

Mirror field optimization model based on thermal power maximization

Zhiyu Pei¹, Zhenze Jin^{2,*}, Ye Chen¹, Yu Wang¹

¹College of Physics and Electronic Information Engineering, Zhejiang Normal University, Jinhua, China, 321000

²School of Mathematical Sciences, Zhejiang Normal University, Jinhua, China, 321000

*Corresponding author: jzz20040203@outlook.com

Abstract. Optimization of the heliostat field is very important in order to improve the performance of solar power tower (SPT) and reduce the construction cost. In this paper, we simulate and analyze the reflection relationship of the heliostats in the heliostat field and find the adjacent heliostats to the current heliostat by solving the distance matrix. The neighboring heliostats are projected onto the plane of the current heliostat, and the shadow-free region of the heliostat is obtained by the ray-tracing method and by considering the sunlight as a cone of light. The shadow blocking efficiency can be calculated based on the number of light strips in the region, and similarly the truncation efficiency can be calculated. Based on the circular heliostat field, a new cross-density distribution model based on the circular heliostat field is proposed. Under the condition of confirming the annual average thermal power, the distribution of the heliostat mirror field with the maximum thermal power per unit area, the optimal installation height and mirror size can be obtained.

Keywords: Fixed-sun mirror field; cross-density-minimum distribution; conical light; ray-tracing method; heuristic search.

1. Introduction

Tower solar power generation is a novel technology of clean energy that has low carbon emissions and environmental impacts. The heliostat field is the essential component of the SPT, which can adjust the azimuth and pitch angles of the mirrors by two rotating axes, so that the reflected light is directed to the center of the collector. The collector is the key component of the tower type power station, which can concentrate the reflected light on the heat transfer medium, store the solar energy as heat energy, and convert it into electric energy through heat exchange. The heliostat field consists of a large number of heliostats, and its thermal power output and optical efficiency depend on various factors, such as the position, size, installation height, and reflectivity of the heliostat [1]. A significant part of the SPT cost is allocated for the heliostat field, so the optimal design of this element is crucial.

This model is based on the theoretical analysis of the process of transforming electromagnetic energy into thermal energy in the heliostat field, and the goal is to optimize the maximum value of the annual average thermal power output per unit mirror area. The model has certain adaptability. When the longitude, latitude and altitude change, by changing the position coordinates of the absorption tower, the size, installation height and position of each heliostat, the model can re-adapt to the new environment and obtain a new optimal solution, so as to improve the solar energy utilization rate in the heliostat field, which has certain reference significance for the actual construction layout.

2. Model description

In this paper, a new heliostat field distribution is proposed using a direct model based on the Monte Carlo method, employing the cross-densest distribution, which is further combined with a heuristic search to achieve the optimization task.

2.1. About Tapered Light Modeling

The formation of conical rays of sunlight is due to refraction by the atmosphere. When sunlight enters the atmosphere, the light is refracted, i.e., it changes its direction of propagation. Due to the uneven density distribution of the atmosphere, the angle of refraction of light rays varies at different locations. This leads to a conical propagation of sunlight through the atmosphere.

In this problem we consider the sun rays as conical rays and consider each conical equivalent as 12 rays. This is shown in the figure1 below:

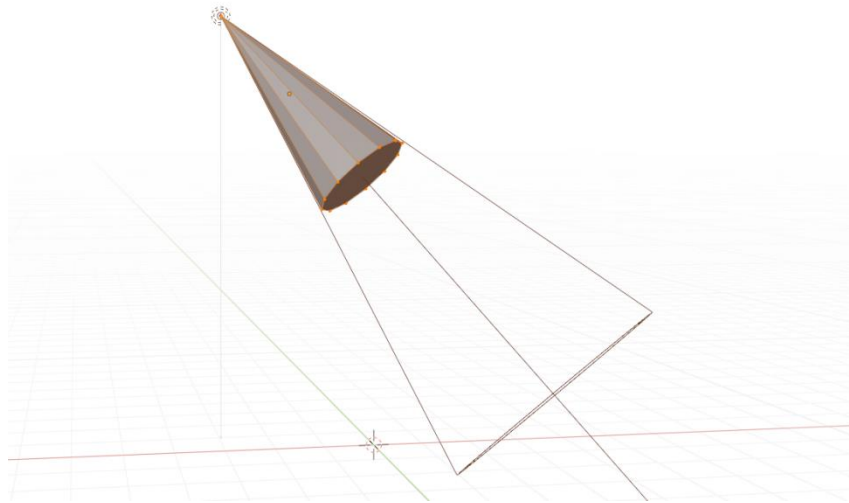


Figure 1 Schematic diagram of a conical ray

2.2. Shadow Occlusion problem

Establish a model of reflection and absorption of solar light with an absorption tower as the center and heliostats distributed around a circle. On this basis, the problem of incident occlusion and reflection occlusion is considered. For a certain heliostat, it will be occluded by the heliostat in front of the incident direction, and when the heliostat reflects the sun light to the collector, it will block the heliostat in the direction of the reflected light. The distribution of the occluded heliostat is shown in Figure 2. The occlusion problem of the current directional mirror caused by 8 nearby directional mirrors is considered. It is assumed that the light reflection is total reflection, without considering the energy loss caused by reflection, and the occluded part is considered as no reflected light.

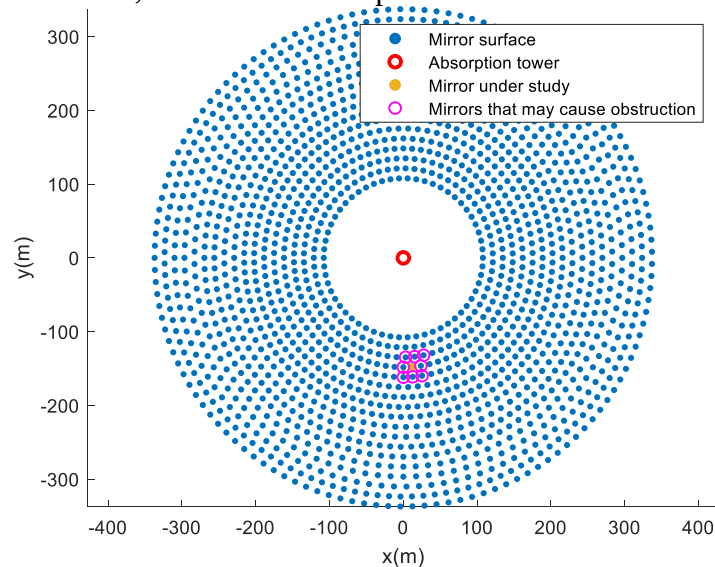


Figure 2 Enlarged view of an area in the circumference of the heliostat field

To find the shadow area of the adjacent heliostat on the current one, the adjacent heliostat is projected onto the plane of the current one along the direction of the solar ray or the heliostat reflection. The center of the current heliostat is the coordinate origin, with the axis pointing east, the axis pointing north, and the axis normal to the local ground, the coordinate system should be rotated and transformed to make the $x'S_2y'$ plane coincide with the plane of the current heliostat, as shown in Figure 3.

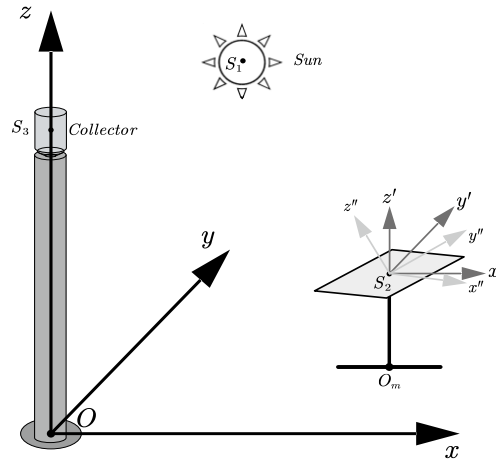


Figure 3 Schematic diagram of coordinate system transformation

Let the coordinates of any point P in space be (x_p, y_p, z_p) , and after rotation the coordinates can be expressed as

$$(x_p'', y_p'', z_p'')^T = M_z(\beta)M_y(\gamma)M_x(\varphi) \cdot (x_p', y_p', z_p')^T \quad (1)$$

In the formula, $M_z(\beta), M_y(\gamma), M_x(\varphi)$ is the Euler rotation matrix. Contains unknowns β, φ, γ . β is the counterclockwise rotation Angle of the x and y axes when the coordinate system rotates around the z axis; φ is the counterclockwise rotation Angle of the y axis and the z axis when the coordinate system rotates around the x axis. The heliostat plane unit normal vector R_N is used to solve:

$$\left\{ \begin{array}{l} \sin \varphi = \sqrt{1 - \cos^2 \varphi} \\ \cos \varphi = \frac{R_{Nz}}{\sqrt{R_{Nx}^2 + R_{Ny}^2 + R_{Nz}^2}} \end{array} \right\}, \left\{ \begin{array}{l} \sin \beta = \frac{R_{Ny}}{\sqrt{R_{Nx}^2 + R_{Ny}^2}} \\ \cos \beta = \frac{R_{Nx}}{\sqrt{R_{Nx}^2 + R_{Ny}^2}} \end{array} \right. \quad (2)$$

According to Equation (1) and (2), the projection of the adjacent heliostat on the current heliostat can be determined, and the area of the shadow part can be obtained. The calculation method of multiple adjacent heliostats is the same as that of a single adjacent heliostat, and it also needs to be calculated by projection into the same plane.

2.3. Field optical efficiency of the heliostat

The optical efficiency of the heliostat field is commonly modeled in the academic literature as the product of the instantaneous optical efficiency factors that reflect the optical behavior of the heliostat field. This instantaneous optical efficiency enables the calculation of the annual optical efficiency [4].

$$\eta = \eta_{sb} \eta_{\cos} \eta_{at} \eta_{trunc} \eta_{ref} \quad (3)$$

With:

η_{ref} Specular reflectance: It can be a constant, for example 0.92.

η_{sb} Shadow occlusion efficiency[2]: It is the fraction of the single heliostat's total area that is not blocked by other heliostats, as in Equation (4). Use the Monte Carlo algorithm to find the area, in the current heliostat area, sprinkle points uniformly at random, and the ratio of the number of sprinkled points characterizes the ratio of the area.

$$\eta_{sb} = \frac{S_r}{S_{all}} \quad (4)$$

η_{cos} Cosine efficiency: It is the cosine value of the Angle between the incident rays of the sun and the normal of the heliostat, as shown in the Equation (5)

$$\eta_{cos} = \cos \theta = \frac{\vec{R}_{in} \cdot \vec{R}_N}{|\vec{R}_{in}| |\vec{R}_N|} \quad (5)$$

η_{at} Atmospheric transmittance: The heliostat reflects solar radiation to the receiver, but some of it is lost due to the scattering and absorption of the atmosphere.

$$\eta_{at} = \begin{cases} 0.99321 - 0.0001176d_{HR} + 1.97 \times 10^{-8} \times d_{HR}^2 & d \leq 1000m \\ \exp(-0.0001106d) & d > 1000m \end{cases} \quad (6)$$

d_{HR} represents the distance from the center of the mirror to the center of the collector.

η_{trunc} Collector truncation efficiency: It is the ratio of the energy received by the collector and the total reflection energy of the mirror minus the shadow shielding loss energy, and considering that the sun is a cone light, the reflection will form a spot on the collector.

2.4. Estimating the Annual Energy Harvesting

To estimate the yearly energy collected, the yearly heliostat field light efficiency is also considered. The equation for the output thermal power is:

$$E_{field} = DNI \sum_i^N A_i \eta_i \quad (7)$$

Where DNI is the normal direct radiation irradiance, N is the total number of heliostats, A_i is the lighting area of the heliostat i , and η_i is the optical efficiency of the heliostat i .

The annual energy estimate depends on the DNI value at every moment, which is a function of direct normal irradiance (DNI).

$$DNI = G_0 [a + b e^{-\frac{c}{\sin \alpha_s}}] \quad (8)$$

Where α_s is the solar altitude Angle[3], the values of a, b, c are all related to the altitude, and G_0 are solar constants. The formula takes into account the non-uniformity of the annual distribution of solar resources.

2.5. Cross-densest distribution model

The objective function of the optimization process is represented by the annual heliostat field light efficiency and the annual collected heat energy. Increasing the non-occlusion area of the heliostat, which is related to the heliostat field layout, is necessary to improve these two values. In this paper, the cross-dense distribution is used to enhance the heliostat field distribution model[5][6].

The power per unit area decreases as the heliostat moves away from the coordinate origin, so the heliostat should be densely arranged near the collector site. In order to reduce the shadow occlusion loss, we establish a heliostat field model with the Cross-densest distribution, which is specifically shown as the cross distribution and the densest distribution. As shown in figure 4.

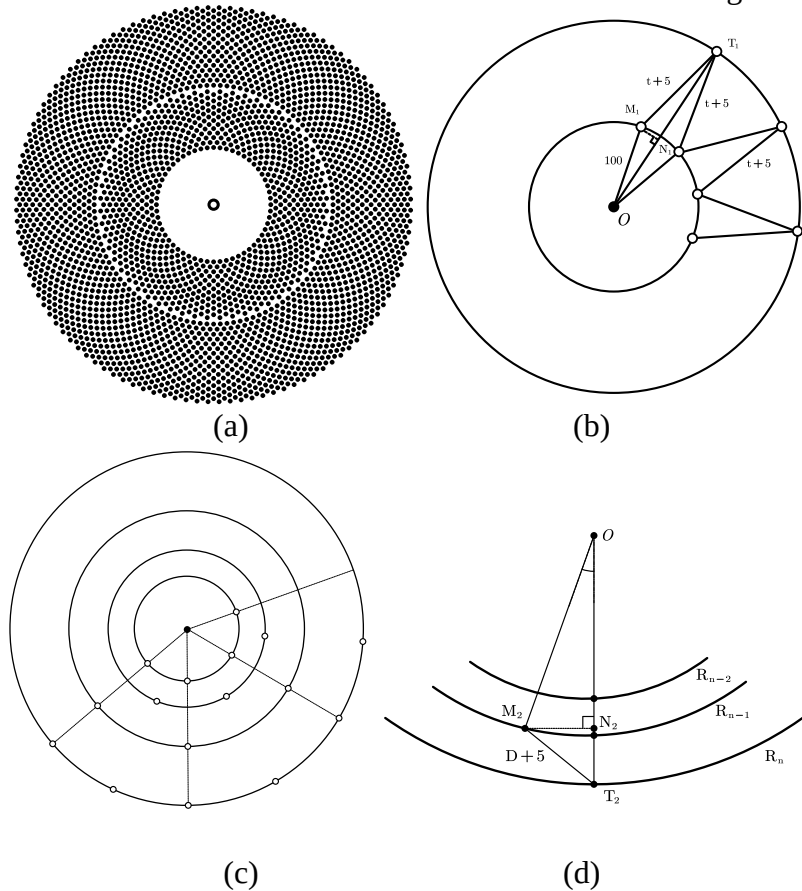


Figure 4 (a) Model diagram of the cross-densest distribution of heliostat mirror fields

(b)Diagram of the second layer inside the absorption tower

(c)Diagram of the arrangement pattern

(d)Diagram of the third layer inside the absorption tower

The densest distribution is divided into two points: one is near the absorber side distribution, increasing the heliostat power per unit area; Second, in the same ring, the distribution of as many heliostats as possible can ensure that the circle near the absorption tower side is arranged more heliostats, increasing the power per unit area of the heliostat.

The cross distribution is close to the innermost layer of the absorption tower circumferential distribution as many heliostats as possible, and the spacing is

$$d = t + 5 \quad (9)$$

In the formula, t is the mirror height, which is conducive to increasing the shadow occlusion efficiency.

The second layer takes the points $t + 5$ away from the two adjacent points of the first layer as shown in Figure 4(b)

There are two cases of distribution after the third layer:

(1) The spacing between the n layer distribution and the $n - 1$ layer distribution is less than $2(t + 5)$.

As shown in Figure 4(c), the radius of the n layer needs to be satisfied:

$$R_n = \max\{R_{n-2} + t + 5, OT_2\} \quad (10)$$

(2) The spacing between the n layer distribution and the $n-1$ layer distribution is equal or greater than $2(t+5)$.

The arrangement is along the direction of the extension line connecting the center of the circle and the distribution points on the $n-1$ layer, and the length is $t+5$, and the spacing is arranged on the n layer. Add the points at $t+5$ from the two adjacent points of the $n-1$ layer.

3. Results

To evaluate our method, we design a circular heliostator field in a circular region with a center at 98.5° E, 39.4° N, an altitude of 3000m, and a radius of 350m. The design aims to optimize the light efficiency and annual thermal energy of the heliostat field, and achieve the maximum value of the annual average output power per unit mirror area, given that the heliostat field's average annual output power is $60MW$.

3.1. Unified optimisation of heliostat parameters

Sometimes the length, width and height of all heliostats are fixed for the whole mirror field in order to reduce the construction cost and difficulty, in which case we can optimise them. The local solar elevation and azimuth angles can be determined based on latitude and longitude. For simplicity of calculation, the "annual average" indices are at 9:00, 10:30, 12:00, 13:30 and 15:00 local time on the 21st day of each month.

The heliostat mirrors are arranged using a cross-densest distribution and the optimal solution is found using the Matlab programme. The height and width of the mirrors were searched in the range of $[2,8]$ in large steps, with the height being less than the width. The optimal solution was determined by the initial variation rule and the range was searched further using heuristic search method.

We first optimise the position of the absorber tower of the heliostat, due to the rotation of the earth, the lens efficiency of the mirror field is left-right symmetric, then the x-coordinate of the absorber tower should be zero, but due to latitude, the y-coordinate can be optimised, we traverse the $y=-250:10:250(m)$, m to get the highest thermal power at $y=-10(m)$, then fix the position of its absorber tower as $(0,-10)$.

Then optimise the height of the mirror, taking into account the maintenance problem, the minimum height is set to 2m, we selected the length and width of the mirror for $4*4(m)$, so that the height of $z=2:0.5:6(m)$ change, get the trend of the following chart, the visible mirror in the lower the height of the position, the higher the efficiency, to get the height of the expression $z: z = \max\{2, D/2\}$. As shown in figure 5:

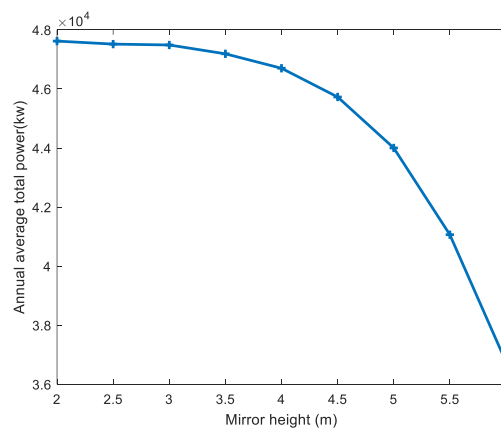


Figure 5 Relationship between mirror height and annual average total power

Finally the length and width of the mirror are optimised and traversed with the aid of a heuristic search, first letting the length be searched in the range of $D=2:0.5:6(m)$, $L=2:0.5:D(m)$, and obtaining the total power and the power per unit area as in the following two figures[7][8]. As shown in figure 6:

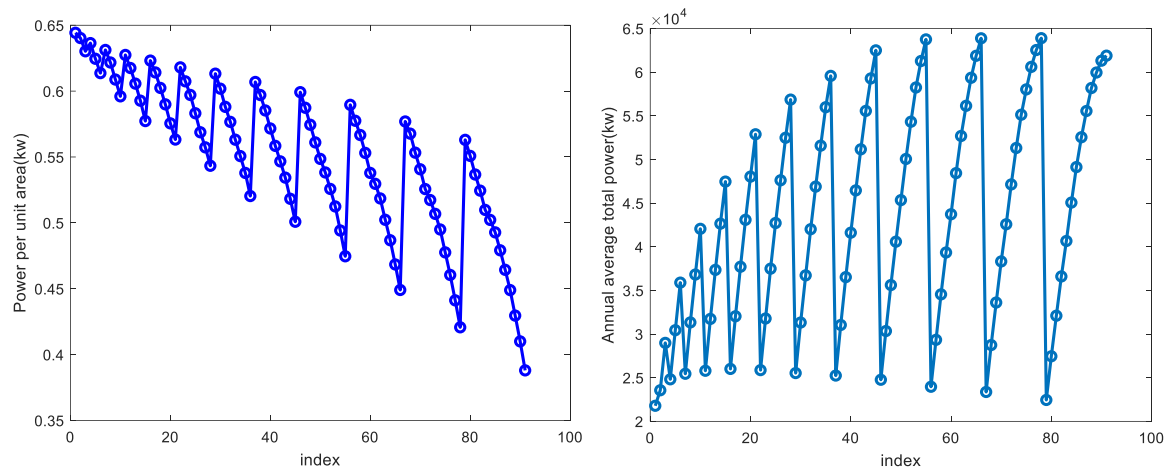


Figure 6 Total power and power per unit area graph

The horizontal coordinates are traversal indexes corresponding to different values of L,D. The maximum power per unit area to satisfy the total power requirement is obtained at D=5.6:6.4(m), so the search is done in the range of D=5.6:0.1:6.4(m) and L=5.6:0.1:D(m) as shown in the following two figures. As shown in figure 7:

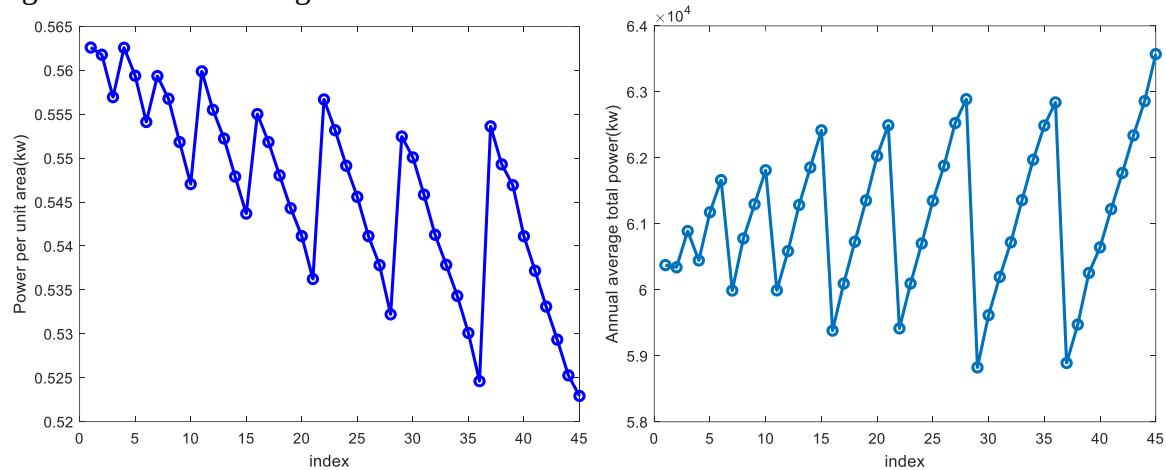


Figure 7 Final total power and power per unit area graph

The final optimal solution obtained is L=D=5.6(m), z=2.8(m). The results are shown in Table 1 below:

Table 1 Table of harmonised optimised design parameters

Absorption tower position coordinates	Heliostat Dimensions (W x H)	Heliostat mounting height (metres)	Total number of heliographs
(0,-10)	5.6 x 5.6	2.8	3422

3.2. Optimisation of differences in heliostat parameters

In order to pursue higher thermal power per unit, sometimes the parameters of all the heliostats can be different, and here different length, width and height are designed for different radii, and here we refer to the focusing principle of concave mirrors to adopt the design of mirror field height with the middle low and the surroundings high, considering that the internal heliostat is more serious in the phenomenon of obstruction, and in order to place more heliostats, the size of the internal heliostat is set to be small while the external is larger[9]. The heuristic search is still used to obtain the optimal solution, and the variation of length, width and height with radius is shown below: As shown in figure 8:

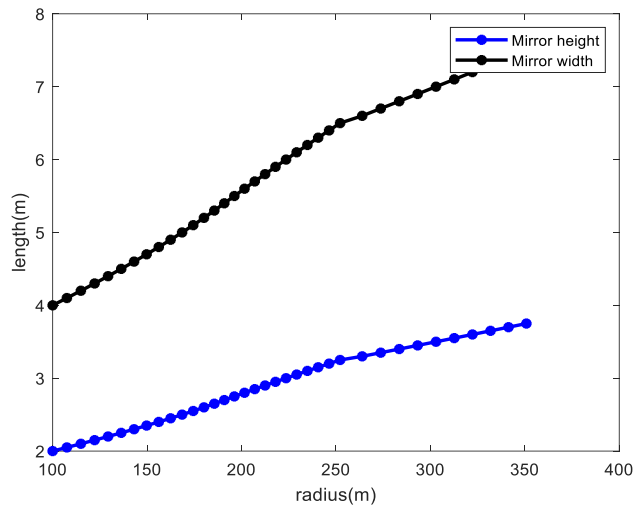


Figure 8 Plot of length and height as a function of radius

It is not repeated here. Getting from inside to outside $D=L=4:0.1:7.5$ (m) $z=2:0.05:3.75$ (m)[10]. As shown in table 2

Table 2 Differential Optimisation Design Parameters

Absorption tower position coordinates	Heliostat Dimensions (W x H)	Heliostat mounting height (metres)	Total number of heliographs
(0,-10)	Near to far: 4*4~7.5*7.5	Near to far: 2~3.75	3033

3.3. Comparison of results

Table 3 shows the conventional radial heliostat field parameters. For the same distribution area, the radial heliostat field has only 1745 heliostats. The average optical efficiency and output power are shown in Table 4. Table 5 shows the annual average optical efficiency and output power for the uniform optimization. Table 6 shows the annual average optical efficiency and output power for the differential optimization. In comparison, the annual average output thermal power is nearly doubled when using the cross-density distribution model, the output power per unit area is increased, and the output power from the differential optimization is also improved over the output power from the uniform optimization. The final results are shown in the following four tables.

Table 3 Conventional radial heliostat field parameters

Absorption tower position coordinates	Heliostat Dimensions (W x H)	Heliostat mounting height (metres)	Total number of heliographs
(0,0)	6*6	4	1745

Table 4 Annual average optical efficiency and output power table

Average annual optical efficiency	Average annual cosine efficiency	Average annual shadow shading efficiency	annual average truncation efficiency	Average annual output fired power generation (MW)	Average annual heat output per unit area of reflector ² (kW/m)
0.5482	0.7566	0.9458	0. 8992	33.569	0.5344

Table 5 Unified optimised annual average optical efficiency and output power table

Average annual optical efficiency	Average annual cosine efficiency	Average annual shadow shading efficiency	annual average truncation efficiency	Average annual output fired power generation (MW)	Average annual heat output per unit area of reflector ² (kW/m)
0.5757	0.7527	0.8239	0.9527	60.369	0.5625

Table 6 Differential Optimisation Annual Average Optical Efficiency and Output Power Table

Average annual optical efficiency	Average annual cosine efficiency	Average annual shadow shading efficiency	annual average truncation efficiency	Average annual output fired power generation (MW)	Average annual heat output per unit area of reflector ² (kW/m)
0.6062	0.7642	0.8369	0.9573	60.405	0.5908

Finally, the distribution of the three mirror fields and the thermograms of heat power per unit area at each location are shown to visualise the three mirror fields. As shown in figure 9.

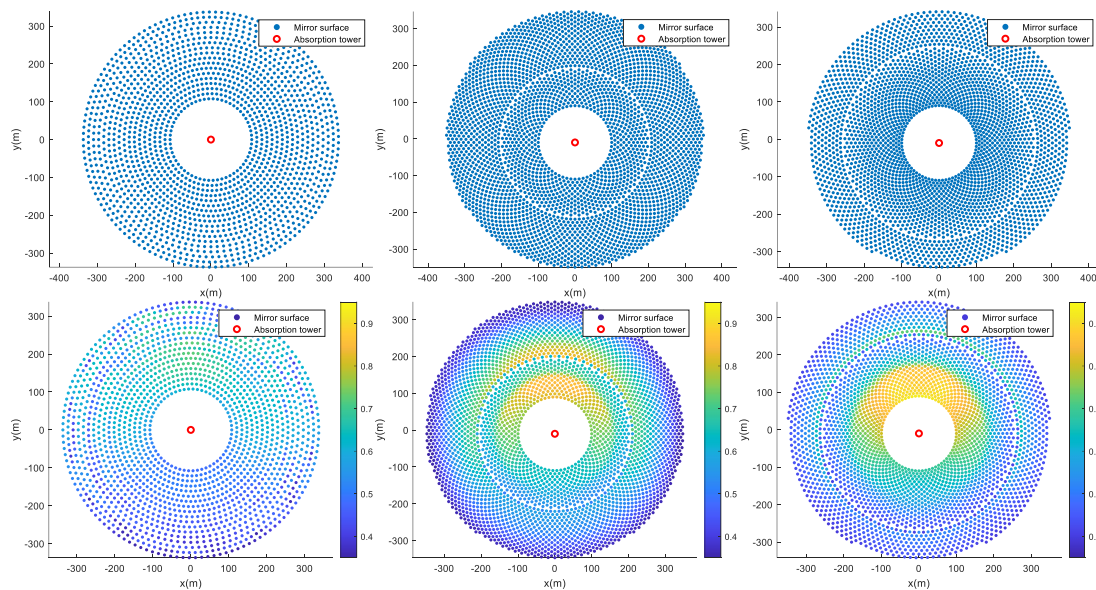


Figure 9 Distribution maps of the three mirror fields and thermograms of the thermal power per unit area at each location

4. Conclusion

The model is based on practical considerations and takes into account as many important factors as possible in order to obtain a reasonable model, such as the effect of conical light compared with parallel light, and the masking after specular reflection. The model is more realistic and has practical application. The ray tracing method is used to solve the occlusion area of the shaded part, which effectively reduces the limitation of geometrical conditions. Meanwhile, the accuracy is improved by increasing the number of trials, which facilitates the model solution. On the basis of the circular distribution of the heliostat field, the maximum output thermal power of the heliostat field is nearly doubled compared with the original.

In this paper, a unified coordinate system for the heliostat field is established, and a series of models, such as the solar ray model, the shadow masking model, and the heliostat physical model, are built on this basis. The shadow masking model adopts the ray tracing method, which takes into account the non-parallel characteristics of sunlight. The objective functions are optical efficiency, annual average output thermal power and unit area output thermal power. The effects of heliostat layout, mounting height and mirror size on heliostats were investigated. A cross-densest distribution model is proposed, which can optimise the unit output thermal power with a fixed total thermal power.

In addition, the proposed model has a certain degree of adaptability. When the longitude, latitude and altitude change, by adjusting the positional coordinates of the absorption tower, the size, installation height and location of each heliostat, the layout and configuration of the heliostat field under different conditions can be optimised to make full use of the solar energy resources in different geographic environments, which will provide useful references and guidance for the actual construction.

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