

Mathematical Modelling and Optimisation in Multibeam Bathymetry: A Study of Coverage Width, Overlap Rate and Course Optimisation

Zhiwei Bai^{*}, Xiaoxin Zhang, Yuying Sun

College of Intelligence and Information Engineering, Shandong University of Traditional Chinese Medicine, Jinan, China, 250355

^{*} Corresponding Author Email: baizhiwei1313@163.com

Abstract. This paper focuses on the field of multibeam bathymetry, firstly, a two-dimensional mathematical model with coverage width, overlap rate, and seawater depth as the objectives is established by using the trigonometric properties, the terrain slope factor and the effect of different survey line distances from the centre on multibeam bathymetry. Matlab software was used to solve the model, and the expressions for the coverage width and the overlap rate of adjacent strips were obtained. Secondly, for the multibeam bathymetry along a certain direction, based on the three-dimensional geometric seawater depth model, the coverage width model corresponding to different survey line directions was obtained by using the classification discussion method. Finally, through the optimisation model and traversal algorithm, the optimisation model that meets the requirement of the overlap rate of adjacent strips is established with the objective of minimising the total routes, and solved by the cyclic traversal idea. This study provides a comprehensive mathematical model and optimisation method for multibeam bathymetry, and lays a theoretical foundation for practical applications in related fields.

Keywords: Multibeam bathymetry, mathematical modelling, optimization models.

1. Introduction

Single-beam bathymetry is widely used in the measurement of ocean depth, which is calculated from the speed of sound waves travelling through seawater. Sound waves move with constant velocity in seawater of the same density and contact with different interfaces produces reflective motion. Through this principle, the time used to record the sound wave from the ship transducer to send a signal to the sea and receive the signal, combined with the speed of sound wave propagation in the sea and the time used to calculate the depth of the sea water [1-3].

Multibeam bathymetry has multiple transducers and was developed based on the premise of single-beam, which allows multiple beams to be emitted and received at the same time, solving the shortcomings of single-beam bathymetry in terms of low efficiency. In seawater with flat topography, it is able to strip the seabed and obtain the coverage width [4-5]. The coverage width, w , is related to the transducer opening angle, θ , and the water depth, D . Choosing different water depths to design the line spacing will affect the overlap rate and efficiency. In this paper, in the designated rectangular sea area, with the goal of the shortest total route, the survey line is designed to completely cover the sea area, and at the same time meet the overlap rate of adjacent strips within the range of 10%~20%. Based on the optimisation model and traversal algorithm, this paper determines the parameters such as the direction of the survey line, the depth of the sea area and the opening angle [6].

2. Modelling and solving

2.1. Modelling of 2D mathematical geometry

In multibeam bathymetry, the relationship between line segments and the relationship between line segments and angles plays a crucial role in solving the two-dimensional mathematical geometric model. Therefore, we make the following additions to the graph given in the question, citing the three angles α_1 , α_2 and α_3 , and the three-line segments m_1 , m_2 and x (Fig. 1).

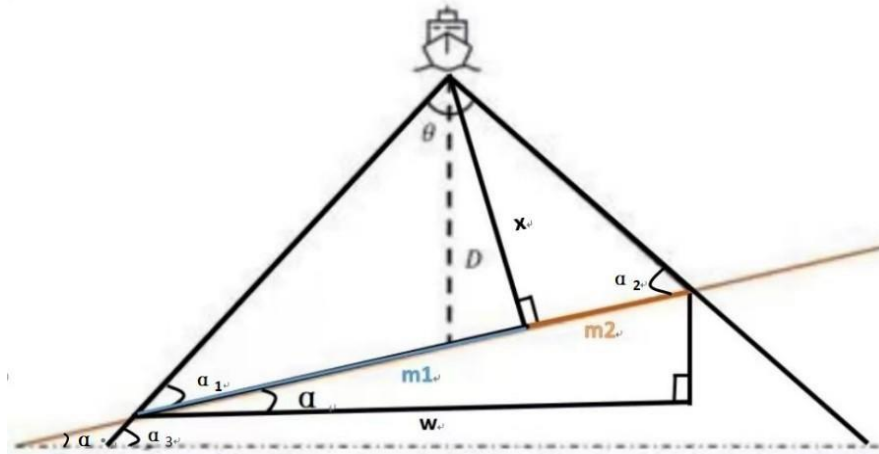


Fig. 1 Coverage width of multibeam bathymetry and overlap rate between neighbouring strip.

For the solution of this problem, we use the properties of trigonometric functions and consider the problem in terms of the relationship between line segments and angles, and the relationship between angles and angles, respectively [7-8].

2.1.1 Modelling of coverage width based on trigonometric properties.

When the lines of multibeam bathymetry are parallel to each other and the seabed topography is flat, the sound waves emitted will cover the width w equally, where α is the angle. where m is the length of the line segment.

$$\alpha_3 = \frac{\pi - \theta}{2} \quad (1)$$

2.1.2 Modelling of overlap rates between neighbouring strips based on trigonometric properties and topographic factors.

It is possible to derive the overlap between neighbouring strips of the terrain η_1 . where $w-d$ is the width of overlap and w is the coverage width. The overlap rate η_1 between neighbouring strips in uneven terrain is the overlap rate/coverage width.

$$\eta_1 = \frac{w-d}{d} \quad (2)$$

2.1.3 Modelling of centroid distance and seawater depth based on trigonometric properties.

According to the question, quote α an angle, Δh , b , h three-line segments (see Fig. 2)

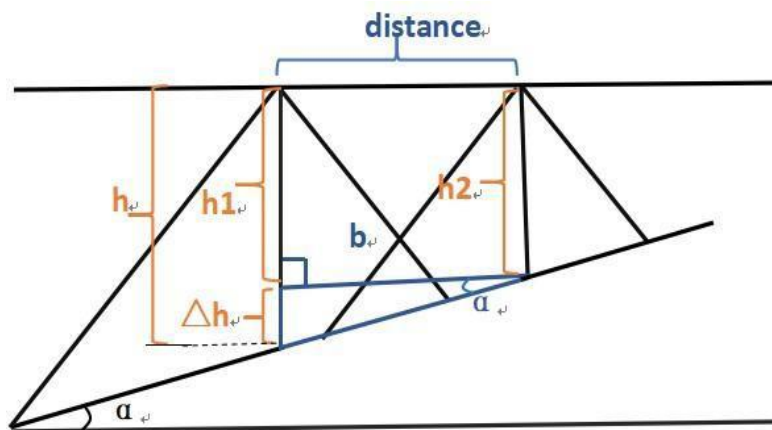


Fig. 2 Relationship between the distance of the survey line from the centre and the depth of the sea water

The relationship between Δh and b can be derived from the tangent theorem (see the following equation)

$$\tan \alpha = \frac{\Delta h}{b}; \Delta h = \tan \alpha b \quad (3)$$

Where Δh is the height difference. According to the information given in the question, it is known that the depth of the sea at the centre of the sea is 70m. Set this point as the origin and this point as the relative point, the route along the slope of the perpendicular to the plane to the shallow water b m is recorded as b , otherwise it is recorded as $-$. An expression for the depth of the sea at different distances from the centre of the line can be obtained (see the following formula) .

$$h = 70 - \Delta h = 70 - \tan \alpha b \quad (4)$$

where b is the distance of the line from the centre point.

2.2. Mathematical modelling of three-dimensional geometry

Assuming that the ship departs from the centre of the sea area A' (whose projection point on the slope is A), the direction of its line of sight is Z m (fig.3). The angle between the direction of the survey line and the projection of the normal direction of the sea floor slope on the horizontal plane is β . (taking the centre point as the relative point, Z metres travelling towards the depth of the water is recorded as minus Z , and vice versa is recorded as plus Z) and arrive at the point O' (whose projection point on the slope is O), through which a line parallel to the slope is made to intersect with the point P , and the line OP can be found to be equal to the depth of the point A and is recorded as the point B .

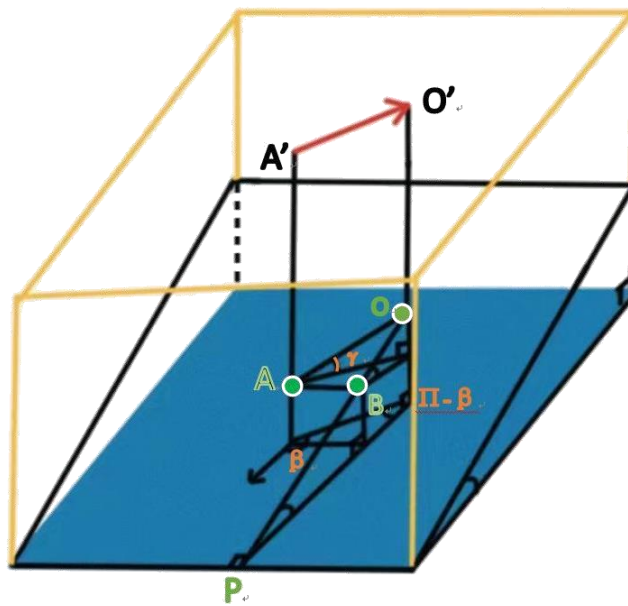


Fig. 3 Coverage width of cubic geometry

The three plane analytical diagrams of the three-dimensional geometry are listed as the projection of the bottom surface of the seabed, the plane of measurement over the points O and P , and the tangent plane where the angle γ is located. O , P , and the tangent plane of the angle γ to be solved [9-10].

Figure 4 shows a ship from the centre of the sea area A' (seabed projection as a) along the course in the direction of β to reach the point O' (seabed projection as o'') in the plane projection and the depth B (seabed projection as b) which is equal to A' in the slope projection A . (the seabed projection is o'') and the projection of A' on the slope at depth B (the seabed projection is b). (b) The top plan view of the projections. According to the above, it is known that the ship is travelling along the course in the direction of β for Z metres, i.e. ao'' is Z metres. The angle between the direction of the survey line and the normal direction of the seabed slope is β . According to this condition and the cosine formula, we can find the distance E of bo'' .

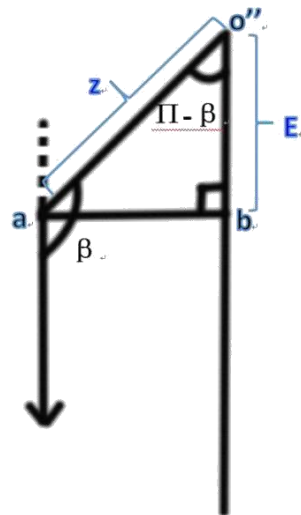


Fig. 4 Plan view of the ground projection of the seabed

According to this condition and the cosine formula, the bo'' distance E can be found.

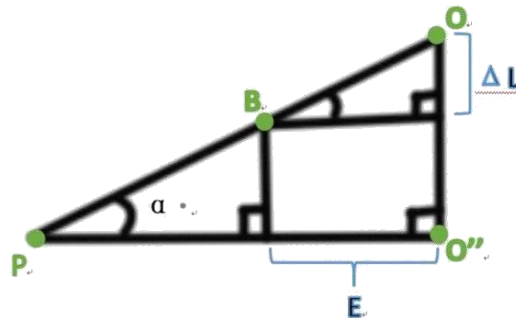


Fig. 5 Side view

Figure 5 shows a side view of the slope projection O of a ship travelling Z metres along a course in the direction β to reach a point O' where B is a point of equal depth to A . The depth of B is a point of equal depth to A . B is a point of equal depth to A . The following equation can be found by the tangent theorem to find the height difference ΔL .

2.3. Establishment of mathematical model of three-dimensional geometry

Consider a rectangular sea area with a length of 2 nautical miles from north to south and a width of 4 nautical miles from east to west, the depth of the sea water at the centre of the sea area is 110m, the depth is deep in the west and shallow in the east, and the gradient is 1.5° , and the angle of opening of the multibeam transducers is 120° . Design a set of survey lines with the shortest measurement length that can completely cover the entire sea area to be surveyed, and the overlap rate between adjacent strips meets the requirement of 10%~20%. The total measuring line length should be shortest with respect to $f = (\beta, d)$, and the spacing d to make the total measuring line length shortest. Multi-objective planning problem with two decision variables to find the optimal β . Assume that the travelling directions of the routes are all parallel to each other. From the meaning of the problem, we know that the neighbour spacing d is relatively small in shallow water areas, and the neighbour spacing d is relatively larger in deep water areas, i.e., the neighbour spacing between each route should be different. According to the symmetry of the sea area and the symmetry of the heading, β can be narrowed from $(0^\circ, 360^\circ)$ to $(0^\circ, 180^\circ)$. The schematic diagram is shown in figure 6:

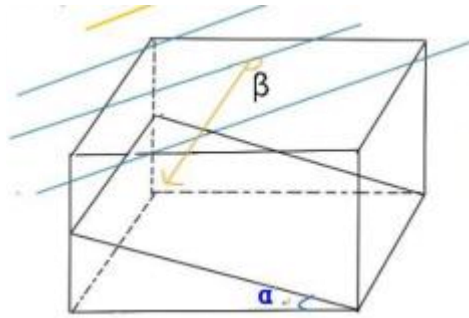


Fig. 6 Schematic diagram of question three

Decision variables: (1) The angle β between the direction of the survey line and the projection of the normal direction of the submarine slope on the horizontal plane.

(2) The spacing d between individual neighbouring survey lines.

objective function: $f_y = \min \sum_{i=1}^n l_i$

restrictive condition: $20\% \geq \eta = \frac{m-d}{w} \geq 10\%$

The angle β affects the length of each route when the angle β is unknown, and since the area of the sea is known, the d variable affects the total number of routes.

When making the angle β between the direction of the survey line and the projection of the normal direction of the seabed slope on the horizontal plane, the plane perpendicular to each route is roughly shown in Figure 7 as follows.

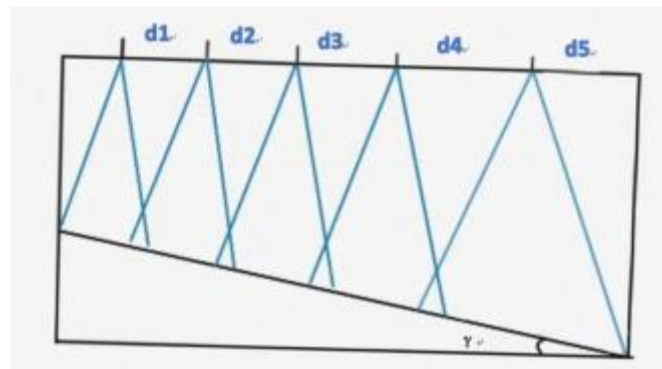


Fig. 7 Sections of planes perpendicular to each course

Algorithm: Due to the existence of two unknown variables, in order to simplify the problem, we adopt a fixed β -value, i.e., the β -value is traversed in $(0, 180)$, so as to solve the total number of routes, denoted as n . The sum of these n routes is denoted as f_i and saved in a set F . When the β -value is traversed once in $(0^\circ, 180^\circ)$, the shortest total route distance corresponding to each angle of the β -value is stored in the set F . The minimum global route distance is found by traversing the set F to find the minimum global route distance. The global minimum total route length is found by traversing the set F . The β -value corresponding to this length and d is the global optimal solution of the problem. According to the optimisation model, the global optimal solution β is found to be 45° and the minimum measured total route length is 15.478 nautical miles.

3. Conclusions

In conclusion, this paper has successfully addressed the challenges in multibeam bathymetry by establishing comprehensive mathematical models and optimization methods. The two-dimensional mathematical model considered coverage width, overlap rate, and seawater depth as objectives, utilizing trigonometric properties and topographic factors. The MATLAB software facilitated solving the model, leading to expressions for coverage width and overlap rate. Subsequently, a three-dimensional geometric seawater depth model was developed, providing a coverage width model for different survey line directions. The optimization model aimed at minimizing total routes while

meeting the overlap rate requirement, employing a traversal algorithm. The study's findings contribute significantly to the field of multibeam bathymetry, offering a solid theoretical foundation and practical applications. The established models and optimization methods pave the way for efficient survey planning in diverse marine environments. The presented algorithms, including the cyclic traversal idea, demonstrate the feasibility of optimizing multibeam bathymetry surveys. Overall, this research provides valuable insights and tools for practitioners and researchers engaged in marine mapping and exploration.

References

- [1] WANG Fengfan, MA Yong. Design and realisation of polar ocean multibeam survey line laying system [J]. Marine Information Technology and Application, 2023, 38 (03): 158-162+186.
- [2] Shan Jianchen, Li Shanshan, Yang Qian et al. Design of line density for shipborne sonar bathymetry with satellite altimetry [J]. Journal of Surveying and Mapping Science and Technology, 2021, 38 (05): 454-460+465.
- [3] Bai Zhihong, Xiao Hu, Yan Wisdom, et al. A method of segmental scanning planning survey line based on digital elevation data in complex mountainous areas[C]// Petroleum Physical Exploration Professional Committee of Chinese Petroleum Society, Exploration Geophysics Committee of Chinese Geophysical Society. Proceedings of the 2021 Symposium on Physical Exploration Technology of the Chinese Petroleum Society. Acquisition Technology Centre of China Petroleum Oriental Geophysical Company; Changqing Physical Exploration Division of China Petroleum Oriental Geophysical Company; 2021: 4.
- [4] Zhang YJ, Zhang N, Wang P et al. Introduction and application of Survey Box~+ [J]. Physical Exploration Equipment, 2021, 31 (04): 247-249+252.
- [5] Gao Qingyun, Li Xiangguang, Liu Wenge et al. Research and design of quality inspection method of wireline tester [J]. China Inspection and Testing, 2020, 28 (05): 29-30.
- [6] Zhang YJ, He Q, Wu JH et al. The solution of survey line design adapted to the development of physical exploration [J]. Physical exploration equipment, 2020, 30 (04): 238-241.
- [7] SHANG Ke. Research on three-dimensional ground-penetrating radar detection method and intelligent comparison method of time-shift data for urban roads [D]. China University of Geosciences (Beijing), 2020.
- [8] Xu Zhaohui. Observation of surface subsidence in Yangcun coal mine air-mining area and analysis of results [J]. Coal Technology, 2019, 38 (05): 71-74. 4
- [9] Guangsou Xu, Guojun Zhai, Taiqi Wu, et al. Key technical issues of airborne laser bathymetry operation [J]. Ocean Surveying and Mapping, 2019, 39 (02): 45-49+66.
- [10] Jiao Guchao. Research on multi-channel transient electromagnetic detection method for underground pipelines [D]. China University of Mining and Technology, 2018.