

Methodological Study on Heliostat Configuration Optimization in Tower Solar Thermal Systems

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Abstract. In response to the increasing demand for sustainable energy solutions, this study focuses on optimizing heliostat fields, crucial for solar power generation, to boost efficiency and reduce costs in line with national new energy policies. Initially, a model using Monte Carlo and ray tracing methods was developed to assess the heliostat field's lighting rate. This model showed an average annual optical efficiency of 25.54% and a thermal output of 20.61MW. Further, employing a grid search method, the ideal heliostat field configuration was identified, resulting in an average annual thermal output of 70.58MW. This research provides important insights for the future construction of heliostat fields, targeting operational efficiency and cost reduction.

Keywords: Tower type photothermal power generation, Energy structure transformation, Monte Carlo method, Grid search method.

1. Introduction

In recent years, with the continuous growth of global energy demand and the increasing importance of environmental protection, the development and utilization of renewable energy have become a hot topic in the international community. Among the many renewable energy technologies, solar thermal power generation has received a lot of attention due to its efficient, sustainable, and environmentally friendly characteristics. Tower solar thermal power generation system, as an important way of solar energy utilization, generates heat energy by concentrating solar energy, and then converting it into electricity. In this process, the configuration and performance of the heliostat have a decisive impact on the efficiency of the system.

This study employ the Monte Carlo method and grid search. The Monte Carlo method, introduced by S.M. Ulam and J. von Neumann, is utilized across medicine, finance, and engineering. Dou^[1] used it to assess the uncertainty in pig liver of four isoxazoline drugs (Avrana, Frelana, Sarolana, Lotilana). The grid search method, beneficial in architecture and the internet, was improved by Yuan^[2] to optimize support vector machine parameters, enhancing model generalization and accuracy over BP neural networks for predicting surrounding rock data. Shen^[3] improved it with adaptive annealing factors for a warping preparation workshop's scheduling, overcoming the traditional version's inefficiencies and tendency towards local optima.

In this study, this paper establish a preliminary model for the ring heliostat field, incorporating factors such as mirror reflectivity, shadow blocking efficiency, and collector truncation efficiency that influence the field's daylighting rate. From the insights gained from this initial model, trying to employ the grid search method to determine the optimal heliostat field configuration and the maximum annual average output thermal power per unit area. This is achieved by adhering to a predefined standard for the heliostat field's average annual output thermal power. The study focuses on optimizing the field's efficiency and output, highlighting the significance of considering various physical and operational factors in the design and arrangement of heliostat fields.

2. Modeling of Heliostat field Dynamics

Tower solar thermal power generation is a technology that converts solar energy into heat energy and drives a turbine to generate electricity through heat energy. The heliostat is one of the core components of the heliostat field for tower photothermal power generation. The following is the working flow of the heliostat field^[4] :

In the operation of a heliostat field, sunlight is reflected by the heliostats with high reflectivity towards a receiver located atop a tower, concentrating the light to achieve high temperatures. The receiver, a device filled with a special working fluid such as oil or salt, converts the focused sunlight into heat, significantly increasing the fluid's temperature and releasing a substantial amount of heat energy. Building on this understanding and an initial assessment of the solar light reception rate, a ring heliostat field model was developed, positioning the absorber centrally and establishing specific coordinates for all heliostats to preliminarily determine the field's annual average thermal efficiency.

1.1. Establishment of the coordinate system

The key to photothermal power generation in the heliostat field is the reflection of solar rays. To better describe the working process of the heliostat, this paper first establish the mirror field coordinate system XYZ for the circular heliostat field and heliostat, and the mirror coordinate system xyz for each mirror.

Suppose that the center position of the circular heliostats mirror field is the origin O, the positive direction of the X axis is due east, the positive direction of the Y axis is due north, and the vertical upward direction of the ground is the positive direction of the Z axis, the mirror field coordinate system XYZ is established.

Suppose the center of the heliostat is the origin o, the upward direction parallel to the left and right sides of the mirror is the positive direction of the x-axis, the right direction parallel to the upper and lower sides of the mirror is the positive direction of the y axis, and the direction perpendicular to the mirror facing out is the positive direction of the z-axis, and the mirror coordinate system xyz is established.

1.2. Model preparation

Before considering the efficiency of the heliostat field, this study must first describe one of the main rays of the sun. Before calculating the following factors, we need to analyze the incident main light, reflected main light, normal, pitch Angle, and azimuth Angle of the heliostat. According to the relevant literature^[5], this study can get the relevant formulas of the above factors.

Sun altitude Angle formula:

$$\sin \alpha = \cos \delta \cos \varphi \cos \omega + \sin \delta \sin \varphi \quad (1)$$

Where α is the solar altitude Angle, δ is the direct solar latitude, φ is the local latitude, and ω is the solar hour Angle.

For the declination Angle of the sun, the formula is:

$$\sin \delta = \sin \frac{2\pi D}{365} \sin \left(\frac{2\pi}{365} 23.45 \right) \quad (2)$$

Solar hour Angle formula:

$$\omega = \frac{\pi}{12} (ST - 12) \quad (3)$$

Where ST indicates the local time

From the above correlation variables, we can derive the azimuth of the sun:

$$\cos \gamma = \frac{\sin \delta - \sin \varphi \sin \alpha}{\cos \alpha \cos \varphi} \quad (4)$$

1.3. Establishment of annual mean optical efficiency model of helioscope field

We first studied the I-plane heliostat in the heliostat field. According to relevant literature^[11], the annual average optical efficiency of the heliostat field is affected by many factors, including shadow

occlusion efficiency, cosine efficiency, atmospheric transmittance, collector truncation efficiency mirror reflectivity, etc. The calculation formula is as follows:

$$\eta = \eta_{sb} \eta_{\cos} \eta_{at} \eta_{trunc} \eta_{ref} \quad (5)$$

Among them, η represents the optical efficiency of the heliostatic field, η_{sb} represents the shadow occlusion efficiency, $\eta_{\cos}$ represents the cosine efficiency, η_{at} represents the atmospheric projection rate, η_{trunc} represents the collector truncation efficiency, and η_{ref} represents the mirror reflectivity.

Then start to look at how each factor is calculated:

Mirror reflectance is related to the material and cleanliness of the mirror, according to relevant literature, in most cases, mirror reflectance is a constant, this study take its value of 0.92. For this, this study only need to assume the coordinate relationship between the sun's position and time to get the direction of the sun's incident light at a certain period, and then analyze other factors more accurately^[6].

When the light propagates in the atmosphere, it may be affected by the content of atmospheric composition, resulting in a certain degree of energy attenuation. According to relevant literature, the atmospheric transmittance is related to the center distance of the mirror center island collector^[7]. The calculation formula is

$$(\eta_{at})_i = 0.99321 - 0.0001176(d_{hr})_i + 1.97 \times 10^{-8}(d_{hr})_i^2 \quad (6)$$

Where $(d_{hr})_i$ represents the distance between the center of the heliostat i and the center of the collector.

The cosine efficiency is the cosine of the Angle between the incident light on the specified day mirror and the normal direction of the mirror reflection point, calculated by the formula:

$$(\eta_{\cos})_i = \cos \theta_i = \frac{|n_i \cdot v_i|}{\|n_i\| \|v_i\|} \quad (7)$$

Where θ_i represents the acute Angle between the direction vector v_i of the incident light at the center of the heliostat i block and the normal vector n_i of the heliostat.

The loss of shadow occlusion mainly includes two parts: the shading of the absorber and collector to the sunlight and the shading of the front heliostat to the back heliostat. We need to consider separately:

(1) The shielding area of the absorber and collector

To facilitate calculation, we assume that the absorption tower and the collector constitute a complete cylinder, the height of the absorption tower and the height of the collector, the diameter of the bottom surface is the diameter of the collector so that the shadow of the regular cylinder at any time is always a rectangle, the length of the rectangle is determined by the height of the cylinder and the height of the sun. The ratio of this area to the total area of the circular heliostat field is the ratio of shelter loss caused by the shadow of the absorber and the collector, calculated by the formula:

$$S_t = \frac{(H_1 + H_2)}{\tan \partial_t} d \quad (8)$$

Then calculate the occlusion loss ratio η_{sb1} by the ratio to the total area of the circular heliostat field, and the calculation formula is as follows:

$$\eta_{sb1} = \frac{S_t}{S_{mf}} = \frac{(H_1 + H_2) d}{\tan \partial_s S_{mf}} \quad (9)$$

(2) The shielding area of the heliostat

In a heliostat field, heliostats can experience occlusion from those in the preceding row, manifesting in two ways: shadowing, where the front row heliostat casts a shadow over the rear, and blocking of reflected light, where the light reflected from the rear heliostat is obstructed by the front. To address shadow occlusion, it is necessary to determine if incident light on any point of the rear heliostat's coordinate system is interrupted by the front heliostat by transforming coordinates from the geodetic system to the heliostats' local system and identifying intersection points. Similarly, to resolve reflected light blockage, one must calculate potential intersection points within the front heliostat's coordinate system, drawing on the approach used for incident light.

The incident solar light is a beam of conical light with a certain conical Angle^[8]. According to relevant literature, the conical Angle is about 9.3mrad. Therefore, the reflected light is also conical, as the distance increases, the cross-section of the conical light also grows, and finally reaching the collector may form a huge conical light spot, exceeding the collector's receiving range, resulting in the overflow of energy, which affects the truncation efficiency, to simplify the calculation, We assume that all reflected rays carry the same energy and equal to q , the number of rays received by the collector is n_{rec} , the number of rays reflected by the mirror is n_{all} , and the number of rays blocked by the shadow is n_{sb} . Then, $Q_{rec} = n_{rec} \cdot q$, $Q_{all} = n_{all} \cdot q$, $Q_{sb} = n_{sb} \cdot q$ the calculation formula of the collector truncation efficiency is as follows:

$$\eta_{trunc} = \frac{Q_{rec}}{Q_{all} - Q_{sb}} = \frac{n_{rec}}{n_{all} - n_{sb}} \quad (10)$$

For the uncertainty of the falling point of the incident light in the heliostat, we used the ray tracing method^[9] to trace the light, select a large number of rays to illuminate any piece of the heliostat, and count the number of rays blocked by the shadow and the number of rays received by the collector. For the randomness of the conical light, we take the Monte Carlo algorithm to calculate the random incident light vector, and random reflection points, and then determine the reflected light equation, and finally the collector's surface equation and the reflected light, according to whether there is a real solution to determine whether the reflected light falls on the collector.

1.4. Model of annual average output thermal power of heliostat mirror field

Through the above factors, we can calculate the average optical efficiency of the heliostat field every month, and then through the relevant literature, the formula of the average annual output thermal power of the heliostat field is:

$$E_{field} = DNI \cdot \sum_{k=1}^N A_k \eta_k \quad (11)$$

Among them, E_{field} represents the output thermal power of the whole heliostats field at a certain moment, DNI represents the direct irradiance of the normal direction and A_k represents the lighting area of the heliostats mirror on the k plane, that is, the height of the heliostats mirrors multiplied by the width of the mirror. For this preliminary study, we assume that the heliostat has a size of 8m*8m and a height of 5m. η_k represents the optical efficiency of the K -plane heliostat at this time, and N represents the total number of heliostat.

Normal direct irradiance DNI ^[10], refers to the solar radiation energy received per unit area and per unit time by the plane perpendicular to the sun rays on the earth, and its approximate calculation formula is as follows:

$$\begin{cases} DNI = G_0[a + be^{a(-\frac{c}{\sin \delta_s})}] \\ a = 0.4237 - 0.00821(6 - H)^2 \\ b = 0.5055 + 0.00595(6.5 - H)^2 \\ c = 0.2711 + 0.01858(2.5 - H)^2 \end{cases} \quad (12)$$

Where, G_0 indicates the solar constant and the value is generally a fixed value: $1.366 \text{ kw} / \text{m}^2$, H is the local altitude, the unit is kw .

Through the above formula, we can get the efficiency of each factor of the heliostat field, the average optical efficiency, and the output thermal power of the heliostat field every month, and the results are as shown in Table 1 and 2:

Table 1: Monthly efficiency of each factor and output efficiency of the heliostat field

Month	Average Optical efficiency	Average Cosine efficiency	Average Shadow Occlusion efficiency	Average Truncation efficiency	Average mirror output per unit area Thermal power(kW/m^2)
1	0.2267	0.6870	0.7013	0.5471	0.1404
2	0.2547	0.7064	0.7495	0.5623	0.1759
3	0.2673	0.7270	0.7744	0.5487	0.1981
4	0.2720	0.7463	0.7676	0.5454	0.2109
5	0.2726	0.7579	0.7573	0.5468	0.2160
6	0.2733	0.7617	0.7555	0.5463	0.2178
7	0.2726	0.7577	0.7573	0.5469	0.2159
8	0.2720	0.7455	0.7683	0.5455	0.2105
9	0.2670	0.7259	0.7737	0.5498	0.1973
10	0.2526	0.7039	0.7449	0.5629	0.1725
11	0.2242	0.6854	0.6975	0.5452	0.1372
12	0.2098	0.6789	0.6774	0.5291	0.1220

Table 2: Annual average efficiency and annual average output thermal power of each factor in the heliostat field

Annual mean Optical efficiency	Annual mean Cosine efficiency	Annual average shadow Occlusion efficiency	Annual mean Truncation efficiency	Annual average heat output power (MW)	Annual average mirror area per unit area Output thermal power (kW/m^2)
0.2554	0.7236	0.7437	0.5480	20.6100	0.1845

1.5. Results and Discussion

From the image of Figure 1A, this study can deduce that the sun's altitude Angle is about noon, about 100 days from the vernal equinox when it is the largest, and the lighting conditions are the best. The heliostat field displays a high average shadow occlusion efficiency, peaking at 0.7744 in March due to the wide spacing between heliostats that minimizes shadowing losses, which are predominantly influenced by the tower's shadow. Meanwhile, the average cosine efficiency remains relatively consistent throughout the year, with only a 0.1 variation, attributed to the large incident angles and the resultant minor deviations in positioning of the heliostats, thus maintaining a low and stable efficiency. Conversely, the average truncation efficiency is low but stable, with a maximum of 0.5629

in October, explained by the substantial size of the heliostats and the resultant large spillage area. This, coupled with the consistently low truncation efficiency, leads to an overall low average optical efficiency ranging between 0.20-0.30 monthly. Furthermore, the sun's apparent annual motion suggests a symmetry in the "annual average" indices of all factors across the east-west axis of the heliostat field. This is corroborated by the data showing that the annual average optical and cosine efficiencies of heliostats, as seen in Figures 1C and 1D, support this hypothesis. Notably, the northern heliostats relative to the collector exhibit higher annual average optical efficiency than those to the south, indicating a potential avenue for further research into heliostat distribution.

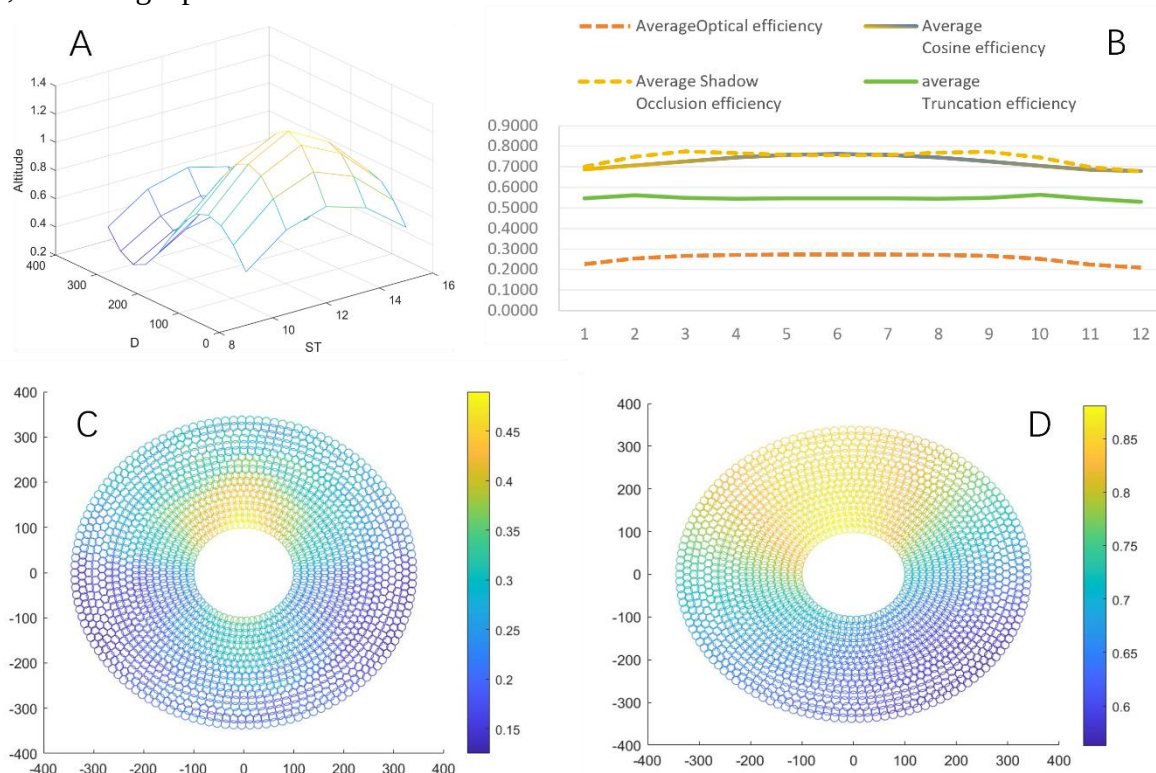


Figure 1: Display of the Factors Influencing the Optical Efficiency of the Heliostat Field. A: The relationship between the solar altitude angle and local time, as well as the distance between the day and the vernal equinox day. B: Changes in the efficiency of various influencing factors in the heliostat field under different months. C: Scatter plot of annual average optical efficiency for each heliostat. D: Scatter plot of annual average cosine efficiency for each heliostat.

2. Grid search optimization for mirror field configuration

According to the previous heliostat field model, we calculated that when each heliostat has a fixed position, a size of 8m*8m, and a height of 5m, the annual average output thermal power of the heliostat is 20.6100MW. Since the average annual optical efficiency of the heliostat field is relatively low, we will then study how to optimize the distribution of the heliostat field under the condition that the size and installation height of each heliostat remain unchanged, so that when the average annual output thermal power of the heliostat field reaches 60MW, the average annual output thermal power per unit area will be as large as possible.

The grid search method is a more commonly used and convenient optimization method, which refers to dividing the parameters to be searched into a network in a certain space range, and searching for the optimal parameters by traversing all the points in the grid. This method can find the global optimal solution when the optimization interval is large enough and the step distance is small enough.

For this study, for the uncertainty of the position of the endotherm tower, to simplify the calculation, we can set it as the origin and discuss the heliostat distribution near the endotherm tower.

Secondly, we assume that the number of heliostats installed is N . Next, we discuss the constraints of each factor:

We can set any point (x_n, y_n, z_n) in the coordinate system as the position of the NTH heliostat, which needs to meet the following conditions:

Because the heliostat is within the scope of the mirror field, the distance from this point to the origin is less than the radius of the mirror field, and the heliostat cannot be installed within 100 meters of the absorption tower. The constraint conditions are:

$$100^2 \leq x_n^2 + y_n^2 \leq 350^2 \quad (13)$$

Assume that the size of the heliostat is $a \times b$, and the installation height is set to h_0 . To simplify unnecessary calculations, this study assume that the mirror side length is between 2m and 15m, the installation height is between 2m and 6m, and the installation height ensures that the horizontal shaft does not touch the ground when rotating. The constraint conditions are:

$$s.t. \begin{cases} 2 \leq a \leq b \leq 15 \\ 2 \leq h_0 \leq 6 \\ a/2 < h_0 \end{cases} \quad (14)$$

The center distance of the adjacent heliostat base is more than 5m more than the height of the mirror. The constraint conditions are:

$$\sqrt{(x_n - x_m)^2 + (y_n - y_m)^2 + (z_n - z_m)^2} > b + 5 \quad (15)$$

Annual average thermal output power: this study require that the rated average thermal power is not less than 60MW. The constraint conditions are:

$$E \geq 60MW \quad (16)$$

In summary, this paper summarized the unified heliostat field optimization design model based on single objective programming, as follows:

$$\begin{aligned} \text{Objective: } E_{\max}(a, b, h_0, N) &= \frac{\sum_{i=1}^{12} \sum_{j=1}^5 \sum_{k=1}^N DNI_{ij} \eta_{ijk}}{60N} \\ s.t. \begin{cases} 100^2 \leq x_n^2 + y_n^2 \leq 350^2 \\ 2 \leq a \leq b \leq 15 \\ 2 \leq h_0 \leq 6 \\ \frac{a}{2} < h_0 \\ \sqrt{(x_n - x_m)^2 + (y_n - y_m)^2 + (z_n - z_m)^2} > b + 5 \\ E_{\text{field}} \geq 60MW \end{cases} \end{aligned} \quad (17)$$

Next, this study only need to use the grid to search the number of heliostats, screen out the unit mirror area, and the maximum annual average thermal output, and finally, this study get the following results are shown in Figure 2. It is easier to see that in the optimal heliostat layout, the annual average optical efficiency of the heliostat in the north of the heliostat field is significantly higher than that of the heliostat in the south. This just confirmed the hypothesis of the heliostat model study.

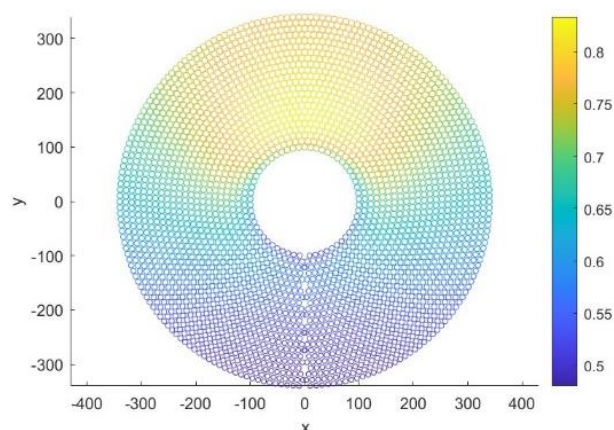


Figure 2: Optimal heliostat layout and average optical efficiency distribution map

3. Conclusions

For the study of heliostat field heliostat arrangement optimization, this paper first understand the working principle of the heliostat field and extract the factors that affect the average annual output power of the heliostat field: Shadow occlusion efficiency, cosine efficiency, atmospheric transmittance, collector truncation efficiency, and mirror reflectivity. Based on these influencing factors, a circular mirror field model was initially established to calculate the average annual output power of 20.61MW. Then, through the result analysis, the heliostat's size, height, and position were optimized for the mirror field. To better explore, this paper first made the size and height of the heliostat consistent to ensure that the rated power of the heliostat field remained unchanged. Through target optimization and grid search method, the optimal mirror field arrangement and the annual average output thermal power of the mirror field at this time were calculated to be 70.58MW. For the above two works, this study can conclude that the arrangement of more heliostats in the north of the heliostats field is helpful to improve the utilization rate of sunlight in the heliostats field, to save the cost appropriately.

This paper provides a research idea and framework for the research of tower photothermal systems in the field of new energy, which is conducive to the construction of new energy modernization, and also proves the feasibility of the Monte Carlo algorithm and grid search.

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