

Efficiency Analysis and Structural Optimization of a Heliostat Mirror Field in a Tower-Type Photovoltaic Power Plant

Hongwei Song, Rende Chen, Xiaozhou Zhou *

School of Automotive and Traffic Engineering, Jiangsu University, Zhenjiang, China, 212013

* Corresponding Author Email: zxz03ppsw@163.com

Abstract. With the growing global demand for sustainable energy, tower-type photovoltaic power plants have received a lot of attention as an efficient way to utilize solar energy. In this thesis, a more perfect model of the heliostat mirror field was established from multiple angles and using various methods. Using Cooper's solar declination formula, the precise solar position was obtained; with the help of the HFLCAL model, the collector truncation efficiency was obtained, and the heliostat field optical efficiency model was obtained, combining it with the genetic algorithm, the arrangement of heliostat mirrors was optimized, and the heliostat field model was simplified, which greatly improves the computational efficiency. Finally, the optimal annual average thermal power output per unit mirror surface area is obtained as follows: the location coordinates of the absorption tower are: (0, -101.5211), the number of heliostat mirrors is: 3653, the interval of heliostat mirror width is [3.5224, 4.7914], the interval of heliostat mirror height is [3.5008, 4.5000], and the interval of heliostat mirror height are [2.2511, 5.9986].

Keywords: Transformation of the energy mix, HFLCAL model, model of heliostat mirror field, genetic algorithm.

1. Introduction

In this paper, it is assumed that the size and mounting height of the heliostat mirror within the heliostat field can be inconsistent, if the parameters of the heliostat field are redesigned by the design requirements, the annual average output thermal power of the heliostat field reaches 60 and the annual average output thermal power of the unit mirror area is as large as possible(Data source:mcm.edu.cn).

In this article, a genetic algorithm is used to optimize how to maximize the annual average optical efficiency and output power of the heliostat field, and the arrangement parameters of the heliostat are solved to obtain the values of the heliostat and the corresponding output power. A genetic algorithm was first proposed by John Holland in the 1970s, the algorithm is a method of searching for the optimal solution by simulating the natural evolution process^[1].

The position of the sun at a specific time is first obtained using the Cooper solar declination angle approximation formula ^[2] in combination with other parameters. Then the heliostat field coordinate system is established, and the mirror normal vector of the heliostat is deduced according to the law of light reflection to obtain the heliostat pitch and azimuth angles. Then the HFLCAL model is used to obtain the collector truncation efficiency ^[3], and the heliostat field optical efficiency model is established, and a single-objective optimization model to maximize the annual average thermal power output per unit mirror area is established.

2. Heliostat Field Analysis in Solar Towers

2.1. Preparation for modeling of the heliostat field

Before establishing the heliostat field model, the earth coordinate system and the heliostat surface coordinate system are first established to lay the foundation for the next model solution.

2.1.1 Coordinate System Establishment

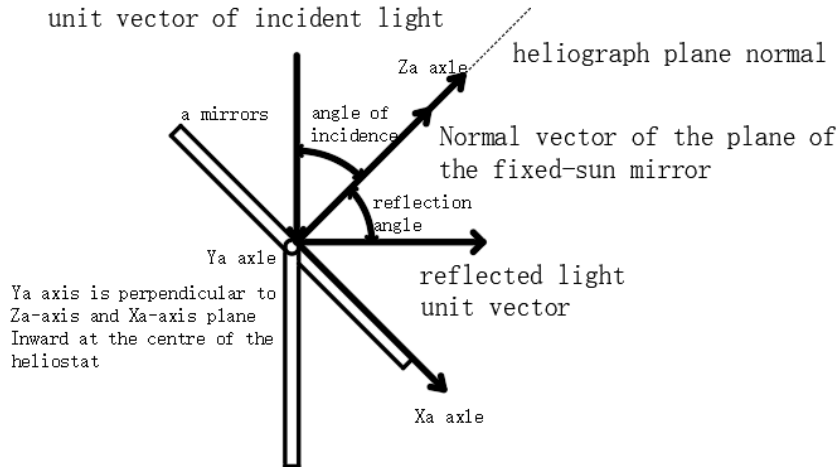


Figure 1. Heliostatic mirror coordinate system

With the center of the circular heliostat mirror field as the origin, the surface plane is the x and y coordinate plane, the east direction as the x-axis positive direction, the north direction as the y-axis positive direction, and perpendicular to the ground upward direction as the positive direction to establish the spatial coordinate system, known as the mirror field coordinate system, also known as the Earth coordinate system.

As shown in figure 1, each heliostat is analyzed and a coordinate system is established on the heliostat, which is known as the heliostat surface coordinate system.

2.1.2 Description analysis and simplification

1)Sunlight is a conical ray

Sunlight is a cone of light rather than parallel rays, so to facilitate the analysis and calculation, in the analysis of incident light and reflected light and solve the corresponding unit vector, the sunlight dispersion of the cone of light is simplified as a cone of light in the axis of the line of light.

2)Simplified diagram of the heliostat, absorption tower, and collector

As shown in figure 2, each fixed-sun mirror size and mounting height is fixed, and the absorption tower is 80m high, to make the fixed-sun mirror, absorption tower, and collector simplified diagram can be obtained:

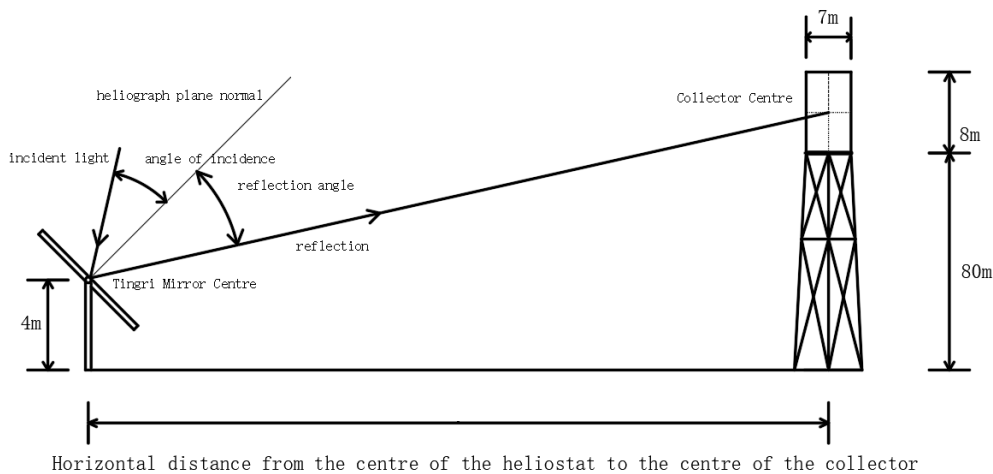


Figure 2. Simplified diagram of the heliostat, absorption tower, and collector

2.2. Modeling of the heliostat field

The analysis leads to a heliostat field model that will consist of four components: solar position, normal radiant irradiance, heliostat optical efficiency, and heliostat field output thermal power. Based

on the Earth coordinate system and the given conditions, The coordinates of the center of the i th heliostat are (x_i, y_i, z_i) , The coordinates of the center of the collector are (x_0, y_0, z_0) , The four parts of the fixed heliostat field model are analyzed and the solution of the fixed heliostat field model will be based on the solution of the four parts.

1)solar position(altitude angle α_s . azimuth γ_s . solar time angle ω . angle of azimuth δ)

Based on the acquired data, the heliostat field is at 98.5°E , 39.4°N , and 3000 m above sea level, analyzed by the established Earth coordinate system.

solar altitude angle α_s calculation formula^[4]:

$$\sin \alpha_s = \cos \delta \cos \varphi \cos \omega + \sin \delta \sin \varphi \tag{1}$$

Φ is the local latitude and is positive at the north latitude, solar azimuth γ_s , calculation formula^[4]:

$$\cos \gamma_s = \frac{\sin \delta - \sin \alpha_s \sin \varphi}{\cos \alpha_s \cos \varphi} \tag{2}$$

solar time angle ω calculation formula:

$$\omega = \frac{\pi}{12} (ST - 12) \tag{3}$$

included among these ST is local time.

angle of azimuth δ formula was calculated using the approximate formula proposed by Cooper in 1969:

$$\delta = 23.45 \sin\left(2\pi \frac{284 + D}{365}\right) \tag{4}$$

included among these D is the yearly cumulative date, if 1 January is the first day, then 1 February is the 32nd day.

2)normal radiation irradiance DNI

Normal radiation irradiance DNI is Approximated by the following equation^[5]:

$$\begin{cases} DNI = G_0 \left[a + b \exp\left(-\frac{c}{\sin \alpha_s}\right) \right] \\ a = 0.4237 - 0.00821(6 - H)^2 \\ b = 0.5055 + 0.00595(6.5 - H)^2 \\ c = 0.2711 + 0.01858(2.5 - H)^2 \end{cases} \tag{5}$$

The formula G_0 is the solar constant, $G_0=1.366\text{kW/m}^2$; H which is the height above sea level. Based on the data it is known that the altitude of the heliostat field is, then $H=3$ (unit km).

3)heliostat optical efficiency η

The optical efficiency corresponding to each heliostat is η_i , the optical efficiency η formula:

$$\eta = \eta_{sb} \eta_{\cos} \eta_{at} \eta_{trunc} \eta_{ref} \tag{6}$$

Five of these parameters were analyzed separately: respectively, specular reflectance η_{ref} . shadow masking efficiency η_{sb} . cosine efficiency η_{\cos} . atmospheric transmittance η_{at} . collector cut-off efficiency η_{trunc} .

4)Thermal power output of the heliostat field E_{field}

The thermal power output of the heliostat field E_{field} formula is:

$$E_{field} = DNI \sum_i^N A_i \eta_i \tag{7}$$

Formulas DNI are the normal radiant irradiance, N is the total number of fixed-sun mirrors, η_i is the optical efficiency of the I heliostat, A_i is the light area of the I heliostat, then $A_i=A \cdot \eta_{sb}$, thus equation (7) is rewritten as:

$$E_{field} = DNI \sum_i^N A \eta_{sb} \eta_i \tag{8}$$

The thermal power output per unit area of the mirror is:

$$E = \frac{E_{field}}{N \times A} \tag{9}$$

2.3. Fixed-sun mirror field model solution

2.3.1 initial conditions

Based on the simplified diagrams of the heliostat, absorption tower, and collector, the initial parameters of some of the variables or symbols are determined, and a few of them can be obtained from the analyses of the data that have been obtained, as follows:

$$\begin{cases} x_0 = 0, y_0 = 0, z_0 = 80 + \frac{1}{2} \times 8 = 84, z_i = 4 \\ \varphi = 39.4^\circ, ST = 9, 10.5, 12, 13.5, 15 \\ G_0 = 1.366, H = 3, N = 1745, LW = 6, LH = 6 \\ N = 1745, \eta_{ref} = 0.92 \end{cases} \tag{10}$$

Note: where specular reflectance can be considered as a constant value $\eta_{ref}=0.92$.

The coordinates of the center of the collector in the Earth coordinate system are $(x_0, y_0, z_0)=(0, 0, 84)$.

The coordinates of each heliostat in the Earth's coordinate system are (x_i, y_i, z_i) .

Step2. Model solving

1) Solve for the vector based on the position of the sun

due to ST_j values corresponding to different local times. The 21st of the 12 months under each period is taken as the yearly accumulation day, which has $k, (j=5, k=12)$.

The five values are shown in table 1:

Table 1. Local time value

| ST_j | ST_1 | ST_2 | ST_3 | ST_4 | ST_5 |
|--------------|--------|--------|--------|--------|--------|
| take a value | 9 | 10.5 | 12 | 13.5 | 15 |

Only one of the time cases, a fixed-heaven mirror, is considered in the model solution analysis. Satisfy the formula:

$$\begin{cases} \sin \alpha_s = \cos \delta \cos \varphi \cos \omega + \sin \delta \sin \varphi \\ \cos \gamma_s = \frac{\sin \delta - \sin \alpha_s \sin \varphi}{\cos \alpha_s \cos \varphi} \\ \omega = \frac{\pi}{12} (ST - 12) \\ \delta = 23.45 \sin(2\pi \frac{284 + D}{365}) \end{cases} \tag{11}$$

As shown in figure 3 , the analytical treatment gives the unit vector of incident light as $\mathbf{n}_r=(x_r, y_r, z_r)$:

$$\begin{cases} x_r = \cos \alpha_s \cos(\gamma_s - 90^\circ) \\ y_r = \cos \alpha_s \sin(\gamma_s - 90^\circ) \\ z_r = \sin \alpha_s \end{cases} \quad (12)$$

The center of the heliostat pointing towards the center of the collector is the direction of the reflected light, and the unit vector of the reflected light can be obtained after analytical processing as $\vec{n}_f=(x_f,y_f,z_f)$:

$$\vec{n}_f = \frac{P_0 - P}{|P_0 - P|} = \frac{(-x_i, -y_i, z_0 - z_i)}{\sqrt{x_i^2 + y_i^2 + (z_0 - z_i)^2}} \quad (13)$$

Based on the analysis of the unit vector of the incident light and the unit vector of the reflected light, the normal vector of the mirror of the corresponding fixed-sun mirror can be obtained as:

$$\vec{n} = \frac{\vec{n}_f - \vec{n}_r}{|\vec{n}_f - \vec{n}_r|} \quad (14)$$

Vector-based solution of the pitch angle of a fixed-sun mirror θ_1 and azimuth angle θ_2 :

$$\begin{cases} \tan \theta_1 = \frac{z_i + \sin \alpha_s \sqrt{m}}{\sqrt{x_i^2 + y_i^2 + (x_i^2 + y_i^2 + z_i^2) \cos^2 \alpha_s - 2 \cos \alpha_s [(x_i \sin \gamma_s - y_i \cos \alpha_i) \sqrt{m}]}} \\ \tan \theta_2 = \frac{x_i - \cos \alpha_s \sin \gamma_s \sqrt{m}}{\sqrt{x_i^2 + y_i^2 + (x_i^2 + y_i^2 + z_i^2) \cos^2 \alpha_s - 2 \cos \alpha_s [(x_i \sin \gamma_s - y_i \cos \alpha_i) \sqrt{m}]}} \end{cases} \quad (15)$$

included among these $m=(x_i^2+y_i^2+z_i^2)^{0.5}$.

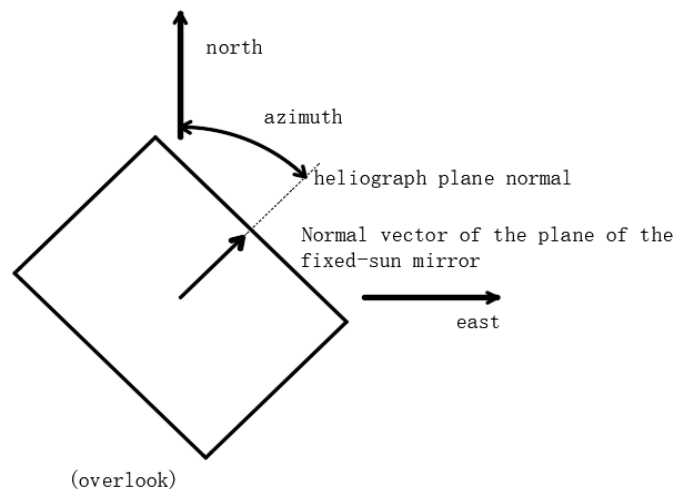


Figure 3. Diagram of azimuth

2) Solving for cosine efficiency η_{cos}

The fixed heliograph coordinate system is analyzed, defining any fixed heliograph as a fixed heliograph A, the three coordinate axes of the fixed heliograph coordinate system are x_A, y_A, z_A , From the optical properties, it is known that the angle of incidence and the angle of reflection concerning the normal of the heliostat are both θ . Let \vec{i} be the unit vector in the opposite direction of the incident light with cosine efficiency^[6]:

$$\begin{cases} \eta_{cos} = \cos \theta = \vec{n} \cdot \vec{i} \\ \vec{i} = (-\cos \alpha_s \sin \gamma_s, -\cos \alpha_s \cos \gamma_s, -\sin \alpha_s) \end{cases} \quad (16)$$

3) Solving for atmospheric transmittance η_{at}

d_{HR} is the distance from the center of the heliostat to the center of the collector, which can be solved by substituting the coordinates of the Earth's coordinate system, and $d_{HR} \leq 1000$, The atmospheric transmittance is calculated as [7]:

$$\begin{cases} \eta_{at} = 0.99321 - 0.0001176d_{HR} + 1.97 \times 10^{-8} \times d_{HR}^2 \\ d_{HR} = \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2 + (z_i - z_0)^2} \end{cases} \quad (17)$$

4) Solving for specular reflectance η_{ref}

The reflectance can be considered a constant, $\eta_{ref} = 0.92$.

5) Solving for shadow shading efficiency η_{sb}

The following idea can be used for the calculation of shadow shading efficiency [8]:

$$\eta_{sb} = 1 - \frac{\text{Shadow and mirror masking area}}{\text{Mirror area}} \quad (18)$$

the earth coordinate system and the heliostat mirror coordinate system are processed under this calculation idea, and the unit matrix of the conversion relationship from the mirror coordinate system to the earth coordinate system is obtained by combining the heliostat pitch angle θ_1 and azimuth angle θ_2 [9].

Analysis can be seen: "shadow" that is, the incident light is blocked; "blocking" that is, the reflected light is blocked. To a certain heliostat A, for example, in the heliostat on the establishment of the coordinate system, in the heliostat A coordinate system under a point $Q_1(x_1, y_1, 0)$, Q_1 projected through light propagation to a heliostat B on the plane to $Q_2(x_2, y_2, 0)$, Determine Q_2 Whether it's in the sun-setting mirror B. If it is in then it is determined that the heliostat B is obscured thus creating a shadow, and the obscuring situation is analyzed along similar lines.

6) Solving for collector cut-off efficiency η_{trunc}

The collector truncation efficiency is solved and the HFLCAL model is used to calculate the collector truncation efficiency η_{trunc} . Analyzed to obtain the calculation formula:

$$\eta_{trunc} = \frac{1}{2\pi\sigma_{tot}^2} \iint_{x,y} \exp\left(-\frac{x^2 + y^2}{\sigma_{tot}^2}\right) dydx \quad (19)$$

$$\begin{cases} \sigma_{tot} = \sqrt{d_{HR}(\sigma_{sun}^2 + \sigma_{bq}^2 + \sigma_{ast}^2 + \sigma_{track}^2)} \\ \sigma_{bq}^2 = 4\sigma_s^2 \\ \sigma_{ast} = \sqrt{\frac{H_t^2 + W_s^2}{8d_{HR}}} \\ H_t = \sqrt{LW \times LH} \left| \frac{d_{HR}}{f} - \cos\theta \right| \\ W_s = \sqrt{LW \times LH} \left| \frac{d_{HR}}{f} \cos\theta - 1 \right| \end{cases} \quad (20)$$

The relevant parameters are: $\sigma_{sun} = 2.51 mrad$, $\sigma_s = 0.94 mrad$, $\sigma_{track} = 0.63 mrad$; where formula (19) is analyzed for a cylindrical heat absorber, formula (20) where LH is the length of the heliostat, LW is the width of the heliostat. Substituting into MATLAB and solving to get the collector truncation efficiency.

7) Thermal power output of the heliostat field E_{field}

Based on the process of solving the fixed heliostat field model, the output thermal power calculation formula of the fixed heliostat field is obtained:

$$\begin{cases} \eta = \eta_{sb}\eta_{\cos}\eta_{at}\eta_{trunc}\eta_{ref} \\ E_{field} = DNI \sum_i^N A_i \eta_i \\ A_i = LW_i \times LH_i \end{cases} \quad (21)$$

Where DNI is the normal radiant irradiance:

$$\begin{cases} DNI = G_0 \left[a + b \exp\left(-\frac{c}{\sin \alpha_s}\right) \right] \\ a = 0.4237 - 0.00821(6 - H)^2 \\ b = 0.5055 + 0.00595(6.5 - H)^2 \\ c = 0.2711 + 0.01858(2.5 - H)^2 \end{cases} \quad (22)$$

Substituting into Matlab and solving, we get the output thermal power of the fixed-sun mirror field E_{field} .

The results of the optical efficiency of each heliostat mirror at different times and, the thermal output power of the heliostat mirror field will be obtained after model solving.

3. Optimizing Heliostat Fields with Complex Parameters

A review of a large amount of relevant literature shows that, under the same northern hemisphere conditions (a power plant in Spain), obtaining the maximum annual average thermal power output per unit area will result in an offset of the absorption tower position in the negative direction of the axis, in other words, due south, in the heliostat field. Based on this scenario, a schematic diagram of the absorption tower offset versus the heliostat distribution was obtained.

As shown in figure 4, based on the optimization model, a schematic diagram of the absorption tower offset and heliostat distribution is established:

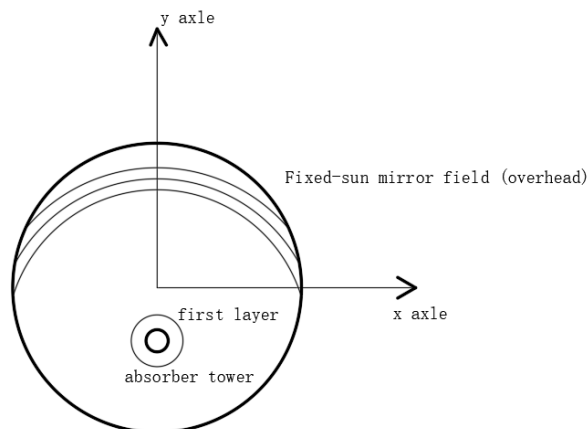


Figure 4. Schematic diagram of absorption tower offset and heliostat distribution

Schematic analysis of the absorption tower offset and heliostat distribution, with a coordinate of 0 on the x-axis of the absorption tower, reduces the number of parameters that need to be analyzed and simplifies the calculation of the model.

Since the width, length and mounting height of any heliostat can be different, the optimization parameters have a higher dimension, which leads to a more difficult solution using a conventional

optimization algorithm. To ensure that the optimization results are robust and do not fall into a local optimum, a genetic algorithm is used to solve the problem^[10].

As shown in table 2. table 3. table 4 , the parameters and model are substituted into MATLAB and the optimization model is solved using the genetic algorithm method to obtain the solution results.

Table 2. Average optical efficiency and output power table on the 21st of each month

| dates | Average optical efficiency | Average cosine efficiency | Average shadow-blocking efficiency | Average cut-off efficiency | Average thermal power output per unit area of mirror(kW/m^2) |
|-------|----------------------------|---------------------------|------------------------------------|----------------------------|--|
| 1.21 | 0.5539 | 0.8209 | 0.8117 | 0.9896 | 1.2131 |
| 2.21 | 0.5452 | 0.7795 | 0.8798 | 0.8975 | 0.9114 |
| 3.21 | 0.5419 | 0.8216 | 0.8752 | 0.9521 | 1.2535 |
| 4.21 | 0.5281 | 0.7788 | 0.8771 | 0.9478 | 1.1158 |
| 5.21 | 0.4989 | 0.6789 | 0.8916 | 0.9358 | 0.8387 |
| 6.21 | 0.6199 | 0.7926 | 0.8317 | 0.9741 | 1.1832 |
| 7.21 | 0.5833 | 0.6582 | 0.8065 | 0.9902 | 0.8818 |
| 8.21 | 0.5714 | 0.7567 | 0.8849 | 0.9564 | 1.0953 |
| 9.21 | 0.5099 | 0.7151 | 0.8633 | 0.9382 | 1.342 |
| 10.21 | 0.5022 | 0.6863 | 0.8930 | 0.9376 | 0.8132 |
| 11.21 | 0.5792 | 0.7068 | 0.8113 | 0.9891 | 1.1015 |
| 12.21 | 0.6091 | 0.8371 | 0.8018 | 0.9803 | 1.0102 |

Table 3. Annual average optical efficiency and output power table

| Average annual optical efficiency | Average annual cosine efficiency | Average annual shading Shading efficiency | Average annual cut-off efficiency | Average annual thermal power output (MW) | Average annual thermal power output per unit area of mirror(kW/m^2) |
|-----------------------------------|----------------------------------|---|-----------------------------------|--|---|
| 0.5536 | 0.7523 | 0.8523 | 0.9734 | 60.5763 | 1.0698 |

Table 4. Design Parameter table

| Absorption tower position coordinates (x,y) | Total number of heliostats | Total area of heliostat(m^2) |
|---|----------------------------|----------------------------------|
| (0, -101.5211) | 3653 | 56623.9484 |

The interval of the value of heliostat mirror width is [3.5224, 4.7914]; the interval of the value of heliostat mirror height is [3.5008, 4.5000]; the interval of the value of heliostat mirror mounting height is [2.2511, 5.9986]. The results of the genetic algorithm run can be obtained after visualization:

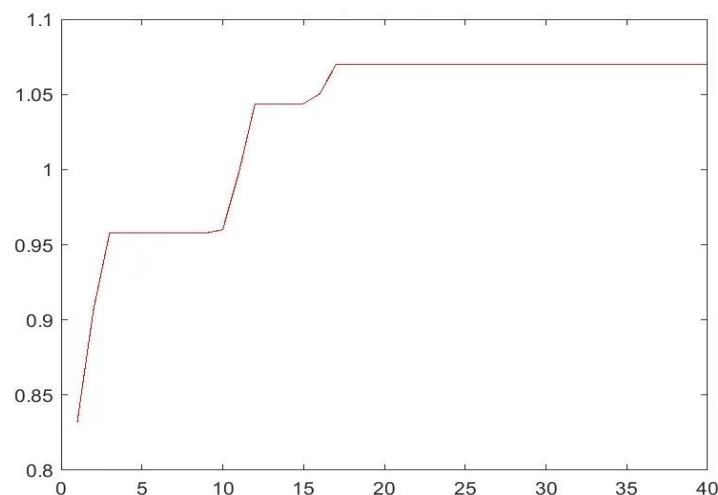


Figure 5. genetic image

As shown in figure 5, analyzing the genetic image shows that the value of the function, i.e. the average annual thermal power output per unit area of mirror, stabilizes after the 17th generation of inheritance and converges at $1.0698kW/m^2$.

4. Conclusions

This paper provides a research idea and framework for research in the field of optimal design of heliostat fields, which can help in the design and construction of heliostat fields, realizing new energy and modern development, and proving the feasibility of using genetic algorithms for the study of heliostat field efficiency and optimization of the structure.

In response to the analysis of the results, the optimal annual average thermal power output per unit mirror area is obtained as 1.0698MW. The coordinates of the absorption tower location are:(0, -101.5211)The number of heliostats is 3653, the interval of heliostat width is [3.5224, 4.7914], the interval of heliostat height is [3.5008, 4.5000], and the interval of heliostat installation height is [2.2511, 5.9986]. In addition to the genetic algorithm, the Cooper solar declination formula is used in the model-building process to obtain the accurate solar position.

In summary, this article adopts multi-model and multi-method, establishes a more perfect mathematical model of the heliostat field, uses a genetic algorithm to solve it, obtains scientific and reasonable results, proves the credibility of the model and calculation method in this article, and provides ideas for the research in the field of optimal design of heliostat field.

References

- [1] Zheng Shuquan. Industrial Intelligence Technology and Application [M]. Shanghai: Shanghai Science and Technology Press, 2019: 250-251
- [2] Coope P.I. The absorption of radiation in solar stills [J]. Solar Energy, 1969, 12(3): 333-346.
- [3] Yang Hongtao. Research on the model and optimal scheduling system of heliostat concentrator[D]. Xi'an University of Technology, 2023. DOI: 10.27398/d.cnki.gxalu.2023.000068.
- [4] Zhuang Yanhao. Derivation of solar altitude angle and solar azimuth angle formulas based on subject integration - An example of senior three geography test questions in ten schools in Jinhua in November 2020[J]. Geography Teaching, 2022(02): 38-40.
- [5] Du Yuhang et al, Impact analysis of different focusing strategies of heliostats in tower-type photovoltaic power plants[J], Journal of Power Engineering, 2020, 40(5):426-432.
- [6] FANG Miaosen, LU Jing, SHEN Zhigang. Modelling of front-end efficiency of tower solar heliostat and its application[J]. Journal of Changzhou Institute of Information Technology,2021,20(03):20-24.

- [7] O. Farges, J.J. Bezia, M. El Hafi, Global optimization of solar power tower systems using a Monte Carlo algorithm: Application to a redesign of the PS10 solar thermal power plant [J], *Renewable Energy*, 2018, 119:345-353.
- [8] HUANG Ju, YAN Zhiguo, DENG Biao et al. Current status of research on mirror field design software for tower solar thermal power plants[J]. *Oriental Electric Review*,2023,37(02):41-47.DOI:10.13661/j.cnki.issn1001-9006.2023.02.014.
- [9] XING Jian,LIU Zhijun,HAN Bing et al. Multispectral radiometric thermometry inversion algorithm based on generalised inverse-coordinate transform[J]. *Spectroscopy and Spectral Analysis*,2023,43(06):1936-1940.
- [10] Zhong Jiahua,Liu Gang. Research on optimal operation of thermal power plant units based on genetic algorithm[J]. *Electrical Technology and Economy*, 2023(10): 4-6.