

Comprehensive Rescue Model for Submersible Based on Monte Carlo Analysis and Genetic Algorithm

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Abstract. In recent years, there has been a growing interest in deep-sea exploration activities, yet significant gaps persist in search and rescue methodologies for deep-sea submersibles. This paper introduces a position prediction and search model for deep-sea submersibles, leveraging Monte Carlo analysis and genetic algorithms for optimization. The underwater motion of the submersible is simulated through dynamic system modeling. Monte Carlo analysis is utilized to model the scattering of the submersible within the specified ranges of longitude [10.25, 20.25], latitude [20.00, 36.50], and depth [2500m-4000m]. The analysis reveals a normal distribution of scattering points from the middle to the end. Genetic algorithms are employed to refine search and rescue paths, iterated 1000 times to identify the optimal path spanning from 540.1km to 225.6km. In comparison to complex numerical methods, this model offers ease of programming and implementation while accurately predicting the submersible's position in three-dimensional space. It provides the most efficient search and rescue route, ensuring swift response in the face of improbable communication or mechanical failures.

Keywords: Dynamic System Modeling, Monte Carlo Simulation, Bayesian Search Theory, Genetic Algorithms.

1. Introduction

The disappearance of the submersible Titan precipitated an urgent search and rescue operation, prompting a reassessment of longstanding safety concerns. The challenge faced by U.S. and Canadian authorities in locating the 6.7-meter-long submersible in an expansive area twice the size of Connecticut and at a depth of four kilometers was compounded by the absence of an emergency radio beacon or a dedicated recovery system[1]. Deep-sea manned submersibles are susceptible to various potential challenges, including loss of communication with the main vessel and mechanical failures such as propulsion loss. The cornerstone of enhancing safety protocols lies in the development of a predictive model capable of accurately tracking the submersible's position over time and determining the optimal search path.

The establishment of a search and rescue decision system holds paramount importance for underwater activities such as submarine sightseeing projects. The increasing prevalence of autonomous underwater vehicles (AUVs) in domains like environmental monitoring, rescue operations, and cluster surveys underscores their distinct advantage in search and rescue missions [2, 3]. AUV path planning involves strategizing the trajectory of the AUV to achieve specific objectives based on provided maps and target points [4]. This meticulous planning is instrumental in ensuring the efficacy and efficiency of search and rescue endeavors in challenging underwater environments.

K. Szwaykowska presents a paper on a feedback-controlled Lagrangian particle tracking algorithm for target tracking at sea [5]. Liu used an improved sparrow search algorithm combining cubic chaotic mapping and Gaussian random walk strategy for path planning [6]. Ai proposed a spatial-temporal SAR region task assignment algorithm based on Genetic Simulated Annealing Algorithm (GSAA) [7]. Xiong used differential evolution (DE) and non-dominated sorting Genetic algorithm -II (NSGAI) to plan the optimal search path [8]. Zhao and Guo used Monte Carlo analysis to predict submarine positions based on the operational requirements of aerial submarine hunting [9, 10]. Xie proposed a target positioning and tracking method based on multiple submersibles [11]. Tu used field experiments combined with Monte Carlo simulations to determine the drift characteristics of humans

in the South China Sea water [12]. Based on the background of maritime search and rescue (SAR), Wu proposed an autonomous coverage path planning algorithm based on deep reinforcement learning for search and rescue of drowning people at sea [13]. Qiu proposes a navigation estimation method based on current prediction models and convolutional neural networks-long short-term memory hybrid neural networks to predict the position of underwater gliders [14]. Wang proposed an underwater three-dimensional path planning algorithm based on adaptive step cuckoo search and particle swarm optimization fusion algorithm [15]. Zhou proposed a path planning approach based on an optimized sparrow search algorithm for underwater vehicles [16]. In the mentioned model algorithms, the Monte Carlo algorithm and genetic algorithm are mainly used for searching targets on the sea surface, but there is a lack of research on locating underwater lost submersibles. The current model is primarily based on the assumption of smooth communication between the submersible and the mother ship, with limited investigation into the search and rescue of lost submersibles, and a deficiency in intelligent algorithmic decisions to reduce mission response time.

We utilizes advanced 3D seabed terrain modeling technology and position prediction techniques based on the Markov chain Monte Carlo (MCMC) method, in conjunction with genetic algorithms, to optimize the Traveling Salesman Problem (TSP) model. This optimization aims to provide an enhanced search route for search and rescue vessels, thereby minimizing search and rescue duration and increasing the likelihood of successful rescue operations.

The second section of this paper primarily outlines the methodology and principles employed in constructing the algorithmic model, while the third section focuses on presenting the positioning and search outcomes derived from the algorithmic model.

2. Construction method of submersible rescue model

2.1. Model condition simplification

In order to facilitate the assumption of power model, this paper assumes that the submersible is only affected by vertical and horizontal directions in water, and does not consider the influence of various forces on the inclined direction.

In addition, it is assumed that the communication device data, transmission data time is accurate and error-free. All data transmissions through the communication device are instantaneous, without delay, and without data loss, errors, or variations in transmission.

2.2. Dynamic systems theory model

Dynamic system theory is used to model the motion of the submersible, which includes the position and velocity of the submersible as state variables. The dynamic system can be expressed as a set of differential equations describing the changes in state variables over time.

2.2.1. Description of Dynamic systems theory model

Assuming that the submersible moves in a three-dimensional plane, its motion can be shown by the following differential equation. By analyzing the force on the submersible in the ocean, the trajectory of the submersible in the ocean is found and analyzed, simulated by applying the dynamic system theory. The force analysis of submersible is shown in Figure 1.

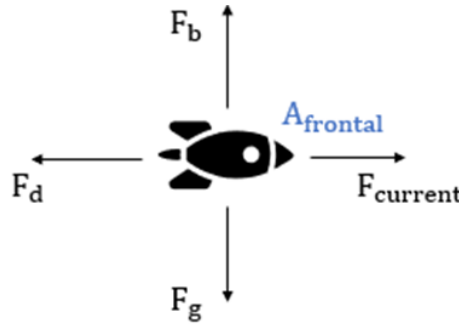


Figure 1. Force analysis diagram of the submersible

2.2.2. Force analysis formula

$$r(t) = (x(t), y(t)) \quad (1)$$

The above equation represents the position of the submersible in the horizontal plane, where $x(t)$ and $y(t)$ are the distances between eastward and northward directions, respectively.

2.2.3. Calculation of forces

(a) Gravity F_g and buoyancy F_b :

$$F_g = m \cdot g \quad (2)$$

$$F_b = \rho_{\text{water}} \cdot V \cdot g \quad (3)$$

(b) Water resistance F_d :

For the horizontal motion of the submersible, the water resistance can be expressed as:

$$F_d = \frac{1}{2} \cdot \rho_{\text{water}} \cdot C_d \cdot A_{\text{frontal}} \cdot |V_{\text{sub}} - V_{\text{current}}(z(t))|^2 \quad (4)$$

V_{sub} is the velocity vector of the submersible relative to the water.

(c) Current force F_{current} :

In fact, in our model, the currents do not exert a direct force, but affect the motion of the submersible by changing its effective velocity. The actual velocity of the submersible is the vector sum of its velocity relative to the water and the velocity of the current.

2.2.4. Movement direction

The equation of motion of the submersible in the horizontal direction can be expressed as:

$$\frac{dr(t)}{dt} = V_{\text{sub}} + V_{\text{current}}(z(t)) \quad (5)$$

Included, $r(t) = (x(t), y(t))$ is the position vector of the submersible, V_{sub} is the velocity vector of the submersible relative to the water, $V_{\text{current}}(z(t))$ Current velocity vector at depth $z(t)$.

The equations of motion of a submersible in the vertical direction can be simplified to its dive and ascent speeds, which depend on the submersible's own propulsion system and control strategy.

2.3. Position Prediction Technology Based on Markov Chain Monte Carlo (MCMC) Method

Vertically, the submersible was found to be vertically balanced by its own gravity and buoyancy as well as by the submersible's own horizontal thrust.

In the horizontal direction, it is balanced by water resistance and current forces as well as the horizontal thrust of the submersible itself. Uncertainty in the prediction arises due to the various differences caused by water density, water resistance, and current forces that are encountered during the dive.

The steps of uncertainty analysis and Monte Carlo analysis are shown in Figure 2.

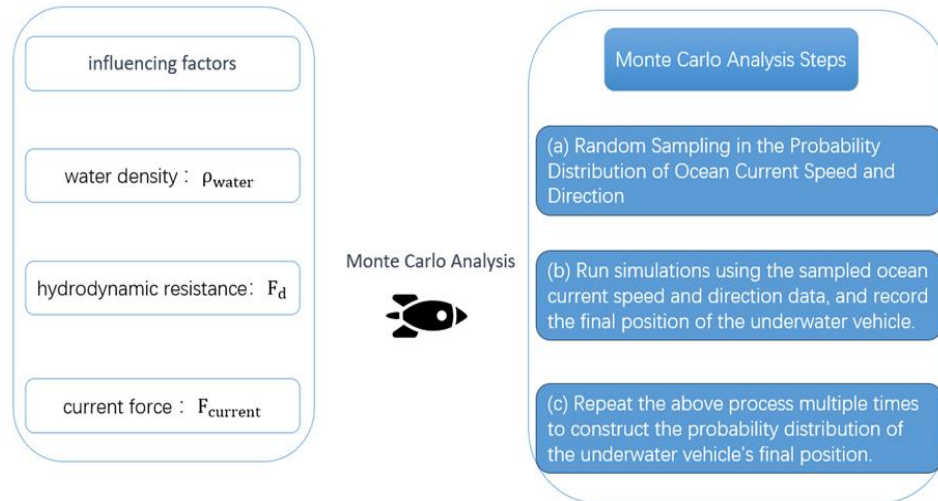


Figure 2. Uncertainty factor analysis and Monte Carlo analysis steps

2.3.1. Uncertainty analysis

(a) Density of water (ρ_{water}): The density of water varies with depth. Deep water layers are usually denser than surface water. This variation may cause the submersible to encounter different water density layers during the dive, which may affect its movement and stability.

(b) Water resistance(F_d): Submersibles will be affected by water resistance during dives, which is related to the shape and speed of the submersible. Submersibles of different shapes and velocities will experience different levels of water resistance, which can lead to uncertainty in predictions.

(c) Current force(F_{current}): The direction and strength of currents may change at different depths and locations. The influence of currents on a submersible during a dive may lead to uncertainty in its trajectory.

To perform an uncertainty analysis, we can use Monte Carlo simulations to evaluate the effects of changes in current speed and direction on the final position of the submersible. In this process, we consider uncertainties in current speed and direction and estimate the effect of these uncertainties on the path of the submersible through multiple simulations.

2.4. Bayesian search theory:

When the submersible has been lost, a search development model is created to set initial deployment points and equipment search patterns to minimize the time to find the missing submersible. Therefore, this research assume that the submersible is a fixed point to facilitate the hypothetical search, and use the information provided by the positioning model to design a search strategy to minimize the search time.(e.g., water density, current forces, fluid dynamics, etc.) and the predicted likely location of the vehicle, update the probability distribution of the search area using Bayesian search theory, optimize search paths and initial deployment points, design search modes as spiral search, parallel search, and fan search modes to predict the submersible location.

2.4.1. Probability distribution model

Let the search area be divided into n networks, and let the prior probability of a diver in each network i be. Use Bayes' theorem to update the probability distribution:

$$P(A_i|B) = \frac{P(B|A_i)P(A_i)}{P(B)} \quad (6)$$

A_i Indicates that the submersible is located in network I , indicates observation information, $P(A_i|B)$ is the posterior probability that the diver is located in network i given the observation B .

2.4.2. Search strategy

(a) Selecting the initial deployment point: the grid with the highest posterior probability is selected as the initial search point.

(b) Determine the search mode, based on the fixed submersible, and the characteristics of the sea environment (such as water density, current force, hydrodynamics, etc.), we decided to choose the spiral search mode as our search mode.

2.5. Time Cumulative Distribution Function Model

To the above model, this research add the determination of the probability of finding a submersible as a function of time and cumulative search results.

Assuming that the probability $P(t)$ denotes the probability of finding the submersible at time t , which can be viewed as a function of the search efficiency, the search coverage, and the movement probability of the submersible.

$$P(t) = 1 - e^{-\omega \cdot A(t)} \quad (7)$$

Where $P(t)$ is the probability of finding a submersible at time t ; ω is a parameter related to the search efficiency, indicating the average probability of searching for a submersible per unit area; and $A(t)$ is the cumulative coverage of the area that has been searched up to time t , which can be an increasing function of time.

2.6. Optimal path model based on genetic algorithm analysis

In the above Bayesian theory, this article have found the optimal pattern from the two-dimensional plane as the spiral search pattern, next the research will assume that the departure will be made from a certain master ship, and then assume that there are five master ships in the vicinity of the lost submersible, and then if they receive the information that the submersible is lost, they will start to actively set out to find the lost submersible. The system we set up will be penalized from a polynomial time algorithm, described by applying graph theory: given a graph where nodes represent nearby master ships and edges represent the distance that each master ship differs from each other, the task is to find a minimally weighted Hamiltonian loop, which we express mathematically is:

$$\min \sum_{i=1}^n \sum_{j=1, i \neq j}^n D_{ij} x_{ij} \quad (8)$$

Where x_{ij} is a decision variable if main ship i to main ship j is in the solution, $x_{ij} = 1$ and otherwise is 0.

In order to solve this problem, we use genetic algorithm to solve the problem, the specific steps of genetic algorithm are as Figure 3:

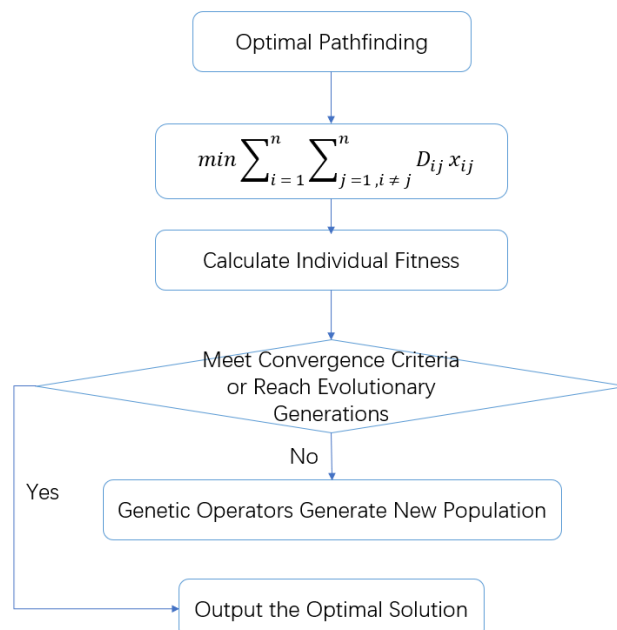


Figure 3. Genetic Algorithm Program Diagram

3. Results

3.1. Dynamic systems theory model

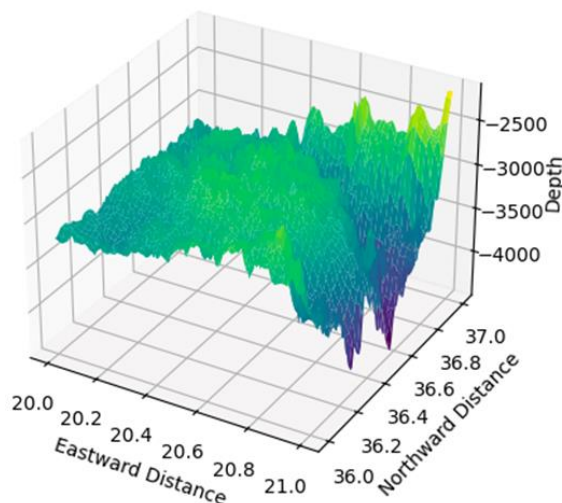


Figure 4. Submersible path prediction chart

Through the analysis of the dynamic system theory model mentioned above, combined with the modeling of the ionian seabed terrain data, we use matlab to obtain the 3D path map of the submersible moving with time. The Figure 4 above shows the path of the submersible subjected to currents over the course of one hour of the simulation, and it represents the time it takes to dive to [2500,4000] meters and return to the surface, updating the submersible's position throughout the process.

3.2. Monte Carlo analysis

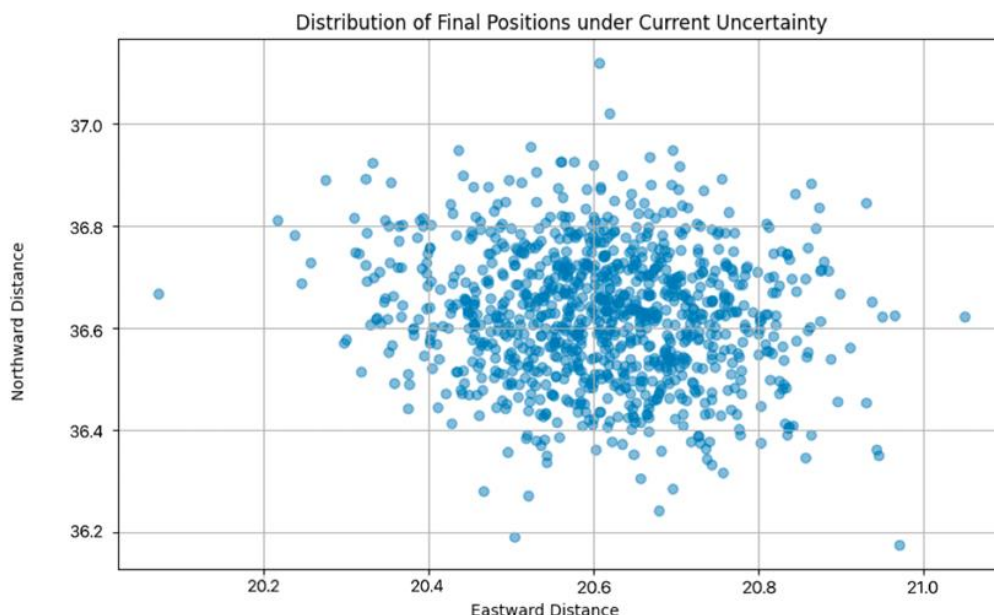


Figure 5. Predictive map of submersible location distribution

We combine the 3D topographic map of the Ionian Sea seabed with Monte Carlo algorithm and matlab to generate the final position distribution under the current uncertainty. The Figure 5 shows the distribution of the final position of the submersible obtained by Monte Carlo simulation taking into account the uncertainty in the current speed and direction. We simulated the scattering of the submersible in the range of longitude [10.25,20.25] latitude [20.00,36.5] depth, [2500m-4000]m.

In this study, MCMC method is applied to simulate the potential location of the lost submersible, and the probability distribution of its location is calculated, which provides key data support for the search and rescue operation.

3.3. Bayesian search theory

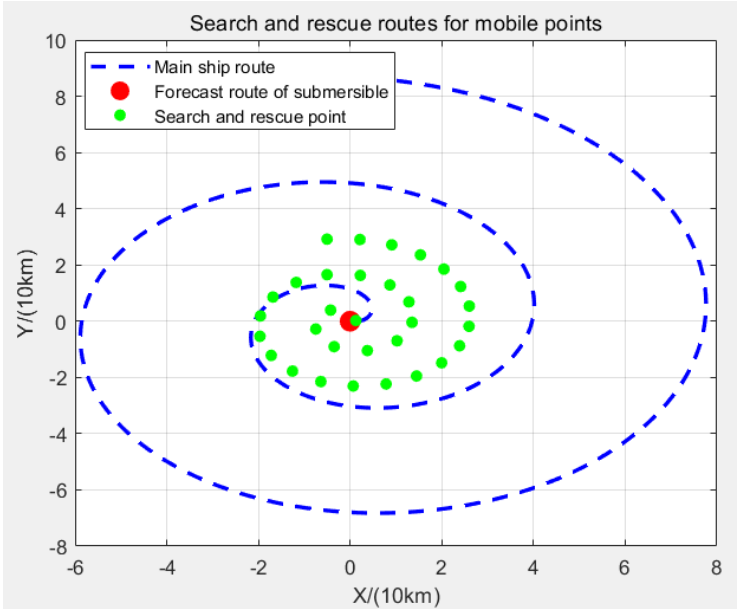


Figure 6. Spiral Search and Rescue Dynamics

We use Bayesian algorithm and matlab to generate the search and rescue path of moving points. The Figure 6 above then shows a grid in which our main vessel makes a departure from the one above, we assume an origin as the predicted location of the lost submersible, and then spiral search from the time of maximum a posteriori probability, the blue dotted line indicates the main vessel movement path, and the blue main vessel, in order to reduce the probability of failure of the search, waits to start fishing and searching until it enters into the range of higher probability, i.e., our green dots, thus increasing the probability of fishing. Shorten the fishing time, however, the above figure is only a simple two-dimensional model, and can not really search for stationary submersibles, the next we will be from the three-dimensional model, specifically to analyze.

3.4. Time Cumulative Distribution Function Model

Table 1. Table of Time Cumulative Distribution Function Model

Time (t/min)	A(t)	P(t)
1	0.00473	0.00995
2	0.00946	0.01980
3	0.01419	0.02955
4	0.01892	0.03921
5	0.02365	0.04877
...		
207	0.97911	0.78325
208	0.98384	0.79712
209	0.98857	0.80912
210	0.99333	0.81316
211	0.99803	0.82368

Based on the analysis of the time cumulative distribution function model, we generate Table 1. The time cumulative distribution function model is shown in Table 1.

3.5. Optimal path model based on genetic algorithm analysis

We do this by setting some parameters of the genetic algorithm, including population size, crossover probability, mutation probability, and so on. We read our data as the coordinates of the main ship of each large ship for 1000 iterations, Crossover and mutation operations are performed on the selected individuals to generate a new generation of individuals. The newly generated individuals are substituted for the original individuals to form a new population. The optimal path to find the stationary submersible is updated each time and recorded.

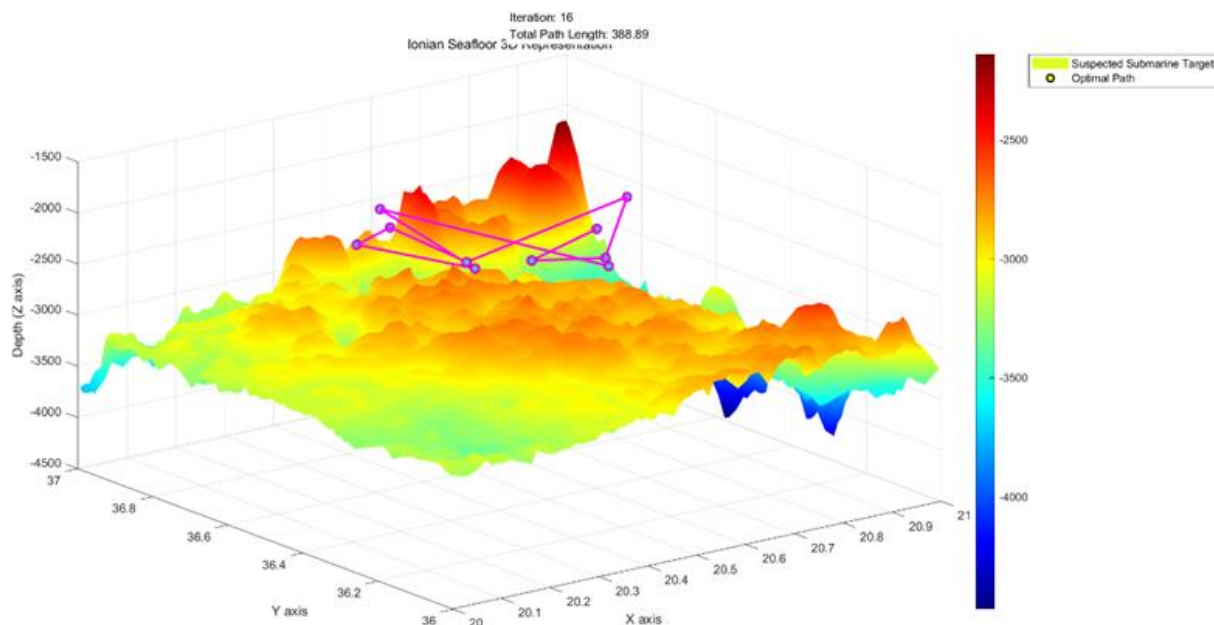


Figure 7. Shortest Path Optimization graph 1

The search path with a total path length of 388.89km after iteration is shown in Figure 7.

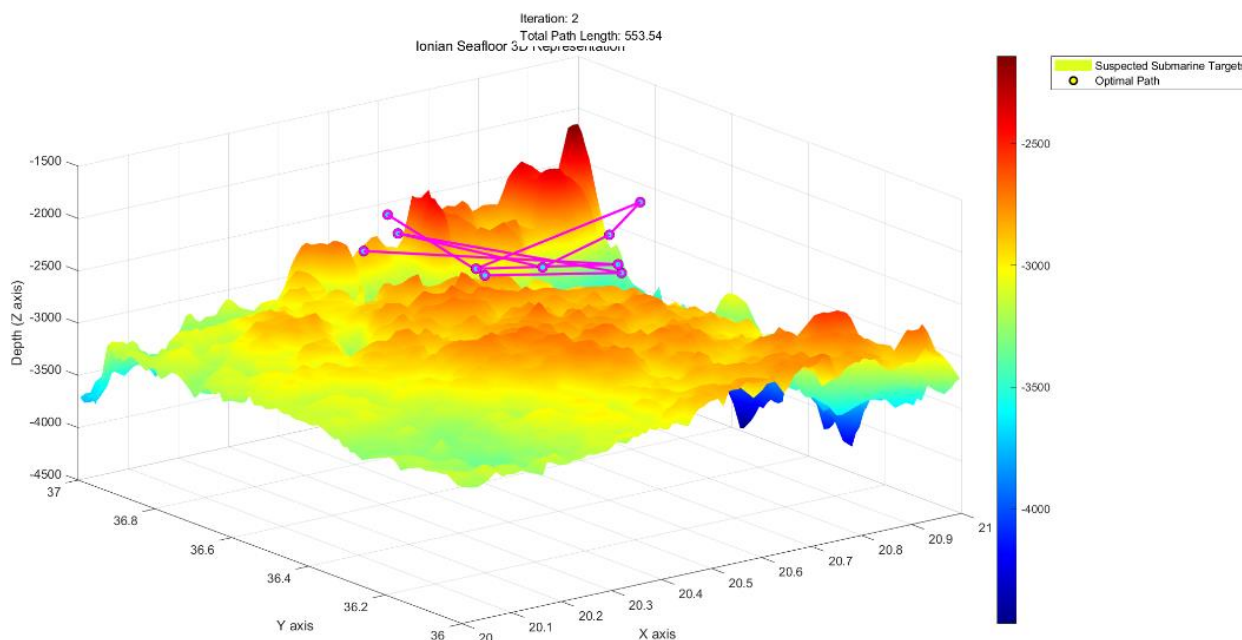


Figure 8. Shortest path optimization graph 2

The search path with a total path length of 553.54km after iteration is shown in Figure 8.

Finally we carry out the initial iteration, the beginning of the path to find the boat is 540.1km to the final iteration of 40 times after the 225.69km, and finally until the iteration of 1000 generations to find the optimal path of the boat remain unchanged, and the search for the maximum reduction in the time to find the lost submersible, but also the optimal path. The above demonstrates that our model

can be used not only for single submersible search and rescue but also for multiple submersible search and rescue.

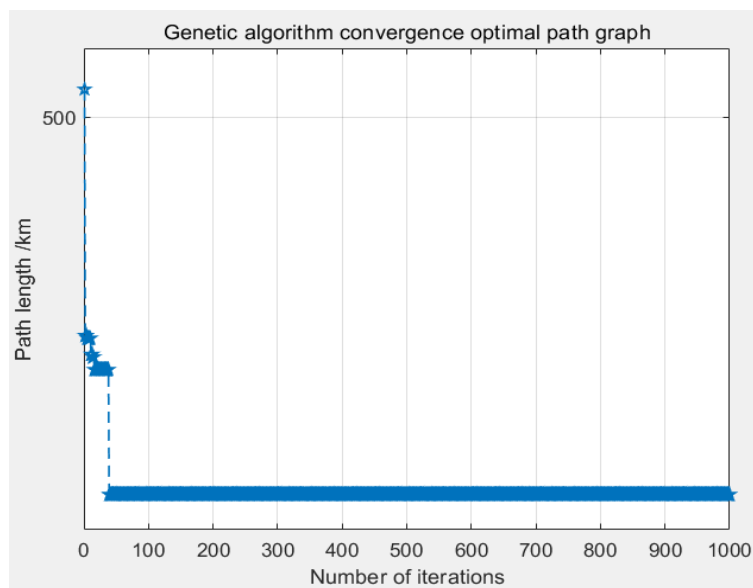


Figure 9. Shortest path iteration effect diagram

The process of obtaining the optimal path through genetic algorithm iteration is shown in Figure 9.

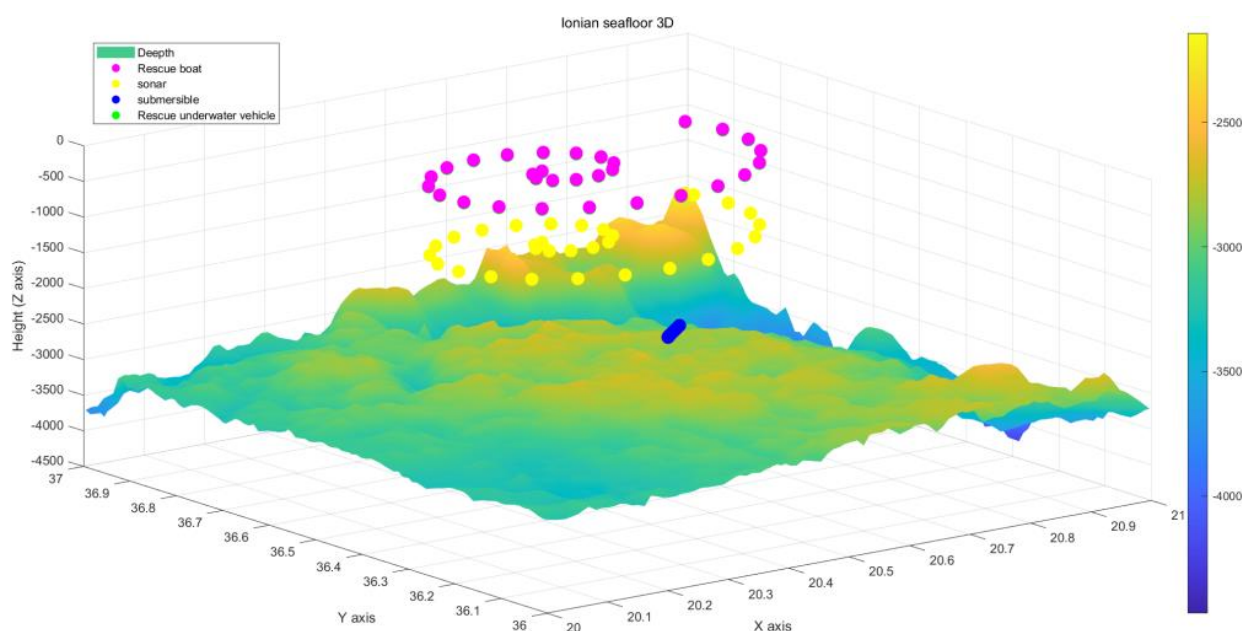


Figure 10. Joint Search and Rescue Chart under multiple submersibles

The path diagram of multi-submarine joint search and rescue is shown in Figure 10.

This paper primarily employs Monte Carlo algorithms and genetic algorithms for the search, localization, and retrieval of a submersible. The Monte Carlo algorithm is characterized by its versatility and flexibility, independent of the specific problem formulation. It finds application across a spectrum of mathematical, physical, and engineering challenges, particularly adept at addressing complex problems that resist analytical solutions. Notably, Monte Carlo algorithms stand out for their relative ease of programming and implementation, relying primarily on random number generation and straightforward statistical analysis.

On the other hand, genetic algorithms exhibit robust global search capabilities, enabling exploration across the search space to circumvent local optima, making them well-suited for tackling intricate nonlinear problems. Our algorithmic approach involves the concurrent evaluation of multiple

solutions, harnessing parallelism to expedite convergence towards an optimal or near-optimal solution.

Despite these strengths, our algorithm model is not without limitations. Monte Carlo methods, for example, often exhibit slow convergence, especially when seeking exact solutions, necessitating a significant number of random samples to achieve the desired precision. While genetic algorithms excel at avoiding local optima, they may converge more gradually compared to algorithms tailored for specific problem domains.

4. Conclusion

To address unexpected scenarios involving submersibles in deep-sea settings, such as communication breakdowns, mechanical issues, or propulsion failures, we have developed a submersible rescue model that combines Monte Carlo analysis and genetic algorithms. By utilizing detailed seabed topographic data from GEBCO2023, this study constructs a precise 3D topographic model of the Ionian Sea region. The MCMC method is employed to simulate the likely location of the missing submarine and calculate the probability distribution of its position, providing crucial data for search and rescue operations. Subsequently, a genetic algorithm is used to identify the shortest search and rescue path, enabling the unmanned submarine to swiftly locate each target.

Through the integration of various innovative techniques and approaches, this research presents a comprehensive and effective solution for undersea submarine rescue, demonstrating both theoretical innovation and practical applicability. Looking ahead, this model has the potential to be adapted for different oceanic environments and diverse submarine rescue missions, laying a solid foundation for multi-submarine operations, safety assessments, and regulatory compliance.

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