Saastamoinen and Marini-Murray's Study of the Tropospheric Delay Model in Shijiazhuang

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Abstract. In addition to the non-dispertic medium, the tropospheric medium changes with time, and it is difficult to use a simple and unified mathematical model to determine the degree of change, especially in the long baseline relative positioning, the tropospheric delay between two stations often shows a weak spatial correlation. In this paper, this study will briefly introduce the main methods to reduce or eliminate various kinds of errors in order to build a real-time tropospheric delay model. The main body of this paper is to introduce and discuss the existing direct calculation model and mapping function of tropospheric delay. Based on the meteorological data of Shijiazhuang meteorological station provided by the official website of China Meteorological Administration, the appropriate tropospheric delay correction model and projection function (Saastamoinen +Marini-Murray model) are selected. The hourly tropospheric delay correction model along the propagation path direction in Shijiazhuang city on April 1, 2023 is established, and the accuracy and reliability of the model are analyzed.

Keywords: GNSS positioning error, Tropospheric delay, Delay correction model, Mapping function model.

1. Introduction

With the successful networking of the Beidou-3 satellite, China has become the fourth country or region with a global navigation satellite system in the world [1]. The completion of the three-step Beidou strategy has greatly improved the accuracy and reliability of the Global Navigation Satellite System (GNSS). When using GNSS to determine the three-dimensional coordinates of ground points by means of distance intersection, there are various errors in the system, such as satellite ephemeris error, satellite clock difference, receiver seed clock difference, ionospheric delay, tropospheric delay, and multipath effect [2]. Some of these errors can be accurately estimated and corrected by strict mathematical models because of the obvious regularity of systematic errors, and some errors can be regarded as constant constants in a considerable time range and can be determined by mathematical or mechanical methods. However, in addition to the non-dispertic medium, the tropospheric medium changes dramatically with time, which is difficult to be determined by a simple and unified mathematical model, especially in the long relative positioning of the baseline, the tropospheric path delay between the two stations often shows a weak spatial correlation. Therefore, it is of great significance to establish a real-time tropospheric path delay model in a certain region to improve the accuracy of satellite navigation. This paper aims to establish a real-time tropospheric delay model. The main body of this paper is to introduce and discuss the existing direct calculation model and mapping function of tropospheric delay and use the meteorological data of Shijiazhuang meteorological station provided by the official website of China Meteorological Administration to select the appropriate tropospheric delay correction model and projection function (Saastamoinen+Marini-Murray model). The hourly tropospheric delay correction model along the propagation path direction in Shijiazhuang city on April 1, 2023, is established, and the accuracy and reliability of the model are analyzed.

2. Some tropospheric path delay correction models and mapping function models are introduced and used

2.1. Direct calculation model of tropospheric delay

At present, the calculation methods of tropospheric delay correction are mainly divided into direct calculation method and mapping function calculation method. In the direct calculation method, commonly used models include Hopfield model, Saastamoinen model [3], Blank model, WAAS and EGNOS tropospheric correction model [4] and so on. The core principle of these models is to calculate and correct the delay of satellite signals during the process of passing through the atmosphere, so as to improve the positioning accuracy. In this paper, the empirical correction model of tropospheric delay and the mapping function method of tropospheric delay model are described in detail. At present, the most commonly used models for direct calculation of tropospheric delay are Hopfield model, Saastamoinen model and Blanc model. According to Li Zhao et al., the Hopfield model is slightly better than the Saastamoinen model when the altitude Angle of the observing satellite is large (≥35°), but because the altitude Angle of the satellite is large enough, the fitting results of the three models can actually meet the needs of general positioning. When the altitude Angle of the satellite is moderate (<35°and≥15°), the calculation results of the three models agree best, and the difference is only a few millimeters. When the altitude Angle of the satellite of the station is low (<15°), because the Saastamoinen model is divided into two parts to fit the troposphere, while the other two models only consider the troposphere as a whole, the Saastamoinen model shows obvious superiority, and its average positioning accuracy is about 2dm higher than that of the Hopfield model. Therefore, this paper will choose the Saastamoinen model as the raw data to get the function of the tropospheric delay in the direction of zenith distance.

The Saastamoinen model divides the global atmosphere into three layers, with temperatures falling by 6.5°C per kilometer above 10km in the troposphere. The second level is about 70km from the upper troposphere [5], where the air temperature is constant. The ionosphere is 70km above the ground [6], and the modified refractive index of the atmosphere is calculated as follows:

$$\Delta Z = \int_1^{n_1} \frac{\tan Z}{n} dn (0 \le Z \le 90°) \tag{1}$$

In the integration of the above formula, the greatest contribution of the Sassamonen model is that the integrated function is first developed in accordance with the zenith distance trigonometric function, and the zenith lag of the atmosphere is corrected.

$$\begin{cases} \rho = 0.002277 \times \frac{\left[P_0 + \left(0.05 + \frac{1255}{T_0 + 273.15}\right) e_0\right]}{f(\varphi, h)} \\ e_0 = rh \times 6.11 \times 10^{\frac{7.5 T_0}{T_0 + 273.3}} \\ f(\varphi, h) = 1 - 0.00266 \cos 2\varphi - 0.00028 h \end{cases} \tag{2}$$

$P_0, T_0, e_0, rh, f(\varphi, h), \varphi$ and h include the surface atmospheric pressure, surface air temperature, surface water pressure, relative humidity (0~1), the deviation correction of gravitational acceleration due to the rotation of the Earth [7], the center longitude of the station and the height of the station (km).

Saastamoinen model can calculate the skew delay correction formula as follows:

$$\delta\rho = \frac{0.002277}{\sin E}\left[P_s + \left(\frac{1255}{T_s} + 0.05\right) e_s - B \cos E\right] W(\varphi, H) + \delta R \tag{3}$$

Formula : $W(\varphi, H) = 1 + 0.0026 \cos 2\varphi + 0.0028 h_s$; φ is the latitude of the station. h_s is the elevation of the measuring point; B to h_s file list function formula; δR is the file list function formula for E and h_s.

From the practical point of view, the accuracy of real-time and fast operation is guaranteed, and a brief overview of it is made, and the simplified Sastamonen model is written:

$$\delta\rho = \frac{0.002277}{\sin E}\left[P_s + \left(\frac{1255}{T_s} + 0.05\right)e_s - \frac{a}{tan^2 E}\right] \tag{4}$$

Formula:

$$E' = E + \Delta E \tag{5}$$

$$\Delta E = \frac{16''}{T_s}\left(P_s + \frac{4810}{T_s}e_s\right)\cot E \tag{6}$$

$a = 1.16 - 0.15 \times 10^{-3}h_s + 0.716 \times 10^{-8}h_s^2$; E is the Angle, the air pressure represented by P_s, the air pressure represented by T_s, and the absolute temperature represented by e_s.

In the absence of observational data, the weather parameters required by Hopfield and Saastamoinen models can be obtained by using the DIPOP setting of the University of New Brunswick in Canada, so as to obtain the weather parameters required in the absence of observational data.

$$\begin{cases} T = T_0 - 0.0068 \cdot h \\ P = P_0 \cdot \left(1 - \frac{0.0068}{T_0} \cdot h\right)^5 \\ e = \begin{cases} e_0\left[1 - \frac{0.0068}{T_0} \cdot h\right]^4 (h < 11000m) \\ 0(h \geq 11000m) \end{cases} \end{cases} \tag{7}$$

Standard reference atmospheric parameters P0=1013.25mbar, e0=11.69lmbar, T0=288.15K, h is the altitude (m).

2.2. Calculation of tropospheric delay mapping function

In mapping function calculation, the most commonly used models include Cfa model, Chao model, Mit model, Marini-Murray model, Niell model and GMF model, etc. [8]. The projection function is an empirical model whose accuracy depends on the accuracy of the parent model and the field observation data. This paper discusses the mapping functions. Although the GMF model is the most accurate and theoretically rigorous mapping function model at present, the required parameters are too complicated, and it is difficult to obtain permissions. As an empirical correction model in common positioning (which can also be used as the initial solution in precision positioning), we only need to obtain the positioning results at the decimeter level. The accuracy difference between different mapping function models is only millimeters. Therefore, the mapping function of Marini-Murray model is chosen in this paper.

The calculation method of electromagnetic wave propagation velocity delay in the troposphere can generally be obtained by the product of the zenith delay value and the projection function of the altitude Angle of the satellite, namely:

$$\Delta L(el) = \Delta L_z MF(el) \tag{8}$$

The parameter estimation algorithm of Saastamoinen model mentioned in this paper has high precision. The projection function is an empirical model whose accuracy depends on the accuracy of the parent model and the field observation data. Lists the Marini-Murray model projection function formula to calculate the tropospheric delay.

According to the relevant knowledge of atmospheric physics, the atmospheric refractive index N is divided into dry component and wet component. The lag in the troposphere mainly includes dry gas lag and wet gas lag. When the dry gas lag time is between 80-90%, the lag time is relatively regular, and the accuracy of estimating zenith orientation is about 1%. However, due to the complexity of the moisture lag process and the influence of many factors, the prediction accuracy is only 10-20%. Equation (8) can be expressed by the dry and wet hysteresis components in the zenith direction and the corresponding mapping function:

$$\Delta d_{trop} = \Delta d_{z,d}m_d(E) + \Delta d_{z,w}m_w(E) \tag{9}$$

Formulas $\Delta d_{z,d}$ and $\Delta d_{z,w}$ below represent the dry and wet time-lag components in the zenith direction. $m_d(E)$ and $m_w(E)$ correspond to the corresponding functions of the height Angle E.

The zenith dry-wet delay using Saastamoinen model is as follows:

$$\Delta d_{z,d} = \frac{0.002277P}{f(B,H)} \tag{10}$$

$$\Delta d_{z,w} = \frac{e_s}{f(B,H)}\left(\frac{0.2789}{T_k} + 0.05\right) \tag{11}$$

Where P, e_s, T_k are the atmospheric pressure (mbar), water pressure (mbar) and absolute temperature (K) of the measuring station, respectively, $f(B,H)$ is a function of the geographical latitude B and elevation H of the observatory.

$$f(B,H) = 1 - 0.00266cos^2B - 0.00028H \tag{12}$$

In that way, after selecting the function suitable for oneself, the correction number of tropospheric refraction in the propagation path can be obtained by equations (9), (10) and (11).

The Marini-Murray model is a constant parameter continued fraction mapping function that uses an exponential atmosphere model under the isothermal assumption. The mapping function of the dry and wet components is the same, which is:

$$m_d(E) = m_w(E) = \frac{1+\beta}{\sin E + \frac{\frac{\beta}{1+\beta}}{\sin E + 0.015}} \tag{13}$$

Formula:

$$\beta = \frac{A}{B} \tag{14}$$

$$A = \frac{2.644\times10^{-3}exp(-0.14372h_0)}{f(\varphi,h)} \tag{15}$$

$$f(\varphi,h) = 1 - 0.0026\cos 2\varphi - 0.00031h \tag{16}$$

B is Saastamoinen's formula for total delay value.

3. Programming and implementation of the model

3.1. Data Acquisition

In the Saastamoinen model, four parameters need to be input, such as satellite zenith Angle, air temperature, air pressure and water vapor pressure. The satellite zenith Angle is set as three typical Angle control groups (5°, 15° and 35°), while the data of air temperature, air pressure and water vapor pressure need to be downloaded from relevant websites. This paper chooses to use the hourly air temperature, air pressure and water vapor pressure data of Shijiazhuang Meteorological station provided by the official website of China Meteorological Administration on April 1. Reasons for choosing to use data from the China Meteorological Administration website include:

(1) Authority: As the main functional department of the national meteorological department, the China Meteorological Administration supervises and manages the national meteorological work in accordance with the relevant provisions of the State, so the meteorological data provided by the China Meteorological Administration is highly authoritative.

(2) Convenient and efficient authorization: Most of the meteorological data provided by the official website of the China Meteorological Administration can be obtained through personal real-name registration, and the audit time will not exceed 24 hours.

(3) The data can be provided in EXCEL format, and the latitude and longitude information of the site is given, which is convenient to use the mapping function to obtain the tropospheric delay in the direction of the propagation path.

3.2. Program implementation and modeling

The core calculation part of the code is mainly used to calculate the delay error caused by the refraction effect of the atmosphere when the satellite signal in the direction of the propagation path passes through the atmosphere. The specific explanation is as follows:

(1) For each satellite signal point read in the data, obtain the longitude and latitude of its station position, as well as the information of air pressure, temperature, water vapor pressure, signal elevation Angle E, etc., and convert the elevation Angle E of the signal read into radian system.

(2) The influence of atmospheric pressure and water vapor pressure on the signal refraction effect is calculated, expressed by Ps and es respectively, where Ts is the temperature of the measuring station. Calculate the elevation Angle E corresponding to the change delta_E.

(3) Calculate the influence of atmospheric density on the signal refraction effect. Based on the Saastamoinen model, according to the propagation path length (omitted here) and direction of the signal (since E and delta_E have been calculated previously, the actual propagation direction of the signal, namely d_E, can be calculated), Calculate the change value delta_rho on the propagation path.

(4) According to the height information on the propagation path (since the earth height of the measuring station where the signal is located has been calculated previously, it can be expressed as h0, and f_phih is calculated as the atmospheric compression factor at this location). Then, the parameters A and beta required by the M&M projection function are calculated based on the latitude and longitude information (expressed as B).

(5) According to the above parameters, md_E and mw_E are calculated, which represent the direction factor and height factor of the signal after passing through the propagation path respectively.

(6) The total delay delta_d_trop brought by the atmosphere on the propagation path is calculated, and the influence of atmospheric density and water vapor pressure on the signal is considered.

(7) The obtained delta_d_trop is stored in final1(i) as the atmospheric delay error of the signal point.

3.3. Drawing Functions

The code in Table 1 below is used to draw the curve of tropospheric delay over time calculated by the three parameters on the same graph and use a variety of functions to beautify the image. The specific explanation is as follows:

(1) figure function: Opens a new graph window for drawing a graph.

(2) hold on function: do not change the current graph window, so that you can draw multiple curves in the same graph. plot function: Draw a curve. Here, three plot functions are applied to make curves represented by final1, final2, and (3) final3 arrays, each with different colors. It is important to note here that all three curves should be on the same axes, so there is no need to specify axes in the plot function.

(4) hold off function: close the graph window and release the window resources.

(5) xlabel function and ylabel function: used to add axis labels. Here, labels for the x and y axes are added, respectively, where FontName and FontSize are used to set the font and size.

(6) legend function: Used to add a legend. Here, an array of characters inside curly braces is used to represent labels corresponding to different curves, i.e. 'Parameter 1',' parameter 2' and 'parameter 3'. At the same time, the legend font and size are also set.

(7) axis function: used to set the value range of coordinate axes. Here, the values of the x and y axes are set to [0,25] and [0,50], respectively, to ensure that the drawn curve is fully displayed in the graph window.

Table.1. Generation program of tropospheric delay curve of three parameters over time

```
%construction
figure
hold on
plot(fina1l,'r')
plot(fina12,'b')
plot(fina13,'g')
hold off
xlabe1('time/s','FontName','Song typeface','FontSize',10)
ylabe1('Propagation path direction tropospheric delay/dm','FontName','construction','FontSize',10)
legend({'argument1','argument2’,’argument3'},'FontName','construction','FontSize',10)
axis([0 25 0 50])
```

3.4. Calculation results and analysis

The tropospheric delay of the output zenith distance direction is shown in Figure 1 below. It can be clearly seen that when the elevation Angle of the satellite is 35 degrees, its accuracy can be maintained at decimeter level. When the elevation Angle of the satellite is 15 degrees, its accuracy is about 1 meter. When the elevation Angle of the satellite is 5 degrees, its accuracy changes greatly, up to 4m. In addition, it is noted that the tropospheric delay has a sudden change from the 18th hour to the 19th hour, and the degree of sudden change becomes more obvious with the decrease of the elevation Angle of the satellite. Analysis of meteorological data shows that the average water vapor pressure in the 18th hour is 10.14, while the average water vapor pressure in the 19th hour is 15.16, which accords with the general law that the tropospheric delay increases with the increase of water vapor pressure.

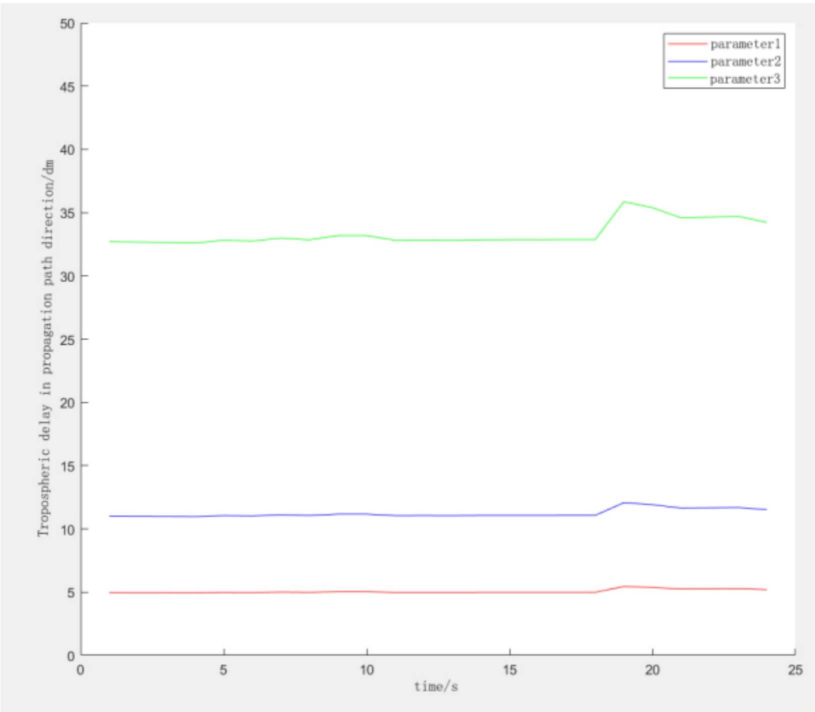

Figure 1. Tropospheric delay output in the zenith distance direction

4. Results

By summarizing the characteristics of various errors affecting satellite navigation and positioning and the ways to weaken and eliminate them, this paper points out the importance of establishing a real-time tropospheric delay correction model. The tropospheric delay correction model and mapping function model are summarized in detail, and the hourly tropospheric delay in the direction of the propagation path in Shijiazhuang area on April 1 is successfully established by using the mapping function of the Saastamoinen model and Marini-Murray model. When the altitude Angle of the satellite is greater than 15 degrees, the accuracy of decimeter level can be basically achieved. It also meets the trend that the tropospheric delay increases with the increase of air pressure, water vapor pressure and temperature, and decreases with the increase of the elevation Angle of the satellite, which basically conforms to the requirements of the theoretical formula, so it has certain application value in practical navigation and positioning. On the one hand, it can be directly used as a result in the ordinary positioning with relatively low precision requirements, on the other hand, it can also be used as an initial solution in the high-precision positioning by using the least square iso adjustment method to obtain the exact solution. For example, it is possible to increase the control group of the Hopfield model and the Blanc model under the same meteorological conditions; It is also possible to combine more mapping functions with direct computational models to increase the diversity of data.

5. Conclusions

At present, the research on tropospheric delay is a popular direction in the field of GNSS, especially the research on the wet component of tropospheric delay, because water vapor is the most active factor in the troposphere. Taking the model studied in this paper as an example, the abrupt change that occurs from the 18th hour to the 19th hour is caused by the change of water vapor. Therefore, the use of more optimized mathematical and physical models for water vapor inversion is the main direction to optimize the tropospheric delay model in the future. For example, Wang Shuaimin from Shandong University proposed in his article on JCR1 area that based on the traditional physical model, machine learning method was introduced to deal with nonlinear surface parameters and a new land water vapor inversion model was constructed [9]. Another example is the paper [10] by Zhang Weixing of Wuhan University, which proposes a PW model of water vapor inversion based on ground based GNSS. At the same time, this is also one of the topics that the author of this paper is interested in. On the road of future scientific research, I hope I can solve this problem and contribute my own strength to the development of China's Beidou satellite navigation system.

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