The Beamline Layout Design Based on the Mathematical Model for Different Water Terrain

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Abstract. Multi-beam sounding is one of the important components of marine surveying. For different water terrain, if the line layout scheme can be reasonably formulated according to the principle of multi-beam sounding, it can greatly improve the sounding efficiency and reduce the loss of manpower, material resources and time while ensuring the accuracy of the sounding results.Based on the different water terrain, the definition of overlap rate and the calculation method in simple cases are given first.For the water with flat bottom terrain, the main selection of isobath, control different spacing and Angle to meet the overlap rate requirements as much as possible.For waters with relatively rugged and changeable underwater terrain, the water area is first rationally divided into several sub-areas, in which the underwater depth and slope change little. Then, the plane fitting is carried out for each sub-area, and the wiring idea of flat underwater terrain is adopted for wiring. For two adjacent areas with similar water depth, the water area is divided into several sub-areas.The starting end of the survey line in the latter area should be made to coincide with the end end of the survey line in the previous area. For the calculation of the coincidence rate, the data points at each depth of the bottom are iterated several times to determine whether they are in the overlapping part of the measurement area of the two survey lines, so as to arrange the multi-beam survey line as far as possible to meet the corresponding requirements.

Keywords: Multi-beam sounding, Isobath, Water division, Three-dimensional plane fitting.

1. Introduction

When the depth of water is measured, the propagation characteristics of sound waves in water can be used to carry out single-beam detection. The principle is that in a uniform medium, the propagation mode of sound waves is uniform linear propagation (Figure.1)[1]. When different interfaces are encountered, reflection will occur, so the depth of the sea water can be obtained from the propagation speed and propagation time of sound waves in the sea water[2].

However, because the single beam survey adopts a single point continuous measurement method, the data obtained along the track is relatively dense and there is no data between the survey lines[3]. Therefore, on this basis, a multi-beam sounding system is developed. As the name implies, when measuring at various points along the track, the system will simultaneously emit multiple beams (as shown in Figure.2), covering as large a plane perpendicular to the track as possible rather than a single point and single line, and then accept the returned multiple beams.This kind of system can overcome the shortcomings of single beam survey, and can measure the full coverage depth strip with the survey ship line as the axis and with a certain width in the sea area where the seabed is flat[4].

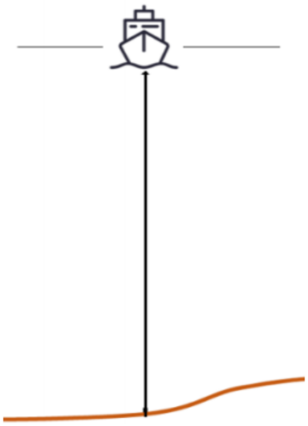

Fig. 1 Schematic diagram of single-beam surveying

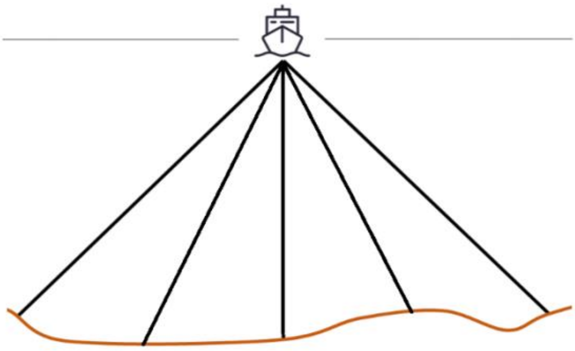

Fig. 2 Schematic diagram of surveying multi-beam

2. Calculation method of overlap ratio between adjacent strips in simple flat terrain

When the multi-beam sounding system is used, the full coverage depth strip with a certain width and the survey line of the measuring ship is the axis is shown in Figure.3.

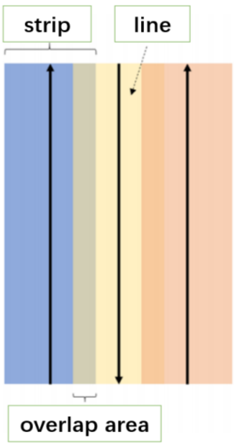

Fig. 3 Line, strip and overlap area

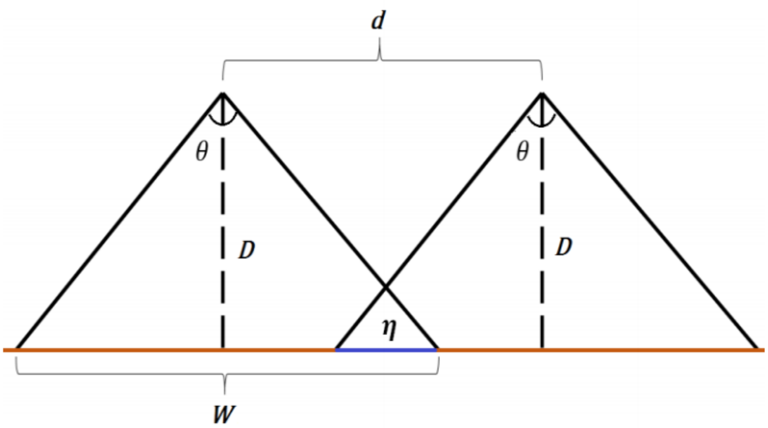

Fig. 4 Relationship between line spacing, coverage width and overlap rate.

The coverage width W of the multi-beam sounding strip is determined by the depth of water D and the opening Angle of the transducer θ. In the case of flat seabed topography and parallel lines, the definition of overlap rate between known adjacent strips is

$$\eta = 1 - \frac{d}{W} \tag{1}$$

Where, d is the distance between two adjacent measurement lines, and W is the coverage width of the strip (FIG.4).If η<0, it indicates that there is a missing test section between two adjacent test lines. When the overlap rate between adjacent strips is controlled to 10%~20%, the measurement convenience and data completeness can be ensured, and the loss of manpower and material resources caused by data redundancy can be effectively avoided.

This paper analyzes and deduces the overlap rate between adjacent strips for the case of seabed slope.

Equation (1) can be reduced to

$$= \frac{W-d}{W} = \frac{\frac{W}{2}+\frac{W}{2}-d}{\frac{W}{2}+\frac{W}{2}} \tag{2}$$

Formula (2) indicates that the actual significance of the overlap rate between adjacent strips is the ratio of the overlap length (numerator) to the sum (denominator) of the width of each covering section between the two strips.

That is, if the two adjacent strips are not the same in terms of cover widths, respectively W_1, W_2, in the case of flat seabed terrain and parallel lines, as shown in FIG.5.

$$\eta = \frac{\frac{W_1}{2}+\frac{W_2}{2}-d}{\frac{W_1}{2}+\frac{W_2}{2}} \tag{3}$$

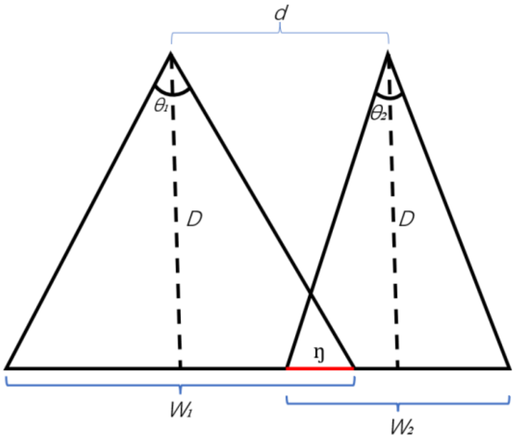

Fig. 5 Schematic diagram of overlap rate with different coverage widths

In the formula, the numerator $\frac{W_1}{2}+\frac{W_2}{2}-d$, is the length of the overlapping part, the covering section is the right half strip of the previous measurement line and the left half strip of the latter measurement line, and the ratio of its width and sum is the denominator $\frac{W_1}{2}+\frac{W_2}{2}$.

3. Construction of mathematical model of line layout under different conditions

3.1. Establishment of coverage width model for multi-beam sounding

The slope of the submarine slope is α, the opening Angle of the multi-beam transducer is θ, the depth of the measurement point from the seabed is D, and the distance between the measurement lines is d, as shown in Figure.6

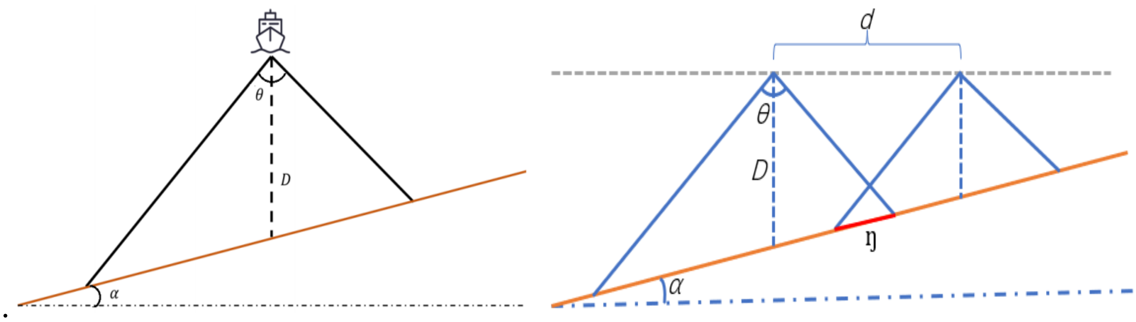

Fig. 6 Schematic diagram of problem 1

According to the analysis of mathematical knowledge and triangular relationship (FIG.7),

$$\begin{cases} \angle 1 = \frac{\pi}{2} - \frac{\theta}{2} - \alpha \\ \angle 2 = \frac{\pi}{2} + \alpha - \frac{\theta}{2} \end{cases} \tag{4}$$

Then by the sine theorem

$$\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC} \tag{5}$$

It is obtained by substituting the corresponding variable.

$$\frac{D}{sin\angle 1} = \frac{D_1}{sin\frac{\theta}{2}} \tag{6}$$

$$\frac{D}{sin\angle 2} = \frac{D_2}{sin\frac{\theta}{2}} \tag{7}$$

Therefore, multiple beam detection coverage widths can be obtained.

$$D_0 = D_1 + D_2 \tag{8}$$

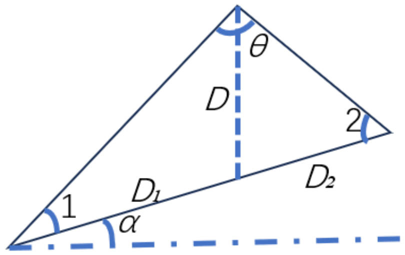

Fig. 7 multi-beam sounding coverage width Angle relation and label.

You can simplify it.

$$D_0 = Dsin\frac{\theta}{2}\left(\frac{1}{sin(\frac{\pi-\theta}{2}-\alpha)} + \frac{1}{sin(\frac{\pi-\theta}{2}+\alpha)}\right) \tag{9}$$

3.2. Establishment of overlapping rate model between adjacent strips

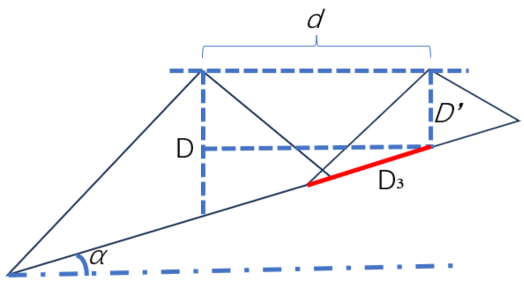

Fig. 8 Relationship between two adjacent strips

Now draw the relationship diagram between the two adjacent strips, as shown in Figure.8. The relationship can be obtained according to the diagram.

$$D' = D - dtan\alpha \tag{10}$$

Similarly, the width of the left half strip of the second measurement line can be found.

$$D_3 = D'sin\frac{\theta}{2}\frac{1}{sin(\frac{\pi-\theta}{2}-\alpha)} \tag{11}$$

Therefore, the width of overlapping parts between adjacent strips.

$$l = D_2 + D_3 - \frac{d}{cos\alpha} \tag{12}$$

Thus, the overlap rate between adjacent strips can be obtained.

$$\eta = \frac{l}{D_2+D_3} \tag{13}$$

I just put in the quantity that I got.

$$\eta = 1 - \frac{d}{cos\alpha sin\frac{\theta}{2}[D\frac{1}{sin(\frac{\pi-\theta}{2}+\alpha)}+(D-dtan\alpha)\frac{1}{sin(\frac{\pi-\theta}{2}-\alpha)}]} \tag{14}$$

3.3. The establishment of spatial stereoscopic model about the direction and orientation of measuring ship

At present, the direction and orientation diagram of the survey ship associated with corresponding indicators on the rectangular sea area to be measured (FIG.9), and the three-dimensional deconstruction diagram of the survey ship at a certain point in a certain direction are drawn based on the parameters specified in the diagram, as shown in FIG.10.

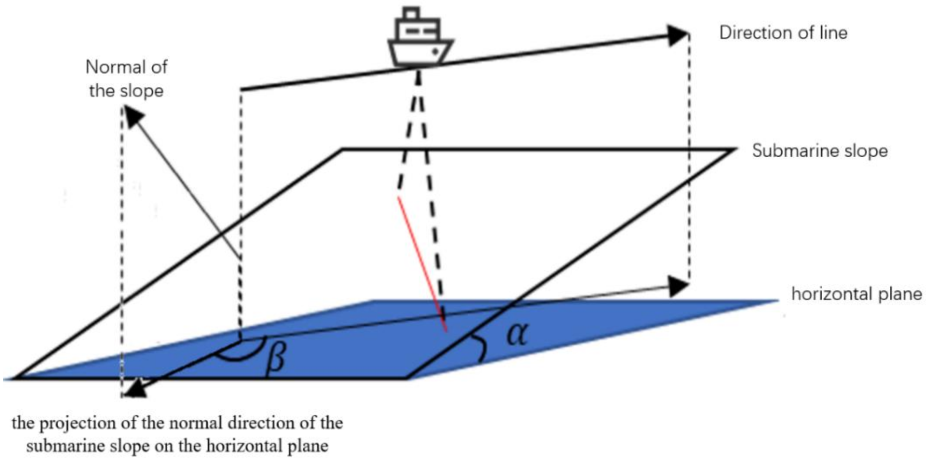

Fig. 9 Schematic diagram of direction and orientation of corresponding indicators

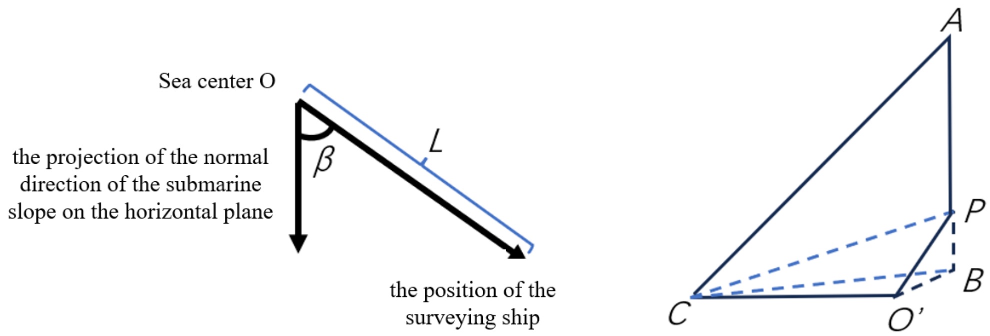

Fig. 10 Deconstruction diagram and three-dimensional diagram of the survey ship in a certain direction and a certain orientation

In FIG.10, point C is the outermost and lowest point that can be measured within the multi-beam sounding surface. Plane CO 'P is the bottom of the submarine slope, and plane BO 'c is parallel to the sailing surface of the surveying ship, that is, the sea level.From the geometric relationship between the triangles, it can be seen that the Angle $\angle BCO' = \beta$, is equal to that formed by the projection of the normal direction of the submarine slope on the horizontal plane and the direction of the measurement line.

The corresponding relation can be obtained from the slope α, the course direction of the measuring ship β and the opening Angle of the multi-beam transducer θ.

$$\begin{cases} O'B = \frac{BP}{tan\alpha} \\ CB = \frac{O'B}{sin\beta} \\ \frac{CB}{BA} = tan\frac{\theta}{2} \\ AB = D + Lcos\beta tan\alpha \end{cases} \tag{15}$$

This can be solved.

$$BP = \frac{tan\frac{\theta}{2}tan\alpha sin\beta(D+Lcos\beta tan\alpha)}{1-tan\frac{\theta}{2}tan\alpha sin(\beta-\frac{\pi}{2})} \quad (16)$$

Thus, it can be concluded that.

$$P = \sqrt{BP^2 + (\frac{BP}{tan\alpha sin\beta})^2}C \quad (17)$$

CP is the coverage length of the deeper half, and the coverage length C 'p of the other shallow half can be obtained in the same way.

3.4. The layout design of survey line when a lot of water information is known

3.4.1 When the sea floor is a flat slope

The information and survey requirements of the existing sea area are as follows: The sea area is rectangular, 2 nautical miles long from north to south and 4 nautical miles wide from east to west. The sea floor is a slope with a slope of 1.5°, and the bottom terrain is considered flat. The sea floor on the west side is deeper than that on the east side.The opening Angle of the multi-beam transducer is 120°, and the overlap rate between adjacent strips should be between 10% and 20%.Design a set of lines as short as possible based on the above conditions.

Considering that the bottom of the sea is a slope, a set of parallel survey lines are designed for mapping needs.Verify the data and connect with the actual situation. It is generally believed in the relevant industries that the survey lines in the sea area should be parallel to each other, and the trend should conform to the isobath line at the bottom of the channel and the trend of geological structure, so as to expand the scope of the survey line as much as possible, improve the coverage rate of the survey line[5], facilitate the layout and perfection of the full coverage depth measurement[6], and maintain the water depth value unchanged along this direction.The change of scanning width of the sounder is relatively stable, which is beneficial to the determination of measuring line spacing[7]. Therefore, the strike of the measurement line should be consistent with the basic strike of the isobath line, that is, parallel straight lines are arranged along the north-south direction.

In order to improve the efficiency of measurement work as far as possible, reduce the loss of manpower and material resources, meet the needs of surveying and mapping work, and ensure that the overlap rate of survey lines is 10% to 20%, now consider the number of survey lines required in each coverage interval.Because the sea area is rectangular, and each measurement line distribution direction is the same, are laid along the north-south direction, so each measurement line length is the same, therefore, the fewer the number of measurement lines, its total measurement length is the shortest.

From the deepest point on the western side of the sea floor, the depth is D_0.Now calculate the distance of the first line that can cover the deepest.

$$d_0 = D_0 tan60° \quad (18)$$

According to the model preparation and analysis in Question 1, the coverage rate η meets the following relation.

$$\frac{W_1+W_2-\frac{d}{cos1.5°}}{W_1+W_2} = \eta \quad (19)$$

Where, W_1 is the coverage width of the right half of the previous measurement line, W_2 is the coverage width of the left half of the measurement line to be measured, and d is the distance between two adjacent measurement lines.

The cable to be measured must meet the following conditions.

$$\begin{cases} D_2 = D_1 - d\tan 1.5° \\ \frac{D_2}{\sin 28.5°} = \frac{W_2}{\sin 60°} \\ \frac{W_1+W_2-\frac{d}{\cos 1.5°}}{W_1+W_2} = \eta \end{cases} \quad (20)$$

From this, the total number of lines to be measured and the spacing of adjacent lines to be measured at different overlapping rates can be calculated, and the mapping lines can be laid to meet the requirements of different overlapping rates.

3.4.2 When the sea floor is a rugged curved surface

An area 4 nautical miles wide from east to west and 5 nautical miles long from north to south has had depth data obtained several years ago using single-beam measurements. Firstly, the data is preprocessed, and the data points near the outlier are replaced by interpolation fitting method. The processed data was visualized to obtain the seafloor topographic map (Figure.11, Figure.12).

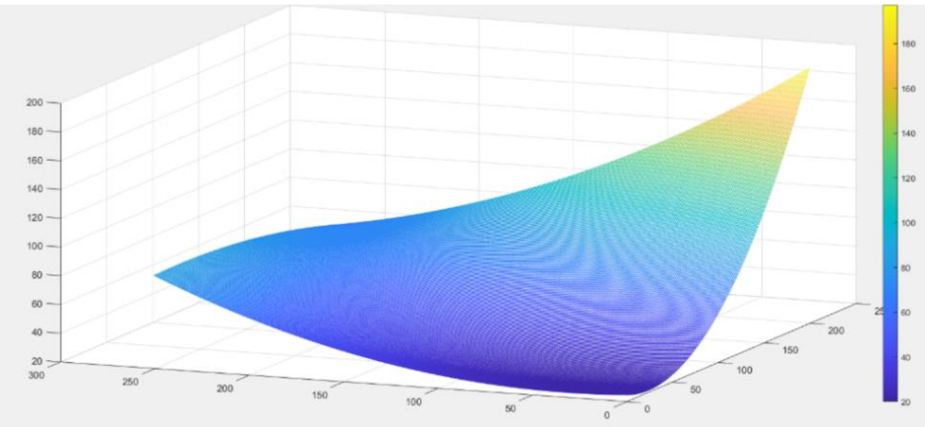

Fig. 11 Submarine topographic map (1)

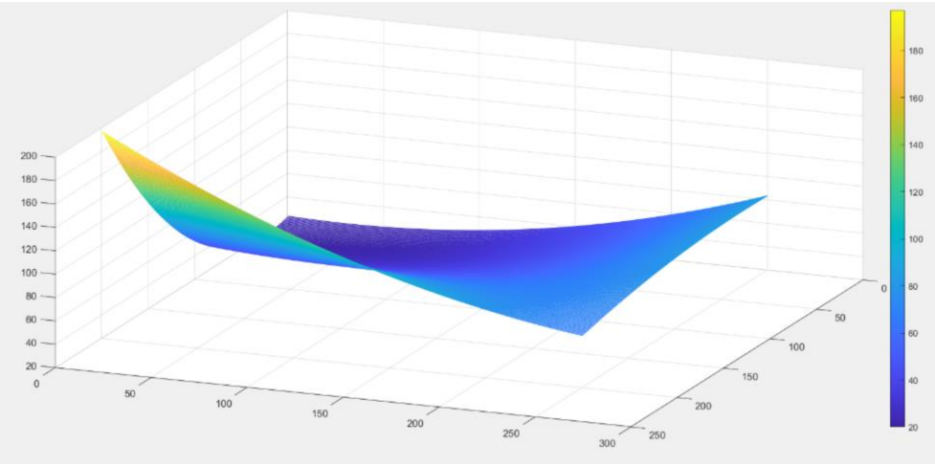

Fig. 12 Seafloor topographic map (2)

After observation, it is found that among the four endpoints of the sea, the southwest and northeast endpoints have the shallowest sea water depth, which is 24.4m and 65.2m respectively. Taking the line of these two points as the dividing line, the seafloor slope on the southeast side is larger, and the seafloor slope on the northwest side is smaller, so it is considered to partition the seafloor on the southeast and northwest side.The segmentation principle is as follows: the southwest corner endpoint is selected as the starting point, and the sampling points are taken every 0.08 nautical miles from east to west, and every 0.10 nautical miles from north to south. The two-fold fitting is performed on the sampling points, and the analytic expression of the function of the vertical direction about the horizontal direction of the sampling point is obtained.

$$y = 0.0055712073380x + 13.38195294 \quad (21)$$

Where x is the distance between the horizontal direction of the sampling point and the starting point, and y is the distance between the sampling point and the starting point in the vertical direction.

As can be seen from the analytic equation, the slope of the line is very small, and it can be approximated as parallel to the sea plane. In this rectangular sea area, the average depth of the line in the vertical direction is about 46m, which is similar to the depth of the northwest corner endpoint.Then the two endpoints of the line are connected with the southwest and northeast endpoints respectively to obtain two common-sided triangles. The original seafloor was thus approximately divided into two triangular slopes.

According to the calculation, the sine value of the Angle between the northwest region of the boundary line and the sea level is 0.00639 (the Angle is about 0.4°), which is very small, so it can be approximately considered parallel to the sea level, and it is regarded as the survey line design under the condition of flat seabed.The sine value of the Angle between the southeast area of the boundary line and the sea level is 0.00871 (the Angle is about 1.5°), which is consistent with the slope of the seabed in the previous three questions, so deal with it with the similar method.

After reviewing the literature, under normal circumstances, the transmitting transducer projects a coverage sector with a width of 10°~160° to the water bottom[8][9], and the accuracy of different pointing angles (half of the opening Angle of the multi-beam transducer) is shown in Figure.13[10]:

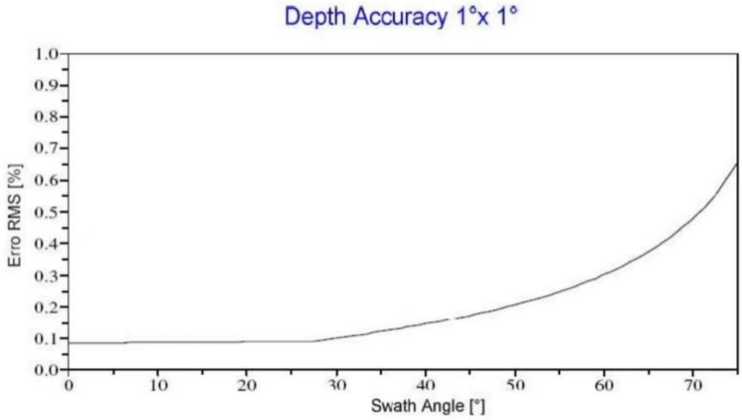

Fig. 13 Relation between accuracy and different pointing angles

As can be seen from the figure:

Pointing Angle 0°~45°: better than water depth ×0.18%;

Pointing Angle 0°~30°: better than water depth ×0.09%;

Pointing Angle 45°~60°: better than water depth ×0.28%;

Pointing Angle 60°~70°: better than water depth ×0.48%.

When the pointing Angle is less than 60°, the accuracy of the sounding data is low; when the pointing Angle is higher than 60°, the data redundancy is large and the overlap rate between adjacent strips is high.

Referring to the first three questions, the layout principle of the measuring line is: 1. the opening Angle of the multi-beam transducer is 120°;2. The directions of the lateral lines are parallel to the isobath lines of the two regions;3. Considering that the diagonal line is the boundary between the two regions, in order to ensure that the measurement lines of the two regions are coherent and that the regions on both sides of the diagonal can be fully covered, a measurement line is laid on the diagonal line;4. the first line to be measured starts from the southeast corner and the northwest corner, and ensures the maximum coverage while ensuring that the last line crosses the diagonal.

The overlap rate is set as η, in order to ensure the coverage of the strip in the sea area to be measured, the length of a survey line is the distance between the intersection point of its adjacent longer survey line and the sea boundary, so as to prevent the formation of triangular areas that the strip cannot cover.

For the case of no coverage, the simple case that the submarine slope is flat cannot be used for calculation. Here, all the seawater depth data points are traversed, and the survey lines on both sides of the location are judged by comparing the vertical distance of each point to the dividing line. The vertical distance between the two survey lines and the location and the depth of the location are compared to determine whether the location is covered. The uncoverage is the ratio of the number of points not covered to the number of all data points.

4. Solution results in each case

4.1. Measurement line arranged parallel to the contour line

In the case of simple submarine slope, the measurement line is arranged parallel to the contour line.

It has been given that the opening Angle of the multi-beam transducer is 120°, the seabed slope is 1.5°, and the sea water depth at the center of the sea is 70m. The corresponding coverage width and coverage rate can be obtained by substituting all the data into the formula.

The data required are listed in Table 1 at an interval of 200m.

Table 1 Simple submarine slope situation, survey lines parallel to the isobath

Measure the distance between the line and the center point /m	- 800.	- 600.	- 400.	- 200.	0	200	400	600	800
Sea depth /m	90.949	85.712	80.474	75.237	70	64.763	59.526	54.288	49.051
Coverage width /m	315.813	297.628	279.442	261.256	243.07	224.884	206.699	188.513	170.327
Overlap with the previous line /%	-	34.684	30.561	25.883	20.529	14.341	7.109	1.458	11.765

4.2. Survey line with an Angle arranged parallel to the isobath line

In the case of simple submarine slope, the survey line with an Angle is arranged parallel to the isobath line.

The dependent variable is set as: (1) the slope $\alpha = 1.5°$(2) the multi-beam transducer opening Angle $\theta = 120°$(3) the depth of sea water in the center of the sea area D=120m.When the Angle of the direction of the survey line is changed and the distance between the survey ship and the center of the sea area is changed, the coverage width is also changed to meet the above derived relationship, so the corresponding calculation results can be obtained, as shown in Table 2.

Table 2 Calculation results of question 2

Coverage width /m		The distance/nautical mile of the measuring vessel from the center of the sea							
		0	0.3	0.6	0.9	1.2	1.5	1.8	2.1
Measurement line direction Angle /°	0	415.692	466.091	516.49	566.889	617.288	667.686	718.085	768.484
	45	416.192	451.872	487.552	523.232	558.912	594.592	630.273	665.953
	90	416.692	416.692	416.692	416.692	416.692	416.692	416.692	416.692
	135	416.192	380.511	344.831	309.151	273.471	237.791	202.11	166.43
	180	415.692	365.293	314.894	264.496	214.097	163.698	113.299	62.9
	225	416.192	380.511	344.831	309.151	273.471	237.791	202.11	166.43
	270	416.692	416.692	416.692	416.692	416.692	416.692	416.692	416.692
	315	416.192	451.872	487.552	523.232	558.912	594.592	630.273	665.953

4.3. Interpretation of the result when the water bottom is a flat slope

According to the preliminary calculation, when η =10%, the minimum number of lines needed is 34, and when η =11%, the minimum number of lines needed is 35.

Then, for the interval of coverage $\eta \in (10\%, 11\%)$, the interval is further reduced, and all values are traversed by successive iterations with 0.1% as the index value. It can be calculated that when $\eta \in (10\%, 10.66\%)$, the total number of measurement lines required is the least, with a total of 34.

For the rest of the overlapping rate interval segments, the same treatment can be performed to obtain the histogram of the overlap rate between adjacent strips with the number of measured lines (see Figure.14).

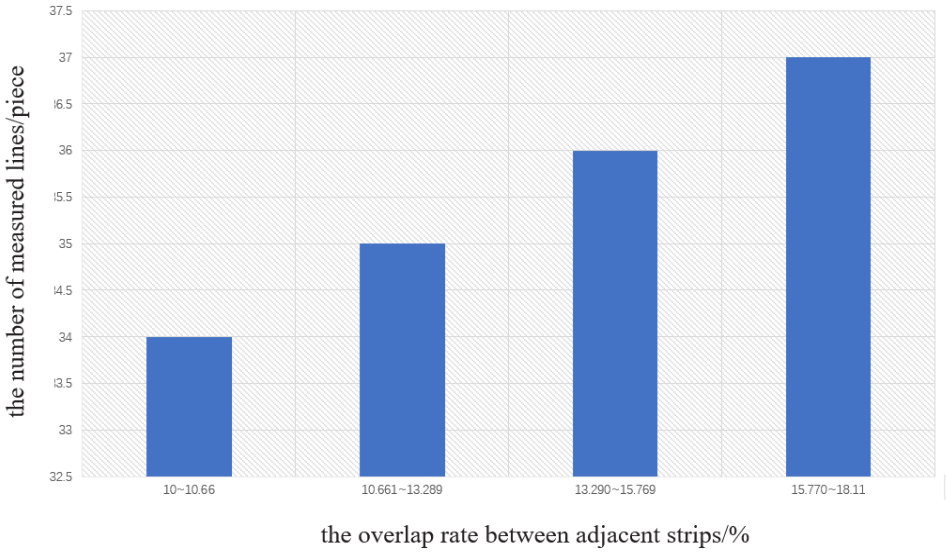

Fig. 14 Line chart with respect to overlap rate between adjacent strips.

4.4. Interpretation of the result when the bottom of the water area is a rugged surface

Using Python calculation can be obtained, in the case of meeting the third layout principle:

1. The overlap rate of the southeast region can only be 13.400% and 17.960%, and the corresponding uncoverage rate is 19.280% and 16.972%, respectively. The total length of the measured line (excluding the length of the dividing line) is 141471.8m and 149030.8m, respectively.The requirement of measuring sea coverage is more important than the requirement of the total length of the lateral line, so the layout of the line with less coverage is adopted, that is, the non-coverage is 16.972% and the overlap rate is 17.960%.

2. The relationship between uncovered coverage, measured line length and overlap rate in northwest China is shown in Figure.15.

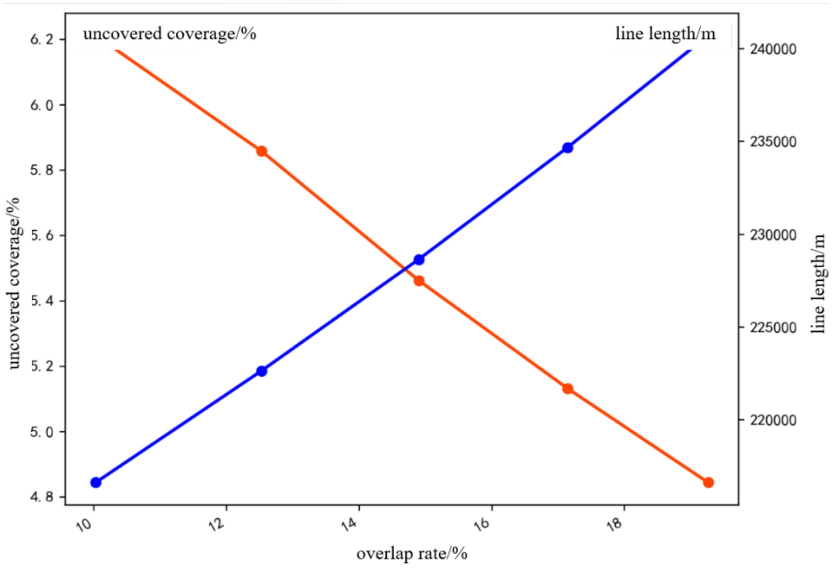

Fig. 15 Relationship between uncovered coverage, line length and overlap rate.

As shown in the figure, the relationship between the length of the measured line and the overlap rate of the measured line is approximately a primary relationship. After comprehensive consideration, the middle value is taken to make the layout of the measured line when the uncovered line and the measured line length are both at a reasonable value, that is, when the uncovered line is 5.462% and the overlap rate is 14.9%.The total length of the measuring line in this area (excluding the length of the dividing line) is 228,652.24m.

Considering the layout of the survey lines in the two regions, the total length of the survey lines can be obtained as 389541.592m, and the percentage of the missing survey area in the total sea area to be measured is 13.948%.

5. Conclusions

For more complex situations, in view of the fact that the above method is only a simple division of the submarine surface, and a plane with a very small slope is regarded as parallel to the sea level and processed, the accuracy of the result is slightly lower. If a certain submarine terrain is relatively rugged, the above method of survey line layout will not be applicable.However, a similar idea can be used to divide the original sea area into several sub-sea areas as small as possible, and the water depth and slope within each sub-area have little difference. Each sub-area is treated as a simple submarine slope after plane fitting, and the parallel survey line layout method with an Angle to the isobarth line is applied to carry out survey line planning.The water depth is detailed between two adjacent sub-areas, so that the beginning end and the end of the two survey lines on it coincide, and then the required results are obtained.The more detailed the division of the bottom of the sea area, the more accurate the results obtained in this way, but the corresponding amount of calculation will be increased.

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