

Combining Kinematic Modeling and TOPSIS For Enhanced Submarine Rescue Missions: Path Prediction and Equipment Assessment

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Abstract. With the growing interest in exploring the unknown underwater world, underwater tourism has become an emerging trend, which also brings about new demands for tourism safety and underwater exploration technologies. In this context, this paper aims to investigate path prediction modeling for submarine failure scenarios with an example in the Ionian Sea. In this paper, a prediction model based on kinematics and dynamics equations is used and combined with a Gaussian Markov process to simulate the current velocity, while an uncertainty perturbation term is introduced to improve the model's realism. To ensure that the submarine can be found in the shortest possible time, this paper establishes the weights of the dimensional and indicator layers based on the analysis hierarchy process (AHP) and entropy weighting method (EWM) and then calculates the scores of the required devices using the TOPSIS method. It was finally determined that the most suitable equipment is the magnetometer, followed by equipment such as a sonar system. This study provides a framework for improving search and rescue efficiency in submarine malfunction situations.

Keywords: Prediction Model, position control, multicriteria decision-making, underwater rescue.

1. Introduction

Interest in exploring the underwater world has increased significantly in recent years, creating unique challenges and opportunities for the world of ocean exploration. With the advancement of science and technology and people's increasing curiosity about the unknown, underwater exploration has become the focus of attention from all walks of life. Exploring the mysterious depths of the ocean is not only a manifestation of the adventurous spirit but also an exploration of mankind's understanding of the natural world and exploration of unknown areas.

However, security challenges come with it, and it becomes particularly important to develop safe and reliable exploration methods and preventive rescue solutions. Against this background, the research core of this paper focuses on the development of submarine path prediction models and rescue assessment models. A prediction model of the submarine's trajectory in the Ionian Sea was established mainly based on kinematic and dynamic equations ^[1]. To determine the required equipment, this article uses the analytic hierarchy process ^[2] proposed by Professor T.L. Saaty and the entropy weight method ^[3] proposed by Shannon to obtain the weights of the dimension layer and indicator layer. These two methods are widely used in quantitative analysis for multi-factor decision-making, analysis and weighting determination. Finally, the final score is obtained through TOPSIS ^[4] proposed by C.L.Hwang and K. Yoon.

The work includes the collection of data from reputable sources such as NOAA, followed by meticulous preprocessing of the acquired data, and the development of a model of seafloor conditions in the Ionian Sea using Digital Elevation Model (DEM) data. By integrating relevant data sets, the study derived a prediction equation for the submarine's trajectory. In addition, the AHP-EWM TOPSIS model is used to evaluate equipment based on multiple criteria such as availability, maintainability, readiness, and cost to effectively identify priorities.

2. Quantitative Analysis of the Underwater Environment's Impact on Submersible Motion

2.1. Get data and preprocess.

This article needs to construct a model of the submarine's motion trajectory on the seabed. In order to construct the kinematics and dynamic equations, it is first necessary to obtain relevant information such as the seabed topography, ocean currents, and seawater density of the Ionian Sea. The main source websites of data are shown in Table 1.

Table 1. Data source summary

Website	URL
NOAA	https://www.ncei.noaa.gov/
Ocean Data View	https://odv.awi.de/
GEBCO	https://www.gebco.net/
USGS	https://earthexplorer.usgs.gov/
CODE	http://wod.iode.org/
Britannica	https://www.britannica.com/

The dataset may contain missing values. We utilize linear regression to impute missing values, aiming to make the most of the information available in the original dataset. The computation is as Equation 1:

$$\hat{y} = bx + a$$

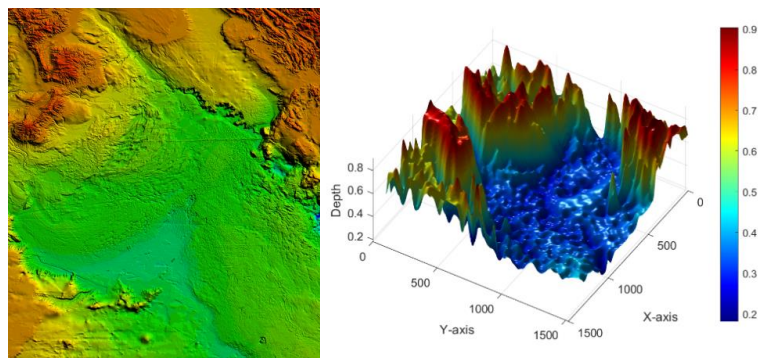
$$b = \frac{n \sum_{i=1}^n x_i y_i - (\sum_{i=1}^n x_i)(\sum_{i=1}^n y_i)}{n \sum_{i=1}^n x_i^2 - (\sum_{i=1}^n x_i)^2} \quad (1)$$

$$a = \bar{y} - b\bar{x}$$

where $\hat{y}$ represents the estimated value of the missing entry, x represents the position of the missing entry. For values that cannot be negative, if $\hat{y} < 0$, we replace missing entries with 0.

2.2. Seabed environment

First, the DEM data of the Ionian Sea was obtained from GEBCO, and then its digital elevation model bathymetry ^[5] was drawn using Global Mapper (shown in Figure 1 (a)). To facilitate observation, we use Matlab to analyze the image and digitally model it (as shown in Figure 1 (b)) so that we can subsequently predict the submarine's location. For ease of observation, we normalize the height to 1.



(a) Digital Elevation Model

(b) Digital Simulation

Figure 1. Seafloor topography environment simulation

2.3. Three-dimensional coordinate diving dynamic model

To simplify the problem and facilitate calculations, this article sets up two coordinate systems: the submarine-fixed coordinate $S - X_S Y_S Z_S$ and the earth-fixed coordinate $E - X_E Y_E Z_E$ (as shown in Figure 2).

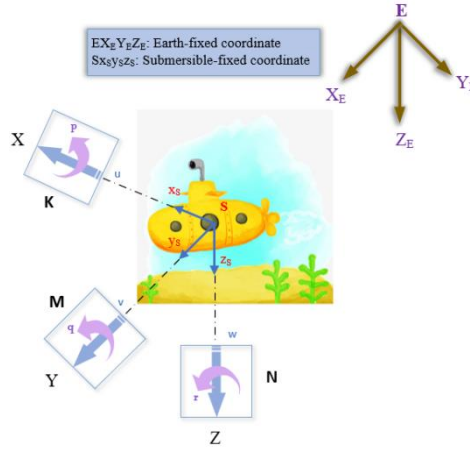


Figure 2. Coordinate System

From this we can also get the following definitions of motion parameters: $\eta = [\eta_1^T \ \eta_2^T]^T$: denotes the linear positions and Euler angles vector of the submarine in the EF frame $E - X_E Y_E Z_E$ describing the linear position $\eta_1 = [x, y, z]$ and the angular position $\eta_2 = [\theta, \varphi, \psi]$, where x, y, z are the linear position of the submarine while, θ, φ and ψ are three Euler angles: roll, pitch and yaw, respectively. And $v = [v_1^T \ v_2^T]^T$ represents the velocities vector in the submarine-fixed coordinate $S - X_S Y_S Z_S$, describing the linear velocity $v_1 = [u, v, w]^T$ and the angular velocity $v_2 = [p, q, r]^T$.

At the same time, the transformation of the coordinate system needs to be taken into consideration when using the two coordinate systems. Therefore, the kinematic equation of the submarine is as Equation 2:

$$\begin{cases} \frac{d\eta_1}{dt} = J_1(\eta_2) v_1 \\ \frac{d\eta_2}{dt} = J_2(\eta_1) v_2 \end{cases} \quad (2)$$

In which

$$J_1(\eta_2) = \begin{bmatrix} c\psi c\varphi & -s\psi c\theta + s\theta s\varphi c\psi & s\psi s\theta + s\varphi c\psi c\theta \\ s\psi c\varphi & c\psi c\theta + s\theta s\psi s\varphi & -c\psi s\theta + s\varphi s\psi c\theta \\ -s\varphi & s\theta c\varphi & c\varphi c\theta \end{bmatrix} \quad (3)$$

$$J_2(\eta_2) = \begin{bmatrix} 1 & s\theta t\varphi & c\theta t\varphi \\ 0 & c\theta & -s\theta \\ 0 & s\theta/c\varphi & c\theta/c\varphi \end{bmatrix} \quad (4)$$

Generally, the motion of the submarine is presented as the six-DOF nonlinear equation ^[6], which can be expressed in matrix form as Equation 5:

$$M \frac{dv}{dt} + C(v)v + D(v)v + G(\eta) = \tau_d \quad (5)$$

Since we are predicting the trajectory of the submarine after damage, we do not consider the propulsion component. In this equation all matrices $\in R^{6 \times 6}$. And among them:

M is the inertial matrix, which is described as the sum of an inertial matrix of the submarine itself M_{RB} and a hydrodynamic additional inertial matrix M_A due to the inertial of the surrounding fluid.

$C(v)$ is the Coriolis and centripetal matrix^[7], which are defined as the sum of a rigid-body Coriolis and centripetal matrix of the submarine $C_{RB}(V)$ and an added-mass Coriolis and centripetal matrix $C_A(v)$.

$D(v)$ is the damping matrix, it is the force generated by the relative motion of an object in a fluid that is opposite to the direction of motion. The resistance of a submarine F_D in water is given by Equation 6.

$$F_D = \frac{1}{2} \rho v^2 C_d A \quad (6)$$

After consulting the data, we assume that the resistance coefficient C_d of the seawater in the Ionian Sea is 5.6[8].

$G(\eta)$ is a matrix about restoring forces and moments, which is a combination of the gravity of the Earth on the submarine and the buoyancy of the Ionian Sea water on the submarine.

The submarine's dynamic model, as expressed in Equation 5, is derived without accounting for the impact of ocean currents. The influence of ocean currents on the submarine's movements is substantial, necessitating an examination of the submarine's response to environmental disturbances.

Given the intricate and irregular nature of ocean currents, modeling and incorporating their effects on the submarine pose challenges. Utilizing the Gauss–Markov process[9], the velocity of ocean currents is represented as:

$$\frac{d\zeta}{dt} = -a\zeta + W_c(t) \quad (7)$$

Where

$$\zeta(t) = [V_c(t), \alpha(t), \beta(t)]^T, a = \begin{bmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{bmatrix}, W_c(t) = [\omega_1(t), \omega_2(t), \omega_3(t)]^T \quad (8)$$

In which $W_c \in R^{3 \times 1}$ is Gaussian white noise, and $a \in R^{3 \times 3} > 0$, and $\zeta \in R^{3 \times 1}$ is the variable vector of the ocean current model that has three components such as ocean current speed in the fluid frame $V_c(t)$, the angle of attack $\alpha(t)$, and the sideslip angle $\beta(t)$

Assuming that the fluid is irrotational, the six components of the ocean current speed vector in the SF frame are expressed as:

$$V_c^E = [v_x \ v_y \ v_z \ 0 \ 0 \ 0]^T \quad (9)$$

Where the linear components of the ocean current in the three axes of the submarine-fixed coordinate can be defined as:

$$\begin{aligned} v_x &= V_c \cos \alpha \cos \beta \\ v_y &= V_c \sin \beta \\ v_z &= V_c \sin \alpha \cos \beta \end{aligned} \quad (10)$$

Using the rotation transformation expressed in Equation 3, the ocean current speed in the submarine-fixed coordinate is defined as:

$$V_c^B = [u_c^B \ v_c^B \ w_c^B \ 0 \ 0 \ 0] = \text{diag}[R^T(\eta), 0_{3 \times 3}] V_c^E \quad (11)$$

Where, u_c^B , v_c^B , and w_c^B represent the ocean current speeds in the surge and sway, and heave motions of the submarine, respectively.

Consequently, the motion of the submarine can be described in terms of the relative speed:

$$v_r = v - V_c^B = [u - u_c^B \quad v - v_c^B \quad w - w_c^B \quad p \quad q \quad r]^T \quad (12)$$

As a result, the dynamic equation motions of the submarine under the influence of the ocean current in the submarine-fixed coordinate are:

$$\begin{aligned} \frac{d\eta}{dt} &= J(\eta)v_r + V_c^E \\ \tau_d &= M \frac{dv}{dt} + C(v)v + D(v)v + G(\eta) \end{aligned} \quad (13)$$

According to this model, we can simulate the movement trajectory of the submarine as shown in Figure 3. Figures a-c are the trajectory of the submarine without considering the influence of ocean currents, and d-f are the trajectory of the submarine after considering the influence of ocean currents and adding disturbances. From the figure, we can see that the effect of ocean currents creates a large uncertainty in the prediction of the submarine's position.

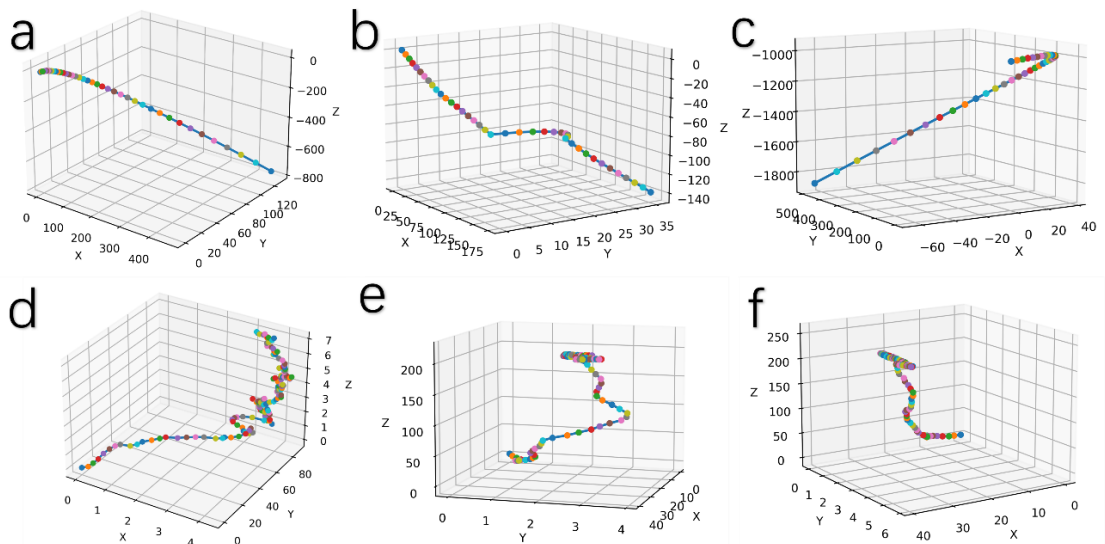


Figure 3. Simulation of the position of the submarine in different situations

From Equation 13 we can get the submarine's position prediction is related to the specific parameters, the uncertainty of the parameters that cannot get completely accurate values include the location of the seawater density, temperature, the specific speed and direction of ocean currents... these are we cannot be sure of the value of the prediction model, which means that we can't completely determine the values for the prediction model. As shown in Figure 4, it will be possible to appear at other points in the prediction of the submarine.

Then, by bringing specific data about the Ionian Sea and the submarine into the model, the value of each constant can be derived. For example, we can get the density ρ of the Ionian Sea to be about 1.018 kg/m^3 , the speed V of the submarine to be about 7 km, which is about 12.96 km/h, 3.6 m/s, and the submarine is assumed here to be a column of uniform mass of 23 meters in length, 2 meters in width and 4 meters in height, weighing 70 tons^[10]. According to the IODE, we can get the velocity of the currents in the Ionian Sea to be about 0.12 m/s. Finally, to obtain a model for predicting the position of the submersible, we need to combine kinematic and kinetic modeling with the seafloor environment to determine how much the submersible impacts the seafloor. We can obtain the results shown in Figure 5, where the Z-axis represents only the relative heights.

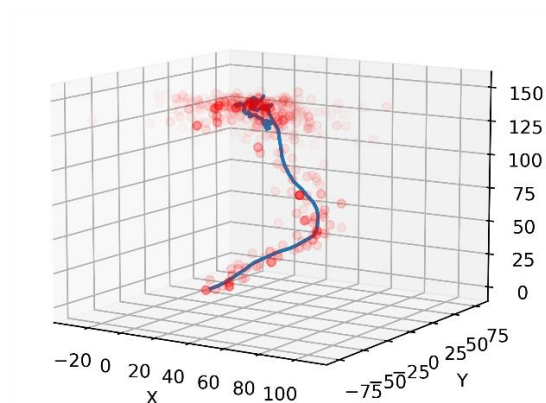


Figure 4. Uncertainty modeling

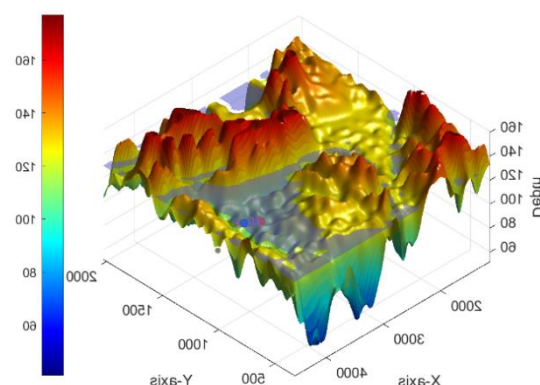


Figure 5. Simulation trajectory diagram

3. Comprehensive Evaluation Method for Search Equipment Selection

To better find the malfunctioning submarine, it is necessary to carry some additional search equipment on the main ship. And which devices to use and their priority becomes a question. Therefore, we first selected some equipment that we considered necessary after searching the data, and then used the TOPSIS model based on AHP-EWM^[11] to score the data by the costs related to the availability, maintenance, preparation, and use of the equipment. Determine the order in which equipment should be selected. This paper takes the time required for preparation and debugging before each dive as the preparation time, which is divided into ten grades from 1 to 10 according to the approximate time, and the average annual maintenance cost is used as the maintenance cost. The final data obtained is shown in Table 2.

Table 2. Devices and Evaluation Indicators

Evaluation Indicators	Indicators			
Devices	availability	maintenance	readiness	price
Side-Scan Sonar Systems	95%	1000 \$/year	3	\$37995
Multibeam Echo Sounders	85%	3000 \$/year	3	\$59999
Remote Operated Vehicles	90%	5000 \$/year	6	\$89995
Magnetometers	70%	1000 \$/year	3	\$24995
Robotic arms and tools	75%	5000 \$/year	9	\$15995

3.1. Original matrix positively

A comprehensive analysis of these four indicators (as shown in Table 3) shows that readiness, price, and maintenance are negative (cost-type) indicators. And availability is a positive (benefit indicator"). The so-called positive matrix is to uniformly convert indicator types into positive indicators. All elements here are positive numbers, so readiness, price, and maintenance are converted using the following formula:

$$X_i = \frac{1}{X_i} \quad (14)$$

3.2. Normalization of the matrix positively

To eliminate the influence of the magnitudes between different indicators, the normalization X consisting of 5 devices, 4 indicators to be evaluated (which has been positive) is positive to obtain a normalization matrix Z , Z in which element of:

$$Z_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^n x_{ij}^2}} \quad Z = \begin{bmatrix} z_{11} & z_{12} & \dots & z_{1j} \\ z_{21} & z_{22} & \dots & z_{2j} \\ \dots & \dots & \dots & \dots \\ z_{i1} & z_{i2} & \dots & z_{ij} \end{bmatrix} \quad (15)$$

3.3. Weighted Combination of Entropy Method and AHP

This paper utilizes a combination of AHP and entropy methods to scientifically determine the weight of each feature. While AHP is commonly used for multi-index evaluation problems, it may introduce subjective biases. The entropy method provides an objective approach by measuring information content in index data. However, relying solely on entropy may neglect valuable knowledge and experience. Therefore, this paper proposes a hybrid approach, combining results from both methods using the Mon formula to consider both subjective and objective indicators in determining the weight vector. The final results are shown in Table 3.

Table 3. Indicator Weights

Indicators	Weight
Availability	0.01100474
Maintenance	0.46953766
Readiness	0.25048229
Price	0.26897531

3.4. Calculate and normalize the score.

Then the corresponding score of each device can be calculated using the TOPSIS method based on these weights and values, that is, the distance between the evaluation object and its maximum value is calculated to measure its relative position. Next, normalize the distance by converting it into a score. The specific calculation steps are shown in Figure 6:

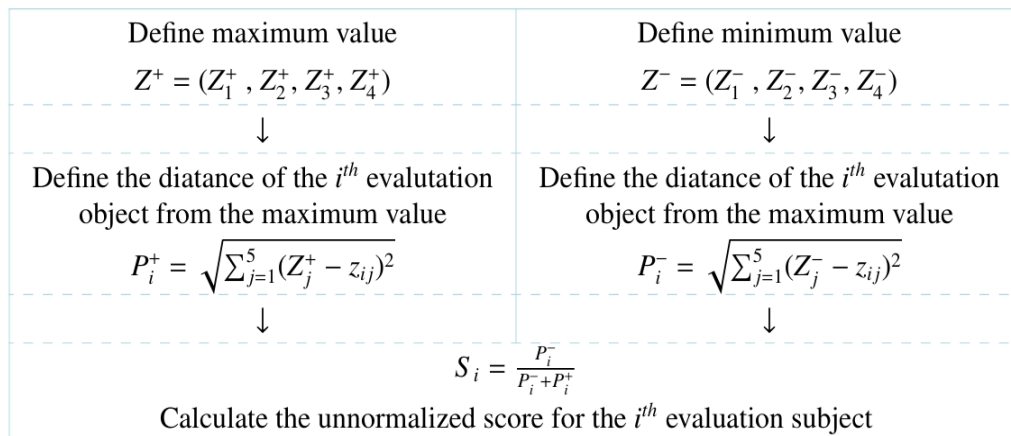


Figure 6. Flow Chart

3.5. Output ranking results

Table 4. Rating Overall Ranking

Rank	Device	Score
1	Magnetometers	0.946813
2	Side-Scan Sonar Systems	0.876266
3	Multibeam Echo Sounder	0.541092
4	Robotic arms and tools	0.335044
5	Remote Operated Vehicle	0.160240

According to the TOPSIS model analysis, the results shown in Table 4 can be obtained. It is recommended that the main ship carries a magnetometer and side scan sonar system as the main additional search equipment. The devices scored highly in terms of availability, maintenance costs and readiness, demonstrating efficiency and reliability for underwater exploration and potential rescue missions in the Ionian Sea. In addition, rescue vessels may consider carrying additional sonar systems, underwater cameras, and communications equipment to supplement the capabilities of the host vessel and address potential emergencies. Equipment selection should be comprehensively considered based on the specific environment and task requirements to ensure that it can effectively cope with various scenarios.

4. Conclusions

This study finally solves part of the problem involved in rescuing submarines through an integrated approach. We successfully develop a dynamic three-dimensional coordinate model based on digital elevation model (DEM) data and ocean current velocities, employing a Gaussian Markov process and introducing uncertainty perturbation terms for realism. By simulating potential trajectories of malfunctioning submersibles in the Ionian Sea, we highlight uncertainties and propose solutions such as onboard equipment for real-time data collection. Additionally, the establishment of the Analytical Hierarchy Process-Entropy Weight Method (AHP-EWM) TOPSIS model prioritizes the selection of search equipment on the main ship, determining the magnetometer as the most suitable device. We recommend additional equipment on rescue vessels to enhance search capabilities. These findings contribute to safer and more effective underwater exploration.

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