Research on Quantifying Sea Lamprey Ecological Impacts Based on Differential Equation Modeling and AHP-TOPSIS Framework

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Abstract. The invasive sea lamprey has posed significant challenges to aquatic ecosystems, affecting biodiversity and the fishing industry. This paper endeavors to quantitatively assess the impacts of sea lamprey population, explicitly accounting for variation in sex ratio, on both prey population and others within ecosystem, thereby facilitating the formulation and implementation of more strategically focused management interventions. The paper achieved the aforementioned objectives by developing a differential equation-based sea lamprey-prey population dynamics model to quantify impacts on prey population, and employing the AHP-TOPSIS framework to quantify the broader ecological impacts. This comprehensive approach provides a holistic perspective on the sea lamprey's cumulative impact on the ecosystem.

Keywords: Differential population dynamics, AHP-TOPSIS, Sea lamprey impacts.

1. Introduction

Since the invasion of the Sea Lamprey (*Petromyzon marinus*) into the Great Lakes from the late 18th to early 19th centuries, its presence has wreaked havoc on local ecosystems and delivered a nearly catastrophic blow to the fishing industry, nearly eradicating the once cornerstone species of lake trout [1]. To manage the invasive sea lamprey and coordinate fishery research and management, the Great Lakes Fishery Commission (GLFC) was established in 1955, launching a comprehensive program in 1956 aimed at reducing sea lamprey populations in the Great Lakes [2]. John W. Heinrich summarized the abundance of sea lamprey in Lake Superior, one of the Great Lakes, from 1957 to 1999 [3]. Sean A. Lewandoski developed a Bayesian state-space model using over 60 years of tributary data from Lake Superior since 1953 to predict the probability of suppressing sea lamprey populations below management targets [4]. However, data on the status of the sea lamprey population before the control program was scarce [5], and no mathematical model has been proposed to quantify the impact of sea lamprey population on ecosystem, taking into account changes in sex ratio.

This paper reveals the nuanced effects of sea lamprey of varying sex ratio on marine environments via two models. The differential equation model reveals the pivotal role of sex ratios in population dynamics, demonstrating that a higher proportion of females initially accelerates population growth and predation, only to later decline due to increased mortality. In contrast, the AHP-TOPSIS framework offers an ecosystem-wide perspective, pinpointing a balanced sea lamprey female ratio as crucial for ecological equilibrium. This work represents a significant step forward in the ongoing battle against the sea lamprey, offering new insights that are critical for the preservation of biodiversity and the recovery of the Great Lakes' fishing industry.

2. Establishment and application of the sea lamprey-prey population dynamics model

2.1. Biological characteristics of sea lamprey

As an invasive species in the Great Lakes, the sea lamprey virtually lacks natural predators and boasts a formidable reproductive capacity, with an average female laying 70,000 eggs, potentially yielding 6,000 or more offspring [6]. Given its status as an invasive entity, not having co-evolved with the native fish populations of the lakes, the parasitic behavior of the sea lamprey often results in the mortality of the majority of its hosts. For the purposes of simplification within our model, we will consider this behavior as predation. The predatory efficiency of the sea lamprey is remarkable, with a single individual capable of killing over 20kg of fish [7]. Notably, the gender ratio within sea lamprey populations is density-dependent. Typically, lower population densities correlate with a higher proportion of females. For instance, prior to control measures, the female proportion of the population was about 35%, while reducing the population to 10% of its pre-control numbers shifted the female ratio to approximately 60% [8]. Moreover, there is some controversy over the differences in predatory efficiency between males and females, though the preponderance of evidence suggests that females, due to the higher energy demands of gonadal development, possess a greater predatory capacity. However, this also leads to increased mortality rates within the population as numbers rise and resources become scarce [9]. The mating system of the sea lamprey is complex, with individuals of different sizes engaging in varying mating behaviors, though generally exhibiting a tendency towards monogamy [10].

2.2. Sea lamprey-prey population dynamics model

Based on the biological characteristics of the sea lamprey, the population dynamics equation for the sea lamprey is established as follows:

$$\frac{dL}{dt} = r_L(L) \cdot L(t) - d_L(L) \cdot L(t) + c(L) \cdot L(t) \cdot P(t) \tag{1}$$

The explanations for each parameter are as follows:

$L(t)$ represents the population size of the sea lamprey at time$t$,$P(t)$ represents the population size of the prey species at time t.The reproduction rate of the sea lamprey, $r_L(L)$ can be expressed as:

$$r_L(L) = r_{L_{max}} - \alpha \cdot \left|R_f(L) - 0.5\right| \tag{2}$$

Where $r_{L_{max}}$is the maximum reproduction rate under a normal gender ratio. α is a sensitivity coefficient characterizing the reproduction rate's sensitivity to changes in the gender ratio, that is, the decrease in $r_L(L)$ as R_f deviates from 0.5.

Note that R_f, the female ratio, can further be expressed as a function of the sea lamprey population, assuming a simple linear relationship:

$$R_f(L) = 0.5 - \delta \cdot \left(\frac{L - L_{baseline}}{L_{baseline}}\right) \tag{3}$$

Where δ is a sensitivity coefficient characterizing the sensitivity of the female ratio to changes in the population size, that is, the decrease in the female ratio as the population size L increases. $L_{bascline}$ is the population size under a normal gender ratio.

The mortality rate of the sea lamprey, $d_L(L)$, can be expressed as:

$$d_L(L) = d_{L_{base}} + \mu \cdot \left(R_f(L) - 0.5\right) \tag{4}$$

Where $d_{L_{base}}$ is the baseline mortality rate when the gender ratio equals 0.5. μ is a sensitivity coefficient characterizing the mortality rate's sensitivity to changes in the gender ratio, that is, the increase in $d_L(L)$ as R_f increases.

The predation efficiency of the sea lamprey, $c(L)$, can be expressed as:

$$c(L) = c_{base} + \beta \cdot \left(R_f(L) - 0.5\right) \tag{5}$$

Where c_{base} is the baseline predation efficiency when the gender ratio equals 0.5. β is a sensitivity coefficient characterizing the predation efficiency's sensitivity to changes in the gender ratio, that is, the increase in $c(L)$ as R_f increases.

The dynamics equation for the prey population is established as follows:

$$\frac{dP}{dt} = r_P \cdot P(t) - e(L) \cdot L(t) \cdot P(t) \quad (6)$$

The explanations for each parameter are as follows:

$P(t)$ is the prey population size at time t, $L(t)$ is the sea lamprey population size at time t, r_P is the reproduction rate of the prey, $e(L)$is the predation pressure coefficient, which can be expressed as:

$$e(L) = e_{base} + \gamma \cdot \left(R_f(L) - 0.5\right) \quad (7)$$

where e_{base} is the baseline predation pressure coefficient when the gender ratio of the sea lamprey population equals 0.5. γ is a sensitivity coefficient characterizing the predation pressure's sensitivity to changes in the gender ratio, meaning the increase in predation pressure on the prey population as R_f increases.

By substituting equations (2), (4), (5), and (7) into equations (1) and (6), we obtain:

$$\frac{dL}{dt} = \left[r_{L_{max}} - \alpha \cdot \left|R_f(L) - 0.5\right|\right] \cdot L(t) - \left[d_{L_{base}} + \mu \cdot \left(R_f(L) - 0.5\right)\right] \cdot L(t) + \left[c_{base} + \beta \cdot \left(R_f(L) - 0.5\right)\right] \cdot L(t) \cdot P(t) \quad (8)$$

$$\frac{dP}{dt} = r_P \cdot P(t) - \left[e_{base} + \gamma \cdot \left(R_f(L) - 0.5\right)\right] \cdot L(t) \cdot P(t) \quad (9)$$

Further substituting equation (3) into the above, we can derive the sea lamprey-prey population dynamics model:

$$\frac{dL}{dt} = \left[r_{L_{max}} - \alpha \cdot \left|0.5 - \delta \cdot \left(\frac{L - L_{bascline}}{L_{bascline}}\right) - 0.5\right|\right] \cdot L - \left[d_{L_{base}} + \mu \cdot \left(0.5 - \delta \cdot \left(\frac{L - L_{baseline}}{L_{baseline}}\right) - 0.5\right)\right] \cdot L + \left[c_{base} + \beta \cdot \left(0.5 - \delta \cdot \left(\frac{L - L_{baseline}}{L_{baseline}}\right) - 0.5\right)\right] \cdot L \cdot P \quad (10)$$

$$\frac{dP}{dt} = r_P \cdot P - \left[e_{base} + \gamma \cdot \left(0.5 - \delta \cdot \left(\frac{L - L_{baseline}}{L_{baseline}}\right) - 0.5\right)\right] \cdot L \cdot P \quad (11)$$

These equations model the dynamics of both the sea lamprey and its prey population, factoring in the effects of gender ratio changes on the sea lamprey's reproduction, mortality, and predation efficiency, as well as the resulting predation pressure on the prey population.

2.3. Application of the population dynamics model

2.3.1. Simulation of the invasion process in lake superior

Lake Superior, one of the Great Lakes, serves as a crucial habitat for the lake trout (Salvelinus namaycush), and the invasive sea lamprey represents the most significant threat to the lake trout fisheries in Lake Superior [11]. To simulate and visually present the entire process from the initial invasion of the sea lamprey into Lake Superior up to its control commencement in 1956—a period for which relevant data are scarce—and to showcase the astonishing threat it poses to its prey populations (here exemplified by the lake trout, its primary prey), we adjusted the parameters in our model as shown in Table.1 and Table.2, resulting in Fig.1 for the invasion process in Lake Superior.

Table 1. Parameter values involved in the simulation of the invasion process

$r_{L_{max}}$	α	$d_{L_{base}}$	μ	c_{base}	β	e_{base}	γ
2e-6	5e-8	5e-7	5e-8	5e-8	5e-8	2e-8	5e-8

Table 2. Parameter values involved in the simulation

δ	$L_{bascline}$	r_P	L_0	P_0
0.1	328000	4e-6	10000	300000

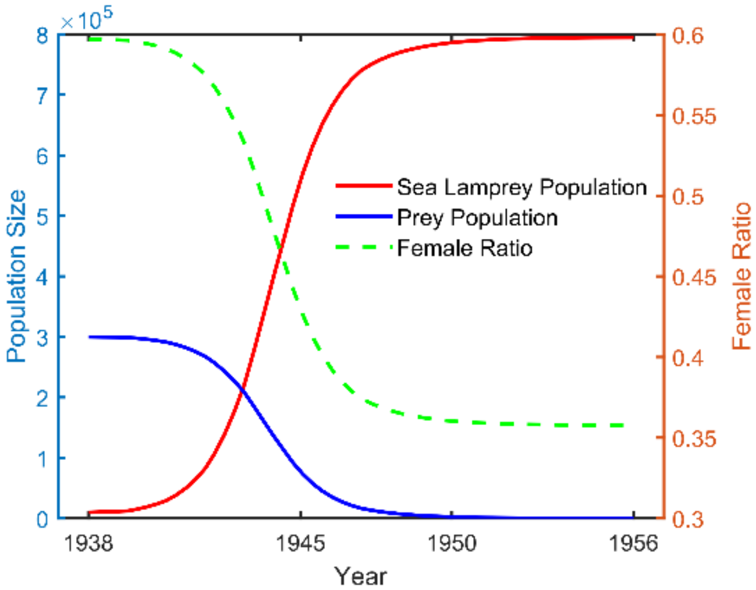

Figure 1. 1938-1956 Sea lamprey-lake trout population dynamics

The figure reveals that from the 1940s to the 1950s, the sea lamprey population expanded rapidly, inflicting catastrophic damage on the lake trout population, even driving it to near endangerment [12], thereby illustrating its severe impact on the fisheries. Notably, prior to the implementation of control measures, as the sea lamprey population increased, the proportion of females gradually declined, ultimately stabilizing at around 35% [8, 9]. The population dynamics graph correlates with historical descriptions provided in previous research.

2.3.2. Assessment of the impact of Sex Ratio on population dynamics

As previously mentioned, the gender ratio of the sea lamprey population is related to its population density. The larger the population, the lower the proportion of females. However, to compare the impact of different gender ratios on the sea lamprey population itself and on its prey population, and to provide a unique perspective for sea lamprey control programs, we artificially fixed the gender ratio during each simulation process, conducting multiple simulations with different gender ratios (Rf=0.35, 0.4, 0.45, 0.5, 0.55, 0.6). To simplify our simulation process, we kept the parameter values consistent with those in 2.3.1, but since we ignored changes in gender ratio in a single simulation, it was necessary to discard the parameters delta and Lbaseline, thus using equations (8) and (9).The result is shown in Fig 2.

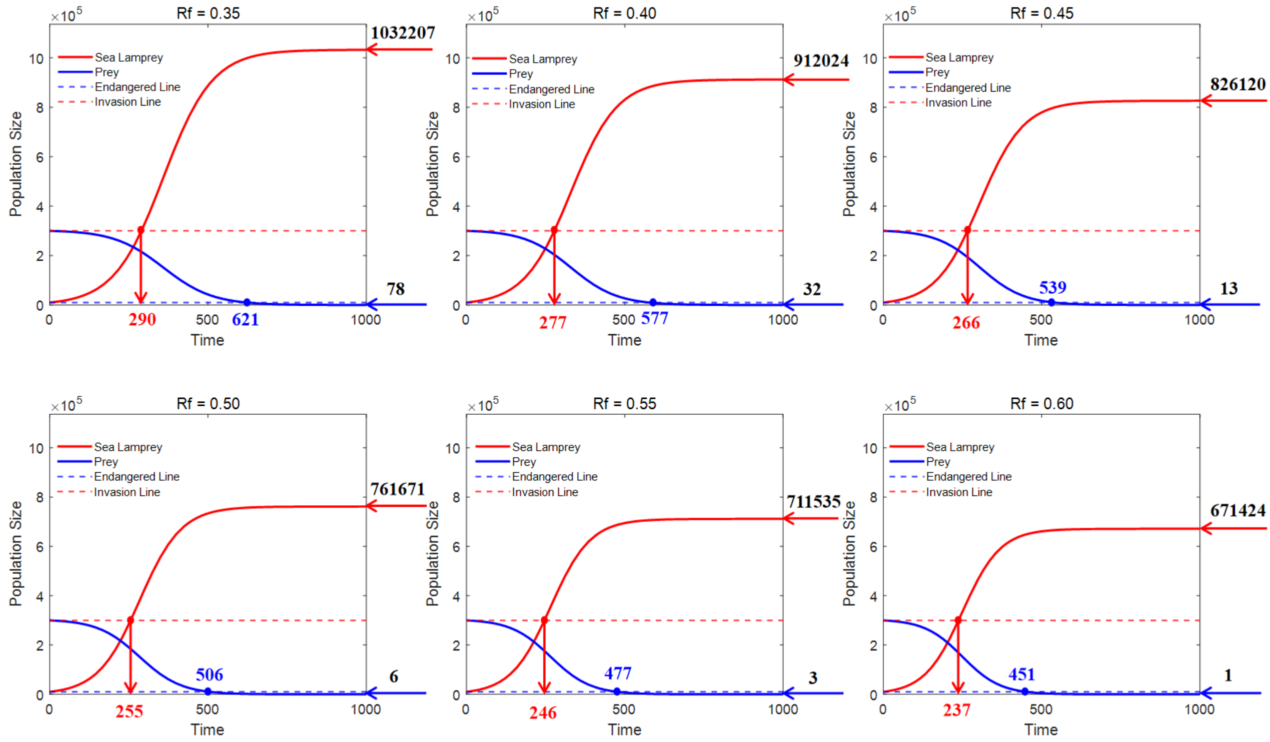

Figure 2. Sea lamprey-prey population dynamics at different sex ratios

From the figure, it is evident that as the proportion of females increases, the population size of the sea lamprey gradually decreases by the end of the simulation, consistent with previous statements. This may be due to the increased population carrying capacity and mortality rate caused by the rise in high-energy-demanding and larger-sized female individuals [6, 9]. However, the intersection point with the invasion line, defined as the population size reaching the invasion standard, moves forward progressively, indicating that the initial growth rate of the population increases with the increase in the number of females. This is the result of the combined effects of the high predation efficiency brought about by the increased female ratio and the gradual normalization of the gender ratio. It is also evident that the predatory benefits brought by the increased female ratio outweigh the decline caused by the dispersion from the normal gender ratio. For the prey population, it is clear that as the female ratio increases, the intersection point with the endangerment line, defined as the population size reaching the endangerment standard, also moves forward, indicating that the increase in the female ratio exerts greater pressure and threat on the prey, leading to its rapid extinction.

3. Analysis of other populations affected degree based on AHP-TOPSIS

In this section, an evaluation model based on AHP - TOPSIS was constructed to predict the possible ecological scenarios through the ecological network of lampreys.

3.1. Construction of index system and model

In order to comprehensively analyze the impacts on other populations in the ecosystem, four ecological scenarios are considered, and several representative influencing factors were selected as evaluation indicators in each scenario. They were used as criterion layer and index layer respectively, and the evaluation index system of sex ratio change was constructed according to this. For each single factor, we examined the effects of different female ratios in lamprey population and assigned values to different levels of its influence.

The values corresponding to the indicators of *Predation* and *ratio of competitors to collaborators* in Table.3 are calculated according to the previous model, but the rest are assigned by the actual situation. The larger the value, the better the scenario.

Table 3. Ecological scenarios evaluation criteria

Female ratios	Evaluation system for the impact on other populations									
	Food Web		Change of Ecological Niche		Parasite Transmission			Competition and Cooperation		
	Predation	Natural enemy	Species richness	Space utilization rate	Species quantity	Life cycle	Effects on the host	The ratio of competitors to collaborators	Competitor growth	Collaborator growth
0.35	0.2	5	3	5	5	1	1	0.6	1	5
0.4	0.5	4	4	4	4	2	2	0.8	2	4
0.5	1.7	3	5	3	3	3	3	1	3	3
0.55	2.2	2	4	2	2	4	4	1.2	4	2
0.6	3	1	3	1	1	5	5	1.5	5	1

3.2. AHP analysis and evaluation

The evaluation system includes target layer A, criterion layer B and index layer C. We evaluate the importance of indicators at each level and construct five judgment matrices: $A-B,\ B_1-C,\ B_2-C,\ B_3-C,\ B_4-C.$

According to the constructed judgment matrix, the maximum eigenvalue $\lambda_{\max}$ of each matrix is obtained, and the judgment matrix has good consistency. Meanwhile, we can get the weight vector ω_i of each judgment matrix through calculation.

3.3. Index determination of TOPSIS

According to the data in Table.1, we construct the initial evaluation matrices. The indicators under these scenarios are divided into benefit indicators and cost indicators. After the positive and standardized transformation of the evaluation matrices, the positive (D^+) and negative (D^-) ideal solution distances corresponding to each male ratio can be obtained.

We define $N_i=\frac{D_i^-}{D_i^+ + D_i^-}$ as the degree to which the sex ratio is close to the best scheme and construct the indicator proximity matrix. The following results are obtained.

$$N=\begin{bmatrix}0.44 & 0.40 & 0.52 & 0.53 & 0.56\\ 0.70 & 0.70 & 0.56 & 0.30 & 0\\ 0.42 & 0.44 & 0.50 & 0.57 & 0.58\\ 0.45 & 0.46 & 0.49 & 0.53 & 0.55\end{bmatrix} \tag{12}$$

3.4. Comprehensive evaluation

The weight value of judgement matrix $A-B$ obtained based on AHP is $\omega_1=[0.182 \quad 0.338 \quad 0.630 \quad 0.675]$. the paper define a new comprehensive evaluation index to calculate the threshold values corresponding to different ecological scenarios under different proportions of female lamprey. The calculation results are as follows.

$$G=\omega_1 N=[0.885 \quad 0.897 \quad 0.930 \quad 0.915 \quad 0.839] \tag{13}$$

From these calculations, we can see that lamprey populations offer the greatest advantage to other species in the ecosystem when the ratio of females is about 0.5.

The proportion of females under this index system is only an estimate. For the proportion of samples in the actual investigation, we can calculate the corresponding G value according to the actual proportion, so as to put it into the corresponding interval to judge the dgree of influence.

4. Conclusion

This paper conclusively demonstrates the differential impacts of sea lamprey on aquatic ecosystems through two distinct models. The differential equation model illuminates how changes in the sea lamprey's sex ratio influence population dynamics: as the proportion of females increases, the sea lamprey population size decreases by the simulation's end due to higher mortality rates from the energy demands of larger female individuals. However, initially, such decrease is offset by a faster initial growth rate and greater predation pressure on prey populations, as indicated by earlier intersections with critical population thresholds. Here, we need to clarify that due to the lack of relevant data to accurately assign the corresponding parameters in differential equations, we only set them to appropriate values to demonstrate how our model operates. Conversely, the AHP-TOPSIS framework broadens the focus to the ecosystem level, identifying a 0.5 female ratio as optimal for ecosystem balance. The framework excels in incorporating a wider range of criteria for a holistic assessment, though it navigates the challenge of subjective criteria weighting. Together, these models underscore the complexity of managing invasive species impacts, advocating for a nuanced, integrated approach that leverages the strengths and addresses the limitations of each modeling technique.

References

[1] Almeida P R, Arakawa H, Aronsuu K, et al. Lamprey fisheries: History, trends and management [J]. Journal of Great Lakes Research, 2021 (S1): 47.

[2] Gaden M, Brant C O, Lambe R. Why a Great Lakes Fishery Commission? The seven-decade pursuit of US fishery treaty [J]. Journal of Great Lakes Research, 2021, 47: S11 - S23.

[3] Heinrich J W, Mullett K M, Hansen M J, et al. Sea lamprey abundance and management in Lake Superior, 1957 to 1999 [J]. Journal of Great Lakes Research, 2003, 29: 566 - 583.

[4] Lewandoski S A, Brenden T O. Forecasting suppression of invasive sea lamprey in Lake Superior [J]. Journal of Applied Ecology, 2022, 59 (8): 2023 - 2035.

[5] Hume J B, Bravener G A, Flinn S, et al. What can commercial fishery data in the Great Lakes reveal about juvenile sea lamprey (Petromyzon marinus) ecology and management? [J]. Journal of Great Lakes Research, 2021, 47: S590 - S603.

[6] Docker M F, Beamish F W H, Yasmin T, et al. The lamprey gonad [M]//Lampreys: Biology, Conservation and Control: Volume 2. Dordrecht: Springer Netherlands, 2019: 1 - 186.

[7] Lennox R J, Bravener G A, Lin H Y, et al. Potential changes to the biology and challenges to the management of invasive sea lamprey Petromyzon marinus in the Laurentian Great Lakes due to climate change [J]. Global change biology, 2020, 26 (3): 1118 - 1137.

[8] Marsden J E, Siefkes M J. Control of Invasive Sea Lamprey in the Great Lakes, Lake Champlain, and Finger Lakes of New York [J]. 2019.

[9] Hume J B, Bravener G A, Flinn S, et al. What can commercial fishery data in the Great Lakes reveal about juvenile sea lamprey (Petromyzon marinus) ecology and management? [J]. Journal of Great Lakes Research, 2021, 47: S590 - S603.

[10] Johnson N S, Buchinger T J, Li W. Reproductive ecology of lampreys [J]. Lampreys: Biology, Conservation and Control: Volume 1, 2015: 265 - 303.

[11] Adams J V, Birceanu O, Chadderton W L, et al. Trade-offs between suppression and eradication of sea lampreys from the Great Lakes [J]. Journal of Great Lakes Research, 2021, 47: S782 - S795.

[12] Wilkie M P, Johnson N S, Docker M F. Invasive species control and management: The sea lamprey story [M]//Fish Physiology. Academic Press, 2022, 39: 489 - 579.