

# Workspace solution of 6-DOF mechanical platform based on imaginary support

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**Abstract.** In this paper, by analyzing the farthest and nearest critical conditions that the platform center can reach on a 6-DOF mechanical platform, the maximum space area that the platform center can move to, that is, the working space, is studied. In this paper, through mechanism analysis, the above platform moves along the direction of the actual support, the three-dimensional model is converted into a two-dimensional plane model by selecting a section, and the respective constraints under five different motion processes are analyzed. Then, the degree of freedom is reduced by defining the unknown, the coordinates of the center of the platform are solved, and the function image is obtained. Then, the concept of "imaginary support" is introduced to ensure the consistency of the model. Finally, the maximum space region that the center of the platform can move to on the 6-DOF mechanical platform is roughly shaped as a quasi-hemispherical structure with symmetry. The working space of the platform center on the 6-DOF mechanical platform obtained in this paper is relatively simple and the model is unified, which can provide data and scientific reference for the manufacture of related instruments in the field of mechanical manufacturing.

**Keywords:** 6-DOF mechanical platform, imaginary support, mechanism analysis, workspace geometry method.

## 1. Introduction

The six-degree of freedom mechanical platform is a parallel mechanism, which is mainly composed of six support rods with servo, upper platform and lower platform, and six universal joints (or spherical pairs) on each side. Among them, the support rod can be an electric rod or a hydraulic cylinder; The lower platform is the base of the whole structure; The upper platform is generally used as a load bearing platform; The main function of the universal joint or spherical pair is to firmly connect the upper and lower platforms. Whether it is a universal joint or a spherical pair, the degree of freedom of the platform is "six", so it is called a six-degree of freedom platform. Through the telescopic movement of the six support rods to change the length of each rod, the upper platform can obtain different air posture, and realize the movement in all directions (lifting, longitudinal, lateral, yaw, pitch, roll), that is, the translation movement, rotation movement or the composite movement of these movements can be realized on the upper platform. Compared with the series mechanism, the 6-DOF mechanical platform has the advantages of high stiffness, low inertia, large bearing capacity, high speed and no accumulated error, so it is more and more widely used in flight attitude simulation, assembly robots, parallel robots, parallel machine tools and entertainment facilities. However, compared with the series structure, the working space of the parallel structure is smaller<sup>[1]</sup>, so it is particularly important to study the working space of the 6-DOF mechanical platform.

Many scholars have done a lot of research on the work space of platform center on 6-DOF mechanical platform. There are usually three methods to solve this problem: geometric method, numerical method and intelligent algorithm. The geometric method is to use the geometric relationship between the upper and lower platform and the support rod to determine the boundary of the work space. For example, Dong Kaijie et al.<sup>[2]</sup> analyzed the workspace of 6-UCU PM on the basis of analyzing the three constraints, and established the workspace model by spherical coordinate search method. In addition, Nigatu<sup>[3]</sup> et al. proposed a 3-DOF parallel robot workspace optimization method based on dimensional homogeneous constraints embedded in Jacobian, which provides a simplified formula and eliminates the complex partial differentiation required in previous methods to

solve the workspace. Moreover, Malyshev<sup>[4]</sup> proposed a dimensional comprehensive optimization design method based on the criterion of maximizing the volume of the workspace based on the method of mechanism Jacobian matrix analysis and connecting rod interference on the basis of considering the singular region. In addition, Huiping S<sup>[5]</sup>, Allaoua B<sup>[6]</sup> and Lu Y<sup>[7]</sup> all adopted the geometric method to solve the workspace. For numerical method, Zarkandi S<sup>[8]</sup> used numerical method to solve the working space of the manipulator in three-dimensional space based on the special design of the manipulator without inverse kinematics and forward kinematics singularity. For intelligent algorithm, it is mainly to use the newly developed intelligent algorithm technology to solve the working space intelligently. For example, Zhao Jie<sup>[9]</sup> proposed a genetic algorithm-based reachable workspace maximization optimization algorithm. Yakovlev<sup>[10]</sup> adopted the method of machine learning - Q-learning to solve the problem of the automatic transition of the arm to the user specified state, taking into account the limitation of the simultaneous movement of the arm link and the minimum and maximum height of the manipulator.

This paper uses geometric method to solve the working space of the 6-DOF mechanical platform, and aims to find a method to solve the structure of the 6-DOF mechanical platform that is suitable for most and easy to realize. Therefore, the concept of "imaginary support" is introduced to solve the working space of the 6-DOF mechanical platform.

## 2. Research methods

### 2.1. Imaginary support

In order to ensure that the maximum space region that the platform center can move to on the 6-DOF mechanical platform can be studied, it is necessary to ensure that the platform can not only move in the direction of the six actual support rods to solve the maximum space region, but also to ensure that the maximum space region is also required to solve when moving in any other direction. If the actual support rods are used to continue the modeling, The model will introduce new variables, and the original actual support model will appear bloated. Therefore, under the premise of ensuring the consistency of the models, the concept of "imaginary support" is proposed, so that the solution of the problem in the maximum space region can be guaranteed to be solved under one model as much as possible, without introducing a new model.

The so-called "imaginary support rod" means that when the platform does not move in the direction of the six actual support rods on the six-degree-of-freedom mechanical platform, two virtual support rods are assumed in the given direction of movement, and the length variation range of the actual support rod model is replaced by solving the length variation range of the virtual support rod, so as to ensure that the model is solved at each stage. It is only necessary to modify the variable range of the support rod.

## 3. Establishment of model

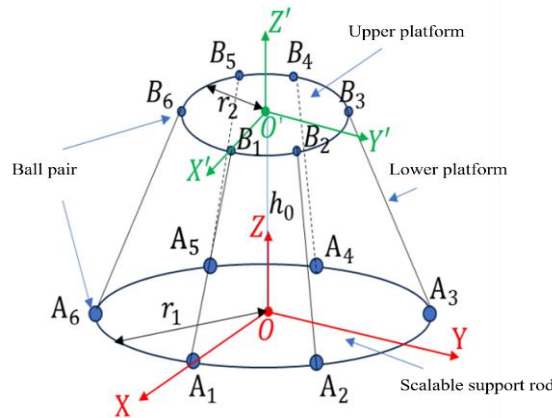
For solving the space region to which the center  $O'$  of the upper platform of the 6-DOF mechanical platform can move, when the model is established, the initial idea can be to determine the range of translation quantity  $X, Y$  and  $Z$  within a certain spatial range, and then select the range of rotation Angle  $\alpha, \beta$  and  $\gamma$  under a certain rotation Angle limitation. The coordinates of the center  $O'$  of the upper platform are solved according to the method of randomly taking values in the domain space. However, because the domain space is too large, the space shape of the center  $O'$  of the upper platform cannot be solved by traversing the domain space.

However, the above method provides a way to solve the problem. This paper follows the ergodic method and reduces the size of the domain space by setting constraints in sections, so that the ergodic method can become feasible.

By observing the position relationship between the center  $O'$  of the upper platform and the intersection point  $B_i$  of the support rod and the upper platform, it can be seen that  $B_i$  always exists in pairs of symmetric points in the center  $O'$  of the upper platform, so along any direction, there will always be mutually symmetric point pairs, but this point pair may not necessarily exist in the actual situation in the process of platform production. For the six-degree-of-freedom mechanical platform, there are three pairs of such actual point pairs:  $B_1, B_4$  and  $B_2, B_5$  and  $B_3, B_6$ , and the movement of the platform in these three directions will only be limited by the length of the six supports. For any other direction, it can be considered that there are a pair of "imaginary support rods", and the length range of the imaginary support rods is solved indirectly through the limiting effect of the imaginary support rods on the length of the actual support rods. A relatively continuous model can be obtained by synthesizing the actual and imaginary supports. The following models are established respectively from the upper platform to the direction of the actual support and from the direction of the imaginary support.

**3.1. Abstract model of six degrees of freedom mechanical platform**

The structure of the 6-DOF mechanical platform is abstracted into the model shown in Figure 1.



**Figure 1** Structure diagram of 6-DOF mechanical platform (initial position)

As shown in Figure 1,  $O$  and  $O'$  are the center of the circle of the lower platform and the upper platform, the radius of the lower platform and the upper platform are  $r_1$  and  $r_2$  respectively, the initial position of the upper platform (the position when the expansion of each support rod is zero) is  $h_0$  in height, and the initial length of the retractable support rod is  $l_0$ .

**3.2. The establishment of the actual support part model**

As an example, the above platform moves along the direction of a pair of  $B_1$  and  $B_4$  in the actual support rod, the model is established.

Because the point  $B_1$  and  $B_4$  are symmetric about the center of the upper platform  $O'$ , the pattern formed by motion along  $\overrightarrow{B_1B_4}$  and motion along  $\overrightarrow{B_4B_1}$  is symmetric about the  $Z$  axis. The model is built by taking the movement along the  $\overrightarrow{B_1B_4}$  direction as an example. In the process of establishing the model, the schematic diagram is a cross-section formed by the plane formed by the four points of point  $B_1, B_4$  and the corresponding point to  $A_1$  and  $A_4$  of the platform below.

Obviously, the shape formed by the center  $O'$  of the upper platform is a solid surface, so only the boundary of this surface needs to be determined. In the process of determining the boundary, the whole process is divided into the determination of the outer boundary and the determination of the lower boundary, and the highest point that can be reached by the above platform is discussed as the starting surface. The establishment of the actual support part model.

1. Determination of outer boundaries

The outer boundary is mainly to find the farthest point from the axis of axis  $Z$ , that is, the farthest position that the center of the upper platform  $O'$  can reach relative to the axis of axis, which requires the support rod to stretch outwards with a high attitude under the condition of satisfying the length constraint as far as possible.

(1) Process One

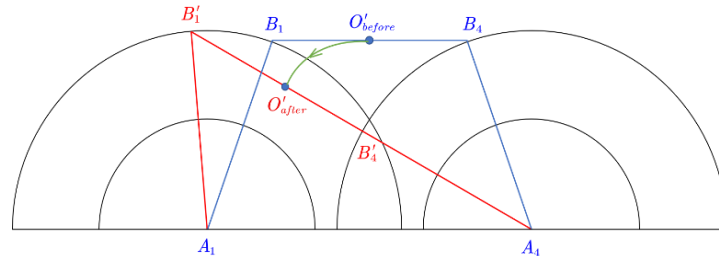


Figure 2 Process one

As shown in Figure 2, when the movement from the highest surface that can be reached by the upper platform to the collineation between the support rod  $A_4B_4$  and  $B_1'$ , the center of the upper platform  $O'$  can reach the farthest position from the axis of  $Z$  in this process. Set the coordinates  $B_1$  of the point as  $(X_{B_1}, Z_{B_1})$ , the coordinates  $B_4$  of the point as  $(X_{B_4}, Z_{B_4})$ , and the coordinates of the point  $O'$  in the center of the upper platform as  $(X, Z)$ . Because the profile is formed by the plane formed by the four points of point  $B_1, B_4$  and the corresponding point  $A_1$  and  $A_4$  of the platform below, the  $Y$  value of the coordinate does not affect the establishment of the model, so the coordinate eliminates the representation of  $Y$  coordinates.

For procedure one, there will be the following new constraints.

1) The length of the support rod  $A_1B_1$  is constrained

During the movement of process 1, the length of the support lever  $A_1B_1$  is always constant, and its length is always the maximum length  $l_0 + dl$ .

$$(X_{B_1} - r_1)^2 + Z_{B_1}^2 = (l_0 + dl)^2 \tag{1}$$

2) The length constraint of the support rod  $A_4B_4$

Similarly, the length of the support lever  $A_4B_4$  also remains unchanged, and its length is also the maximum length  $l_0 + dl$ .

$$(X_{B_4} + r_1)^2 + Z_{B_4}^2 = (l_0 + dl)^2 \tag{2}$$

3) Distance constraint

The distance between point  $B_1$  and  $B_4$  is always the same, and the length is always  $2r_2$ , so there is

$$(X_{B_1} - X_{B_4})^2 + (Z_{B_1} - Z_{B_4})^2 = (2r_2)^2 \tag{3}$$

4) Coordinate constraints

In the process of process one movement, the center of the upper platform  $O'$  is always the midpoint of point  $B_1$  and  $B_4$ , so there is

$$\begin{cases} X = \frac{X_{B_1} + X_{B_4}}{2} \\ Z = \frac{Z_{B_1} + Z_{B_4}}{2} \end{cases} \tag{4}$$

Synthesis formula (1)-(4) can be obtained

$$\left\{ \begin{array}{l} (l_0 + dl)^2 = (X_{B_1} - r_1)^2 + Z_{B_1}^2 \\ (l_0 + dl)^2 = (X_{B_4} + r_1)^2 + Z_{B_4}^2 \\ (2r_2)^2 = (X_{B_1} - X_{B_4})^2 + (Z_{B_1} - Z_{B_4})^2 \\ X = \frac{X_{B_1} + X_{B_4}}{2} \\ Z = \frac{Z_{B_1} + Z_{B_4}}{2} \end{array} \right. \quad (5)$$

By observing the system of equations (5), it can be seen that the system of equations with constraints has six unknowns and five equations in total. Theoretically, the relationship between  $X$  and  $Z$  can be obtained through elimination

$$f(X, Z) = 0 \quad (6)$$

However, the process of eliminating the equations (5) is very complicated, and the relation of equation (6) cannot be given in detail. Therefore, the  $(X, Z)$  coordinate value obtained under the defined unknown can be obtained by determining the range of a certain unknown in the definition domain space and solving other unknowns, and finally presented in the form of function images.

2) Process Two

Process two starts at the end of process one, and the support  $A_1B_1$  then moves downward, its length becoming smaller and smaller, while the length of the support  $A_4B_4$  remains the maximum length, until the support  $A_1B_1$  can no longer fall. The motion process is shown in Figure 3.

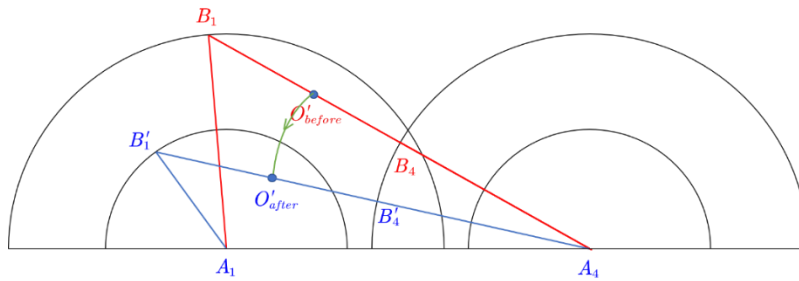


Figure 3 process two

For process two, Pythagorean theorem can be used to directly calculate the motion equation of center of the upper platform  $O'$ , as shown in Figure 4.

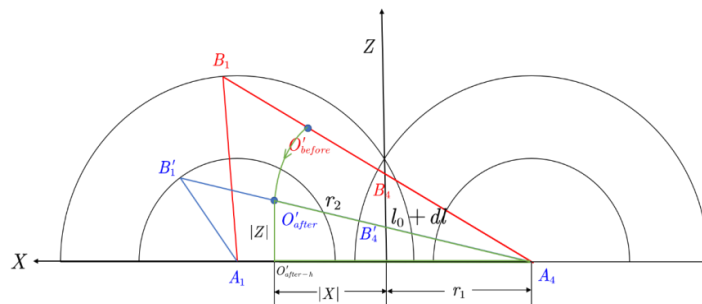


Figure 4 Schematic diagram of solving the motion equation of process two

As shown in Figure 4, the length of the support rod  $A_4B_4$  is still the maximum length, which is  $l_0 + dl$ , so dot  $O'_{after}$  to make the vertical line of the  $X$  axis, and the vertical foot is  $O'_{after-h}$ . In  $Rt\Delta O'_{after}O'_{after-h}A_4$ , the following Pythagorean theorem exists:

$$(r_1 - X)^2 + Z^2 = (r_2 + l_0 + dl)^2 \quad (7)$$

As can be seen from equation (7), the motion process of process two is relatively simple, and the function expression of the center  $O'$  coordinate  $(X, Z)$  of the platform can be directly obtained, and its function image can be directly displayed.

3) Process Three

Process three starts from the end of process two, since the support lever  $A_1B_1$  has reached the lowest point, so let it move upward with the shortest length, in the process of movement, keep the length of the support lever  $A_4B_4$  unchanged until the support lever  $A_1B_1$  parallel to the lower plane reaches a new critical state. The motion process is shown in Figure 5.

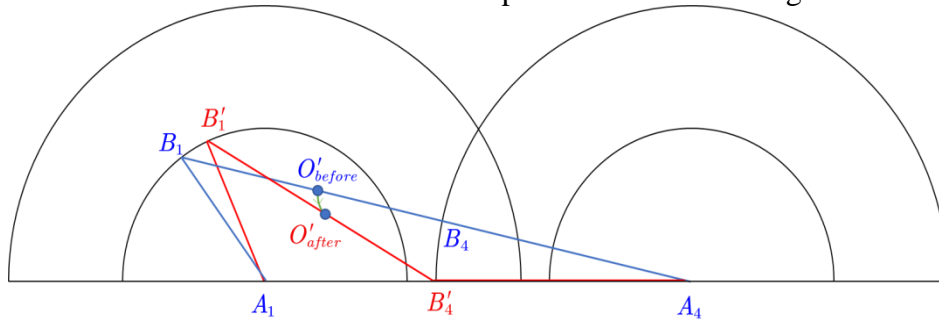


Figure 5 Process three

For process three, there will be the following new constraints.

1) The length constraint of the support rod  $A_1B_1$

During the movement of process three, the length of the support lever  $A_1B_1$  always remains unchanged, and its length is always the original length  $l_0$ .

$$(X_{B_1} - r_1)^2 + Z_{B_1}^2 = l_0^2 \tag{8}$$

2) The length constraint of the support rod  $A_4B_4$

Similarly, the length of the support lever  $A_4B_4$  also remains unchanged, and its length is the maximum length  $l_0 + dl$ .

$$(X_{B_4} + r_1)^2 + Z_{B_4}^2 = (l_0 + dl)^2 \tag{9}$$

3) Distance constraint

The distance between point  $B_1$  and  $B_4$  is always the same, and the length is always  $2r_2$ , so there is

$$(X_{B_4} - X_{B_1})^2 + (Z_{B_4} - Z_{B_1})^2 = (2r_2)^2 \tag{10}$$

4) Coordinate constraints

In the process of process three movement, the center of the upper platform  $O'$  is always the midpoint of point  $B_1$  and  $B_4$ , so there is

$$\begin{cases} X = \frac{X_{B_1} + X_{B_4}}{2} \\ Z = \frac{Z_{B_1} + Z_{B_4}}{2} \end{cases} \tag{11}$$

Synthesis formula (8) - (11) can be obtained

$$\left\{ \begin{array}{l} l_0^2 = (X_{B_1} - r_1)^2 + Z_{B_1}^2 \\ (l_0 + dl)^2 = (X_{B_4} + r_1)^2 + Z_{B_4}^2 \\ (2r_2)^2 = (X_{B_4} - X_{B_1})^2 + (Z_{B_4} - Z_{B_1})^2 \\ X = \frac{X_{B_1} + X_{B_4}}{2} \\ Z = \frac{Z_{B_1} + Z_{B_4}}{2} \end{array} \right. \quad (12)$$

By observing the equations (12), it can be seen that there are six unknowns and five equations. Similarly with the process, if one of the unknowns is valued within the definition range, the coordinate  $(X, Z)$  of the center of the upper platform  $O'$  can also be obtained, and its function image can be drawn at last.

2. Determination of internal boundaries

Different from the outer boundary to find the upper platform center  $O'$  that is farthest from the coordinate axis, the inner boundary is mainly to find the minimum coordinate value of the  $Z$  axis corresponding to the upper platform center  $O'$ , that is, the support rod moves downward as far as possible under the condition of satisfying the length constraint with a low attitude, so that the upper platform center  $O'$  is the lowest relative to the lower platform.

4) Process four

Process four starts from the end of process three and keeps the length of support rod  $A_1B_1$  moving to the right at a minimum. At this time, the length of support rod  $A_4B_4$  gradually decreases from the maximum length until the length of support rod  $aaa$  reaches the minimum value, which indicates that the corresponding state of the platform is a new critical state. The motion process is shown in Figure 6.

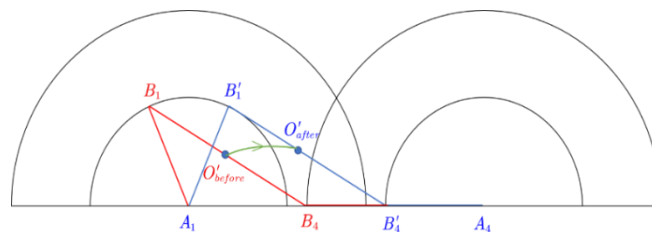


Figure 6 Process four

For process four, there will be the following new constraints.

1) The length constraint of the support rod  $A_1B_1$

During the movement of process four, the length of the support lever  $A_1B_1$  is always unchanged, and its length is always the original length  $l_0$ .

$$(X_{B_1} - r_1)^2 + Z_{B_1}^2 = l_0^2 \quad (13)$$

2) Point  $B_4$  coordinate constraint

During the process four motion, the  $Z$  axis coordinate of the point  $B_4$  coordinate is always zero.

$$Z_{B_4} = 0 \quad (14)$$

During the movement of process four, the  $X$  axis of the coordinate of point  $B_4$  is continuous and relatively uniform, and the length is  $l_0 + dl - r_1$  at the beginning and  $r_1 - l_0$  at the end of the movement of process four. Therefore, during the movement of process 4, the change range of the axis  $X$  of point  $B_4$  is

$$-(l_0 + dl - r_1) \leq X_{B_4} \leq r_1 - l_0 \tag{15}$$

3) Distance constraint

The distance between point  $B_1$  and  $B_4$  is always the same, and the length is always  $2r_2$ , so there is

$$(X_{B_4} - X_{B_1})^2 + (Z_{B_4} - Z_{B_1})^2 = (2r_2)^2 \tag{16}$$

4) Coordinate constraints

In the process of process four movement, the center of the upper platform  $O'$  is always the midpoint of point  $B_1$  and  $B_4$ , so there is

$$\begin{cases} X = \frac{X_{B_1} + X_{B_4}}{2} \\ Z = \frac{Z_{B_1} + Z_{B_4}}{2} \end{cases} \tag{17}$$

The synthesis formula (13)-(17) can be obtained

$$\begin{cases} l_0^2 = (X_{B_1} - r_1)^2 + Z_{B_1}^2 \\ Z_{B_1} = 0 \\ -(l_0 + dl - r_1) \leq X_{B_1} \leq r_1 - l_0 \\ (2r_2)^2 = (X_{B_4} - X_{B_1})^2 + (Z_{B_4} - Z_{B_1})^2 \\ X = \frac{X_{B_1} + X_{B_4}}{2} \\ Z = \frac{Z_{B_1} + Z_{B_4}}{2} \end{cases} \tag{18}$$

By observing the equations (18), it can be seen that there are also six unknowns and five equations, but from the analysis of process 4, it is obvious that the range of coordinates  $X_{B_4}$  in the coordinate  $(X_{B_4}, Z_{B_4})$  of the intersection point between the support lever  $A_4B_4$  and the upper platform is obtained, and  $X_{B_4}$  gradually becomes smaller from  $l_0 + dl - r_1$ , so this range can be used as the range of the unknown in the domain of definition to solve the equations. Finally, the function can be plotted.

5) Process five

Process five starts from the end of process four, keeping the length of the support lever  $A_1B_1$  unchanged, so that it continues to move downward, and then  $A_4B_4$  will continue to move upward, when the upper platform is parallel to the lower platform, the movement ends.

For process five, there will be the following new constraints.

1) The length constraint of the support rod  $A_1B_1$

During the movement of process five, the length of the support lever  $A_1B_1$  is always unchanged, and its length is always the original length  $l_0$ .

$$(X_{B_1} - r_1)^2 + Z_{B_1}^2 = l_0^2 \tag{19}$$

2) The length constraint of the support rod  $A_4B_4$

Similarly, the length of the support lever  $A_4B_4$  also remains unchanged, and its length is also the original length  $l_0$ .

$$(X_{B_4} - r_1)^2 + Z_{B_4}^2 = l_0^2 \tag{20}$$

3) Distance constraint

The distance between point  $B_1$  and  $B_4$  is always the same, and the length is always  $2r_2$ , so there is

$$(X_{B_4} - X_{B_1})^2 + (Z_{B_4} - Z_{B_1})^2 = (2r_2)^2 \tag{21}$$

4) Coordinate constraints

In the process of process five movement, the center of the upper platform  $O'$  is always the midpoint of point  $B_1$  and  $B_4$ , so there is

$$\begin{cases} X = \frac{X_{B_1} + X_{B_4}}{2} \\ Z = \frac{Z_{B_1} + Z_{B_4}}{2} \end{cases} \tag{22}$$

Synthesis formula (19)-(22) can be obtained

$$\begin{cases} l_0^2 = (X_{B_1} - r_1)^2 + Z_{B_1}^2 \\ l_0^2 = (X_{B_4} - r_1)^2 + Z_{B_4}^2 \\ (2r_2)^2 = (X_{B_4} - X_{B_1})^2 + (Z_{B_4} - Z_{B_1})^2 \\ X = \frac{X_{B_1} + X_{B_4}}{2} \\ Z = \frac{Z_{B_1} + Z_{B_4}}{2} \end{cases} \tag{23}$$

The second equation in the equations (23) shows that the length of the support rod  $A_4B_4$  is not a fixed value, and it changes with the change of the support rod  $A_1B_1$ , which is more complicated, but the analysis process is similar to the above processes, so it will not be repeated here.

When the upper platform moves in the direction of  $\overrightarrow{B_4B_1}$ , due to the symmetry of the motion, the range that the center of the upper platform  $O'$  can reach is symmetric with the range in Figure 5 about the axis of  $Z$ .

**3.3. The establishment of part model of imaginary support**

For the more general direction, in order to ensure the consistency of the model, an imaginary support moving in the general direction can be introduced, and the length range of the imaginary support can be obtained through the constraint of the actual support. After the length range of the imaginary support is obtained, the process of establishing the model is the same as that of the actual support, except that the length constraint of the actual support becomes the length constraint of the imaginary support.

The imaginary support length can be solved by making the upper platform shift at a given height. It is necessary to solve the coordinates of the intersection points  $A_{i(vir)}$  and  $B_{i(vir)}$  of the imaginary support  $\overrightarrow{A_{i(vir)}B_{i(vir)}}$  with the upper and lower platforms respectively. The solution method is similar to the actual support bar, except that the center Angle  $\theta$  formed by the adjacent two support bars at the opposite side of the spherical surface of the upper platform is changed into the direction vector  $X'$  and the acute Angle formed by the  $O' - X'Y'Z'$  coordinates in the coordinate system of  $A_{i(vir)}$  and  $B_{i(vir)}$ .

In summary, after specifying the height  $h$  and Angle  $\theta$  of the upper platform, the length variation range of the imaginary support can be solved.

Finally, when the platform moves in the general direction, the shape formed by its movement is similar to the figure of the actual support rod, only the size of the figure has changed to a certain extent, so we will not go into more details here.

#### 4. Model solving

By referring to the size of the 6-DOF mechanical platform, reasonable assumptions are made about the parameters of the 6-DOF mechanical platform. The parameter assumptions are as follows: Bottom radius  $r_1 = 600mm$ , top bottom radius  $r_2 = 360mm$ , the initial length of each support rod is the same,  $l_0 = 400mm$ , the maximum telescopic capacity is  $dl = 320mm$ , the ball pairs located on the upper and lower platforms are evenly distributed on their respective edges, the initial position of the mechanism platform is shown in Figure 1, the initial position height  $h_0 = 320mm$ .

Similar to the process of model building, the solution of the model also needs to be divided into two parts: actual support and imaginary support.

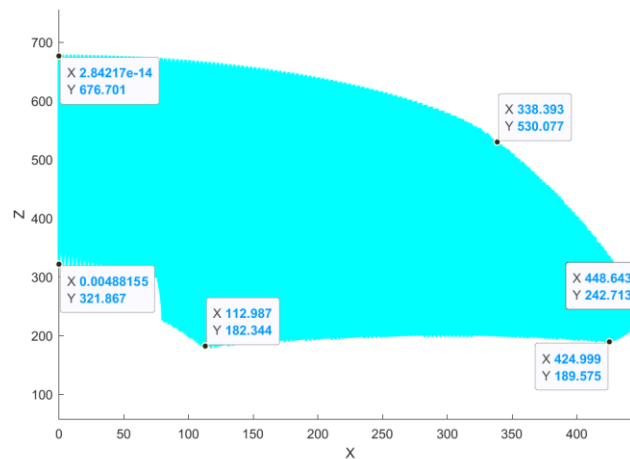
##### 4.1. The solution of the actual support part model

For the equations established by the five processes in the actual support model, it is also necessary to set the range of a parameter in the equations to draw the function image of each process. In this paper, the range of the axis  $X$  of the center coordinate  $O'$  of the above platform is taken as an example to solve the model and draw the function image. The range of the axis  $X$  of the center coordinate  $O'$  of the upper platform in each process is shown in Table 1.

**Table.1.** Range of axis  $X$  on platform centercoordinate  $O'$

Procedure	The range of The point $O'$ coordinate(axis $X$ ,mm)
procedure one	[0, 336]
procedure two	[336, 448]
procedure three	[426, 448]
procedure four	[100, 426]
procedure five	[0, 360]

The graphs formed under the five processes in 3.2 are drawn and spliced respectively, and the boundary curves formed by the two pairs of  $A_i, A_{(i+3)}, B_i$  and  $B_{(i+3)}$  moving along the direction of the actual support rods on the final platform are shown in Figure 7.



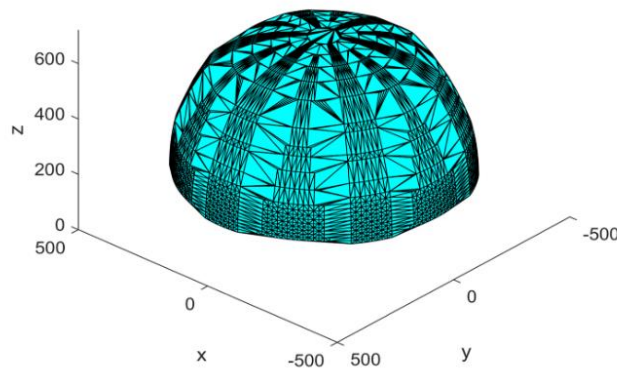
**Figure 7** Boundary curve of actual support rod

#### 4.2. The solution of part model of imaginary support

Similar to the solution steps of the actual support rods, a boundary curve similar to Figure 7 can also be obtained by solving the length range of the imaginary support rods in each process and combining the range of the axis  $X$  of the  $O'$  coordinate of the center of the upper platform in each process.

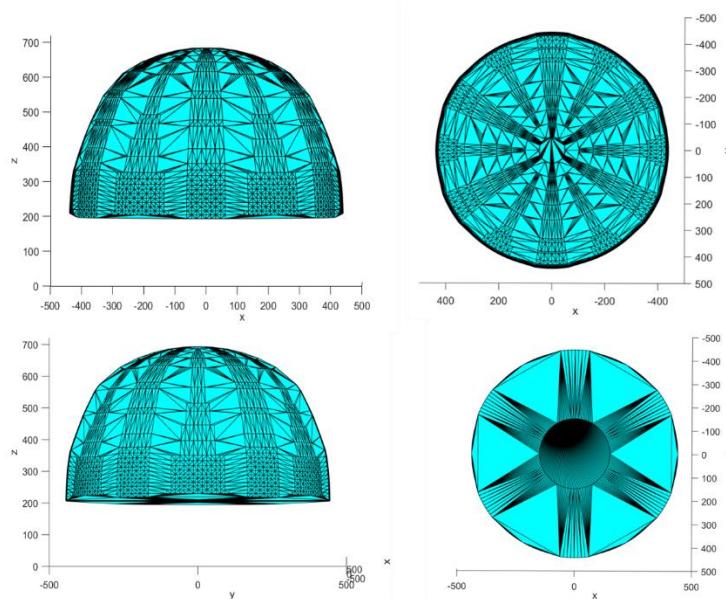
#### 4.3. The synthesis of the results of two parts

Each section is combined to obtain the space area to which the center of the upper platform  $O'$  can move in the final three-dimensional space, as shown in Figure 8.



**Figure 8** Boundary surface

The view from each Angle is shown in Figure 9.



**Figure 9** Boundary surfaces with different viewing angles

## 5. Discussions

The concept of "imaginary support rod" proposed in this paper can solve the working space of the 6-DOF mechanical platform more effectively and quickly than the traditional geometric method and the numerical method with large computation amount and long time. Moreover, the "imaginary support rod" model established in this paper is further expanded based on the actual support rod, and the front and back models are consistent. The model is as simple and general as possible.

However, the model established in this paper also has some defects, for example, the change of platform attitude caused by mechanical reasons is not considered in this paper. If more restrictions on other aspects can be known, more constraints can be provided for more detailed modeling, and the position space solved will be more accurate.

## 6. Conclusions

In this paper, the working space of a 6-DOF mechanical platform is studied by analyzing the farthest and nearest critical conditions that the platform center can reach. Firstly, through mechanism analysis, the above platform moves along the direction of the actual support rods, transforms the three-dimensional model into a two-dimensional plane model by selecting the section, analyzes the respective constraints under five different motion processes, then reduces the degree of freedom by defining the unknowns, solves the coordinates of the center of the upper platform to establish the model, and then calculates according to the coordinate range of the center of the upper platform. Then the function image is obtained by solving the model. Then, the concept of "imaginary support" is introduced to ensure the consistency of the model. Finally, the maximum space region that the center of the platform can move to on the 6-DOF mechanical platform is roughly shaped as a quasi-hemispherical structure with symmetry. The working space of the platform center on the 6-DOF mechanical platform obtained in this paper is relatively simple and the model is unified, which can provide data and scientific reference for the manufacture of related instruments in the field of mechanical manufacturing.

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