

Methodological Study on Enhancing the Efficiency of Heliostat Fields in Tower Solar Thermal Power

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Abstract. In response to environmental pollution and the energy crisis, large-scale tower solar thermal power as a sustainable alternative to traditional energy sources were explored. This article assumed a central absorption tower surrounded by a circular field of 6m×6m heliostats. The calculation of optical efficiency and thermal power output incorporated models for cosine efficiency, shadowing, and truncation efficiency. Key results include an annual average optical efficiency of 0.69, with a thermal power output of 74.61MW and 1.18KW/m² per unit mirror area. By capping the average output at 60MW and optimizing mirror dimensions and heliostat count, this article achieved a maximum annual average output of 1.20KW/m², utilizing 1050 heliostats with a 7m side length, leading to an average power output of 62.07MW. This article provides a research approach and framework for calculating the optical efficiency of a heliostat field, which also has theoretical guidance value for mirror field designers and contributes to the further development of tower-type solar thermal power generation technology.

Keywords: Heliostat field layout, optical efficiency, grid search, global optimization.

1. Introduction

Solar thermal power, utilizing solar heat resources, offers higher efficiency and eco-friendliness than photovoltaic technology [1]. Tower power generation, a key method in this domain, stands out for its high concentration ratio and potential for significant cost reductions. Central to this technology are heliostats, which direct sunlight into the tower's collector for energy conversion, controlled by their azimuth and elevation angles.

This paper first models and calculates the cosine efficiency, shadowing efficiency, truncation efficiency, etc. under the existing heliostat field layout, and then obtains the optical efficiency, thermal power, and other data. Then, under the condition of limiting the annual average output thermal power to 60MW, the mirror parameters (such as side length and quantity) are optimized using the grid search algorithm to obtain the optical efficiency and output thermal power under the new heliostat field layout. The data in this paper are from <http://www.mcm.edu.cn/>.

2. Calculation of Physical Quantities in Circular Heliostat Field

2.1. Basic Physical Quantities

Let D be the number of days counted from the spring equinox, ST be the local time, and ϕ be the local latitude, which is 39.4° in this case, with the north latitude being positive.

Some important physical quantities are shown in the following Table 1 Physical Quantities.

Table 1. Physical Quantities

Physical Quantities	Expression
Solar declination angle	$\sin \delta = \sin \frac{2\pi D}{365} \sin \left(\frac{2\pi}{360} 23.45 \right)$
Solar hour angle	$\omega = \frac{\pi}{12} (ST - 12)$
Solar elevation angle	$\sin \alpha_s = \cos \delta \cos \varphi \cos \omega + \sin \delta \sin \varphi$
Sun azimuth	$\cos \gamma_s = \frac{\sin \delta - \sin \alpha_s \sin \varphi}{\cos \alpha_s \cos \varphi}$
η_{at}	$\eta_{at} = 0.99321 - 0.0001176 d_{HR} + 1.97 \times 10^{-8} \times d_{HR}^2$
η_{ref}	0.92
E_{field}	$E_{field} = DNI \sum_{i=1}^N A_i \eta_i$

2.2. Transformation Matrix between Local Coordinate System and Global Coordinate System

In the calculation of the shading efficiency and collector truncation efficiency in the following, we need to use the conversion between the mirror coordinate system and the global coordinate system [2]. First, let's introduce two coordinate systems:

Mirror Coordinate System: The origin is at the center of the mirror, the mirror normal vector is the z-axis, and the x-axis and y-axis are derived from the origin in the directions parallel to the two mirror edges. Different heliostats have different mirror systems.

Global Coordinate System: The origin is at the center of the circular heliostat field, the positive x-axis is in the east direction, the positive y-axis is in the north direction, and the vertical ground direction is the z-axis.

According to advanced algebra knowledge, we can obtain the transformation matrix from mirror coordinate system a to global coordinate system b as the unit matrix:

$$T = \begin{pmatrix} l_x & l_y & l_z \\ m_x & m_y & m_z \\ n_x & n_y & n_z \end{pmatrix} = \begin{pmatrix} -\sin E_H & -\sin A_H \cos E_H & \cos A_H \cos E_H \\ \cos E_H & -\sin A_H \sin E_H & \cos A_H \sin E_H \\ 0 & \cos A_H & \sin A_H \end{pmatrix} \quad (1)$$

Among them, (l_x, m_x, n_x) , (l_y, m_y, n_y) , and (l_z, m_z, n_z) represent the coordinates of the x-axis, y-axis, and z-axis in the global system under the mirror system, respectively. A_H represents the azimuth angle of the mirror relative to the ground, and E_H represents the elevation angle of the mirror relative to the ground.

With this transformation relationship unit matrix, we can determine the coordinate representation of any ray vector in the local system under the mirror system.

$$\vec{V}_0 = \begin{pmatrix} l_x & l_y & l_z \\ m_x & m_y & m_z \\ n_x & n_y & n_z \end{pmatrix} \vec{V}_H \quad (2)$$

In this equation, $\vec{V}_0$ represents the coordinates of a ray vector in global coordinates.

2.3. Model Establishment

This section's core is to establish an optical efficiency model [3]. In the part where the model is established, the cosine efficiency, shadow masking efficiency, and collector truncation efficiency are finely modeled, and after solving, it is determined whether certain efficiencies can be simplified.

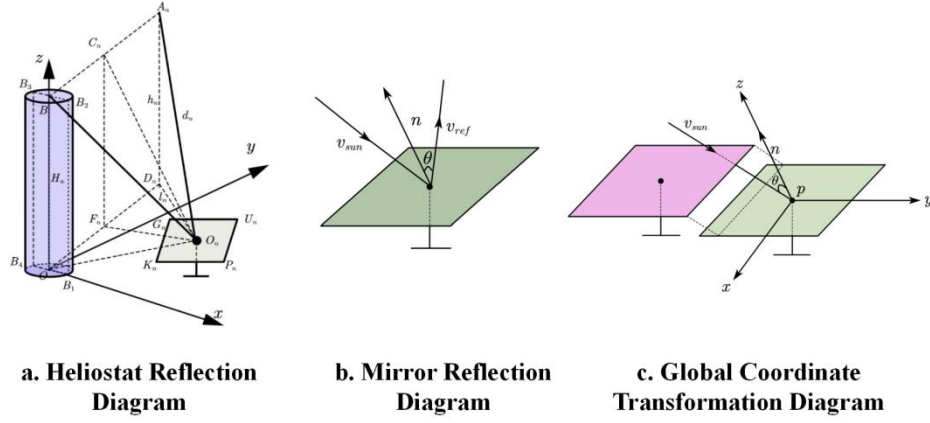


Figure 1. Cosine, Shadow Efficiency Diagram

As shown in Figure 1-a, it is a schematic diagram of a heliostat reflection. Taking the center of the mirror as the coordinate plane parallel to the ground, with the intersection of the plane and the axis of the tower as the coordinate origin, the y-axis points towards the north direction, and the x-axis points towards the east direction.

Therefore, from the analysis of the figure, it can be easily determined that the cosine efficiency calculation formula for the heliostat is as follows:

$$\eta_{\cos} = \cos \theta_n = \frac{\sin \alpha_s \cdot H_n + x_n \cos \alpha_s \cos \gamma_s + d_n}{2d_n} \quad (3)$$

Meanwhile,

$$d_n = \sqrt{H_n^2 + x_n^2 + y_n^2} \quad (4)$$

When sunlight shines, the incident sunlight is blocked by the front sun tracker, causing some areas of the rear sun tracker to not receive the incident sunlight [4]. This is called shadow efficiency η_s . Conversely, the reflected sunlight from the front sun tracker is blocked, causing some of the reflected light from the rear sun tracker to not reach the absorber. This is called occlusion efficiency η_b . The shadow and occlusion efficiency of any sun tracker are as follows $\eta_{sb} = \eta_s \times \eta_b$.

As shown in Figure 1-b, starting from the illuminated point, two unit vectors are drawn in the opposite direction and the outgoing direction of the incident. Let these two vectors be $\vec{v}_{sun}$ and $\vec{v}_{ref}$, and they lie in the same plane as the surface normal vector. They satisfy the following relationship:

$$\vec{v}_{sun} + \vec{v}_{ref} = 2 \times \cos \theta \times \vec{n} \quad (5)$$

Since there are infinitely many points on the sun tracker surface, we need to discretize the continuous into discrete. We use grid segmentation to divide the mirror surface horizontally into n_x parts and vertically into n_y parts. A point in the formed small grid represents whether this small grid is occluded.

Next, we will establish a simple analytical model to determine whether a small grid is occluded by other mirror surfaces, taking shadow efficiency as an example. The schematic diagram is shown in Figure 1-c.

We transform the global coordinates of the intersection point into the coordinates on the other mirror surface by matrix transformation. If the intersection point is inside the mirror, the coordinates of the intersection point in the other mirror coordinate system $(x'_0, y'_0, 0)$ must simultaneously satisfy the following relationship:

$$|x'_0| \leq \frac{L_x}{2}; |y'_0| \leq \frac{L_y}{2} \quad (6)$$

where L_x represents the width of the sun tracker surface and L_y represents the height of the sun tracker surface.

In summary, if the incident light in a small grid on the mirror surface is blocked by other mirror surfaces, the above intersection point should be located inside the occlusion plane, satisfying the above two conditions. Finally, after checking the influence of all other mirror surfaces on the target plane, we remove the occluded small grids. The shadow efficiency is then calculated by dividing the remaining number of small grids by the total number of small grids:

$$\eta_{shw} = \frac{N_{lf}}{N_{al}} \quad (7)$$

The calculation of occlusion efficiency is similar to that of shadow efficiency and will not be repeated here.

The above section assumes that the sunlight near the mirror is treated as parallel light because, within a relatively small range, the divergence angle of sunlight is not significant. However, when it comes to the distance between the mirror and the center of the solar collector, we cannot ignore the properties of sunlight itself. Sunlight is cone-shaped and the half-angle spread of the light cone generated from a point is 4.65 mrad. To facilitate understanding, a schematic diagram is provided here, as the Figure 2 shows:

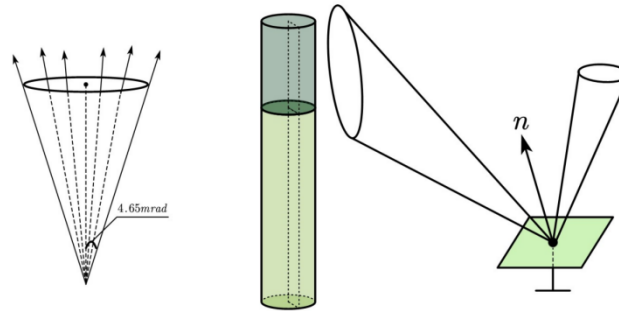


Figure 2. Efficiency of truncation diagram

Since there are countless points on a mirror surface, direct calculation is not possible. We follow the method of calculating shadow occlusion efficiency mentioned above and discretize each mirror surface by dividing it into small grids and representing each grid with a point on it. This transforms the infinite into the finite.

Similarly, for a light cone, there are infinite lines inside which cannot be calculated directly. Therefore, we also discretize it by uniformly tracing several rays within the solar divergence angle as incident rays [5]. The uniform tracing is done by dividing the half-angle expansion direction into equal angles and dividing the azimuthal direction with equal angles as well.

Similarly, we consider whether there is an intersection between the equation of the reflected ray and the cylindrical equation of the central receiver of the tower. When there are two intersection points and the z-coordinate of both points is greater than 76 meters, the ray is absorbed by the receiver. Otherwise, the ray will not be absorbed by the receiver.

Therefore, the receiver absorbs energy, $E_c l = \frac{s}{K}$. Here, s represents the total number of solar rays absorbed by the receiver on all mirror surfaces, and K represents the total number of reflected rays on all mirror surfaces. Furthermore, to simplify the model, we do not consider the case where the conical beam is occluded by other mirror surfaces during reflection. In other words, all reflected rays within the light cone are assumed to reach the receiver intact. Hence, the formula for the overall receiver truncation efficiency is as follows [6]:

$$\eta_{trunc} = \frac{\text{Collector receives energy}}{\text{Mirror reflects energy}} \quad (8)$$

2.4. Results

The calculated results of the physical quantities of the solar concentrator are as follows: the annual average optical efficiency of the concentrator field is 0.45, the annual average output thermal power is 27.57 MW, and the annual average output thermal power per unit mirror surface area is 0.44 kW/m². The data for March 21st is presented below, as Figure 3 shows.

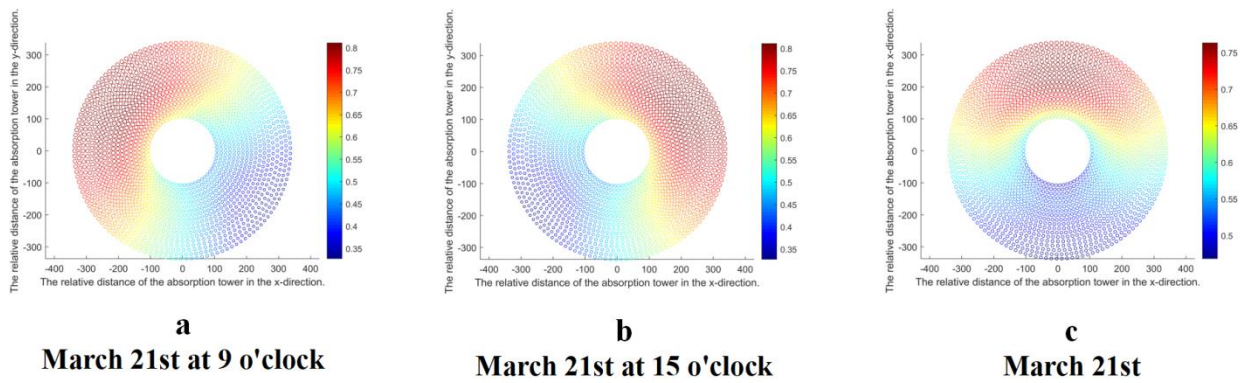


Figure 3. March 21st Optical Efficiency Distribution throughout the day and different periods

The average optical efficiency distribution of the solar concentrator throughout the day on March 21st is shown in Figure 3-c. It can be seen that the optical efficiency distribution throughout the day exhibits an approximately axisymmetric shape. However, the optical efficiency of the solar concentrator field also varies during different periods of the day. Figures 3-a and 3-b respectively show the optical efficiency distribution at 9 am and 3 pm on March 21st.

Based on the above calculations and analysis, it can be concluded that the shadow shading loss is relatively small. Therefore, we will simplify the optical efficiency model in the subsequent analysis.

3. Design of Heliostat Field under Rated Power of 60MW

Due to the dense arrangement of heliostats in the previous section, the shadow occlusion efficiency is very close to 1. Therefore, in this section with a sparse arrangement of the mirror field, we no longer consider the occlusion relationship between mirrors.

For the truncation efficiency of the solar collector, we directly use the calculated values from the circular heliostat field in the previous section. This is mainly because the occlusion between mirrors has a negligible impact on the truncation efficiency of the solar collector. The main efficiency loss is determined by the distance (from the heliostat center to the collector center). From both the perspective of land utilization angle and optical efficiency optimization, the mirror field will be fully occupied [7]. Thus, slight adjustments to the coordinates of the heliostats will not significantly affect the truncation efficiency of the solar collector. Therefore, we use the calculated values from the

previous section as a substitute for the optimization process of the truncation efficiency of the solar collector.

The minimum distance between the mirrors without collision [8] is shown in the following Figure 4:

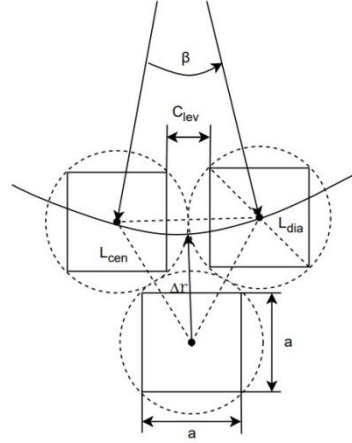


Figure 4. Layout Diagram of the Heliostat Mirrors

From the above figure, we can obtain the overall set of constraints [9] for the mirrors.

$$\begin{cases} \sqrt{2}a \leq d \\ \sqrt{2}a \leq a + C_{lev} \\ l \cdot \cos 30^\circ \leq r \\ \frac{a}{2} \leq h \\ 5 \leq C_{lev} \end{cases} \quad (9)$$

Grid search optimizes model hyperparameters by exhaustively exploring combinations within a predefined range. Users set a finite set of values for each hyperparameter, generating a grid of possibilities. Each combination is tested by training a model and evaluating its performance on a validation set. The combination yielding the lowest validation error is selected as the best for final model training.

3.1. Mathematical model

The goal is to optimize the parameters to maximize the annual average output thermal power per unit of mirror area. Therefore, the objective function F is defined as [10]:

$$Max(F) = \frac{E_{field}}{N \times a^2} \quad (10)$$

Where N represents the total number of solar mirrors and a represents the size of each solar mirror.

Based on the above, the total optimization function model is obtained by combining the objective function with all constraints.

$$\left\{ \begin{array}{l} \text{Objective Function: } \text{Max} = \frac{E_{\text{field}}}{N \times a^2} \\ E_{\text{field}} = \text{DNI} \cdot \sum_i^N A_i \eta_i \geq 60\text{MW} \\ d \geq \sqrt{2}a \\ a + C_{\text{lev}} \geq \sqrt{2}a \\ \Delta r \geq l \cdot \cos 30^\circ \\ h \geq \frac{a}{2} \\ C_{\text{lev}} \geq 5 \end{array} \right. \quad (11)$$

3.2. Results

To implement the network search algorithm using MATLAB software, we have determined that the optimal mirror size is $a=7\text{m}$ when the installation height is 4m . The optimized number of heliostats is 1050, and the rated power of the heliostat field is 62.07MW . At this configuration, the maximum annual average output thermal power per unit mirror area is 0.601 KW/m^2 . The distribution map of the heliostat field at this configuration is shown below, as Figure 5:

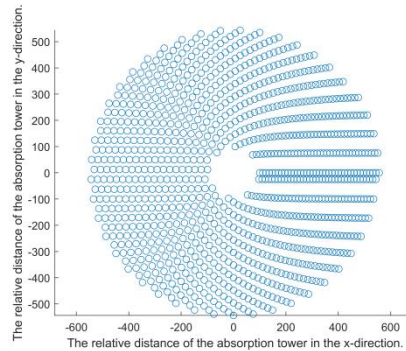


Figure 5. Optimized Distribution Map of Heliostat Fields

4. Conclusions

This article presents a detailed modeling of cosine efficiency, shadowing efficiency, and truncation efficiency, taking into account factors such as conical light. It calculates the optical efficiency and other indicators for multiple periods in a day and obtains a mathematical model for calculating the optical efficiency of a heliostat field. Additionally, using a grid search algorithm, the parameters of the heliostat and the layout of the heliostat field are optimized under the condition of a rated annual average output thermal power of 60MW , resulting in improved optical efficiency.

This article provides a research approach and framework for calculating the optical efficiency of a heliostat field, which also has theoretical guidance value for mirror field designers and contributes to the further development of tower-type solar thermal power generation technology.

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