lampreys. We also developed a model to assess the adaptability of the lamprey population to environmental changes. Additionally, we conducted research on the significant role of lampreys in ecosystem services using an iterative model. Furthermore, we conducted research on the food chain model and predation control based on the lamprey population ecosystem.

To study the individual growth rate of lampreys, we developed a model based on the Von-Bertalanffy model.

Parameters:

λ represents the individual growth rate of lampreys

L^∞ represents the maximum length attainable by lampreys.

The formula for the individual growth rate of lampreys is as follows:

$$\lambda = \frac{d(L^\infty(1-e^{-k(t-t_0)}))}{dt} \quad (8)$$

k represents the growth coefficient of lampreys, and we have set it as $k=0.8$, t_0 represents the initial age of lampreys, and we have set it as $t_0=0.8$.

To study the adaptability of the lamprey population to environmental changes, we established a model based on the logistic model.

Parameters:

$P(t)$ represents the population size at time t .

The formula for the population's adaptability to environmental changes is as follows:

$$P(t+1) = P(t) + rP(t) \left(1 - \frac{P(t)}{K}\right) \quad (9)$$

r represents the intrinsic growth rate of the lamprey population, and we have set it as $r=0.3$, K represents the carrying capacity of the lamprey population, and we have set it as $K=20000$.

3.3. Establishment of Models for Various Inferior Factors

To investigate the pressure exerted by the lamprey on the environment, we developed an environmental pressure model. To assess the risks associated with overfishing on the lamprey population, we established an overfishing risk model. To study the threats to the reproduction of lampreys, we created a reproduction threat model. Additionally, we investigated the impact of predation threats on lampreys through a study based on the lamprey population ecosystem.

To study the pressure imposed by the lamprey on the environment, we constructed an environmental pressure model. The specific parameters of the model come from the Great Lakes Fishery Commission, and part of the data was roughly assessed by us through consulting population materials.

Parameters:

n_t represents the population size of lampreys at time t .

E represents the level of environmental pressure caused by the lamprey.

$$n_{t+1} = R(n_t, E) \times n_t \quad (10)$$

$R(n_t, E)$ represents the growth rate of the lamprey population, and we have set it as $R(n_t, E)=0.1$.

To investigate the impact of overfishing on the lamprey, we developed an overfishing risk model.

Parameters:

n_t represents the population size of lampreys at time t

C represents the fishing intensity

$$n_{t+1} = n_t + G(n_t) - F(C, n_t) \quad (11)$$

$G(n_t)$ represents the growth increment of the lamprey population, which is the product of the population growth rate and n_t , We have set it as $G(n_t) = 0.1n_t$, $F(C, n_t)$ representing the removal function, and we have set it as $F(C, n_t) = 0.1$.

To study the threats to the reproduction of lampreys, we established a reproduction threat model. Parameters:

n_t represents the population size of lampreys at time t .

$$n_{t+1} = n_t \times (1 + b_t - d_t) \quad (12)$$

b_t represents the birth rate of lampreys, and we have set it as $b_t=0.005$, d_t represents the death rate of lampreys, which we have set as $d_t=0.957$.

3.4. Solution results

The results of the solution are shown in Figure 3 and Figure 4.

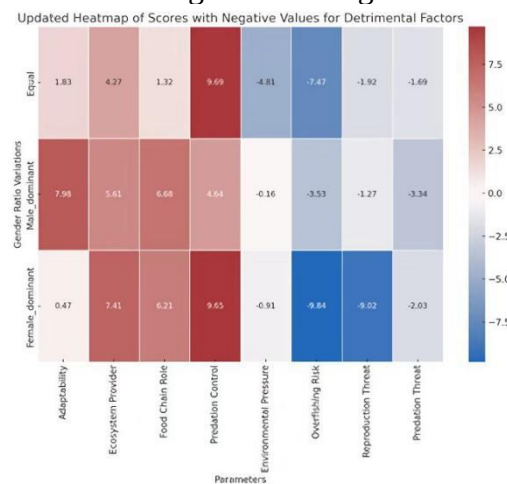


Fig. 3 Updated Heatmap of Scores with Negative Values for Detrimental

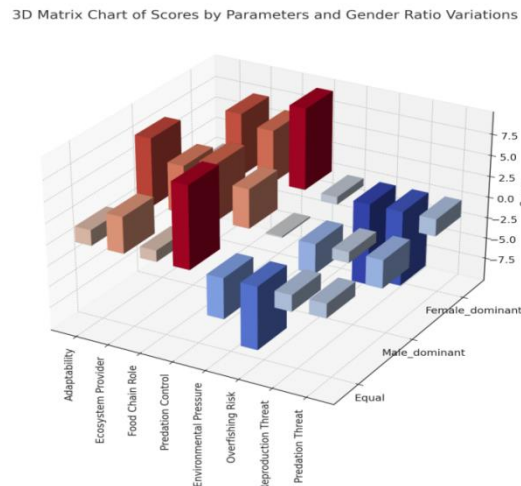


Fig. 4 3D Matrix Chart of Scores by Parameters and Sex Ratio Variations

As shown in Figure 3 and Figure 4, we can draw the following conclusions:

When sex ratios are equal (equal numbers of males and females), the maximum advantage is in Predation Control, and the maximum disadvantage is Overfishing Risk.

When males dominate the sex ratio, the maximum advantage is in Adaptability, and the maximum disadvantages are Overfishing Risk and Predation Threat.

When females dominate the sex ratio, the maximum advantage is in Predation Control, and the maximum disadvantages are Overfishing Risk and Reproduction Threat.

4. Evaluation of the Ecosystem Model

Given that the lamprey primarily appears as a predator in the ecosystem, to address the impact of changes in sex ratios on ecosystem stability, we constructed a food chain: Phyto-plankton → Large Fish → Lampreys. By examining whether changes in the sex ratio of lam-preys affect the stability of this system, including phytoplankton, fish, and lampreys, we aimed to determine the impact on ecosystem stability. We constructed a predator-prey food chain as shown in Figure 5.

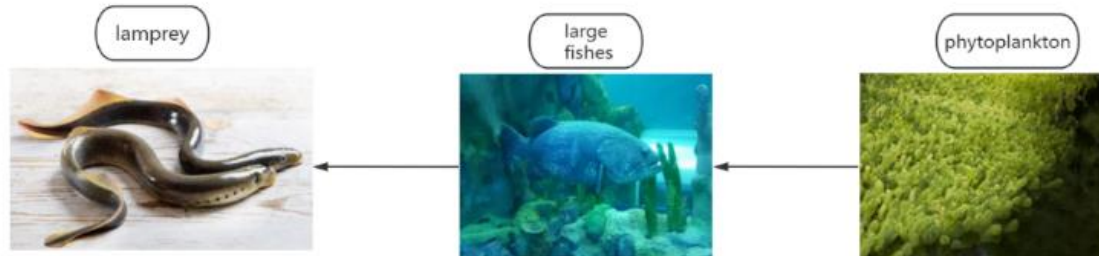


Fig. 5 Food Chain of Phytoplankton, Large Fish and Lampreys

In this food chain, lampreys' prey on fish, and fish prey on phytoplankton. Based on the Lotka-Volterra model, the dynamic equations for this food chain are as follows:

Parameters:

P represents the total number of phytoplankton.

H represents the total number of fish.

L represents the total number of lampreys.

Equations for the lamprey Population Dynamics:

$$\frac{dp}{dt} = R_p \times P \times \left(1 - \frac{P}{K_p}\right) - \alpha_{PH} \times P \times H \quad (13)$$

R_p represents the growth rate of phytoplankton, K_p represents the carrying capacity of phytoplankton, and α_{PH} represents the coefficient of interaction between phytoplankton and fish.

$$\frac{dH}{dt} = R_H \times H \times \left(1 - \frac{H}{K_H}\right) - \alpha_{HL} \times H \times L \quad (14)$$

R_H represents the growth rate of fish, K_H represents the carrying capacity of fish, and α_{HL} represents the coefficient of interaction between fish and lampreys.

$$\frac{dL}{dt} = R_L \times L \times S_m \times S_f - D_L \times L \quad (15)$$

R_L represents the growth rate of the lamprey population, S_m and S_f represent the male and female ratios of lampreys, and D_L represents the mortality rate of lampreys.

We used Python to visualize the results of the model. The parameter settings in Table 2 were applied.

Table 2. Notations Used In This Section

Symbol	Description	Date
R_p	the growth rate of phytoplankton	0.2
K_p	the carrying capacity of phytoplankton	800
α_{PH}	the coefficient of interaction between phytoplankton and fish	0.01
R_H	the growth rate of fish	0.15
K_H	the carrying capacity of fish	500
α_{HL}	the coefficient of interaction between fish and lampreys	0.005
R_L	the growth rate of the lamprey population	0.1
S_m	Male ratio of lampreys	0.8
S_f	Female ratio of lampreys	0.2
D_L	Mortality rate of lampreys	0.957

We used these parameters to define the system and differential equations, and then we visualized the results. The Figure 6 shows the changes in the population sizes of phyto-plankton, fish, and lampreys over time under specific ratios.

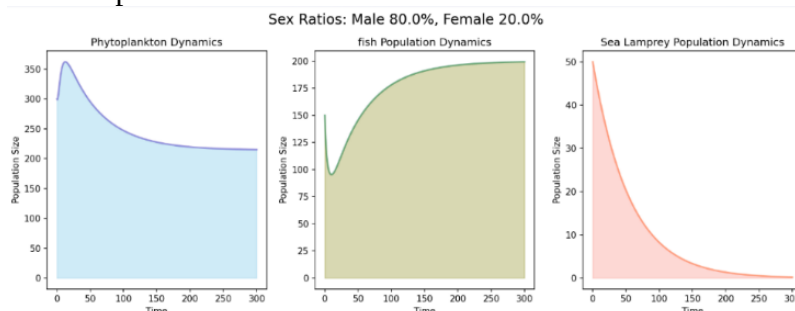


Fig. 6 Population Size Changes of Phytoplankton, Fish, and lampreys Over Time Under a Fixed Sex Ratio

5. Conclusion

In conclusion, our study utilizing the Lotka-Volterra model has elucidated the significant impact of sea lamprey gender ratio variations on the broader ecological dynamics. We have demonstrated that such variations can lead to population decreases in other species within a certain range, emphasizing the interconnectedness of species within ecosystems. Furthermore, our evaluation of the benefits and drawbacks associated with the sea lamprey population has highlighted key advantages in predation control and adaptability, while underscoring critical disadvantages such as overfishing risk and predation threats. This comprehensive analysis contributes to a deeper understanding of the sea lamprey's ecological role and the potential implications of gender ratio shifts within its population.

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