A study of submarine trajectories in the Ionian Sea based on difference equations

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Abstract. This paper addresses the problem of undersea submersible detection, aiming to find a method to determine the three-dimensional coordinates of a submersible within a certain error range. Firstly, the core of the problem is analyzed, and seven key factors affecting the motion path of the submersible are summarized, including initial velocity, drift velocity, current velocity and direction, water depth and topography, time factor, weather conditions, and other factors. Subsequently, a model to establish the equations of motion of the submersible and parameterized current data is proposed, and simulations and calculations are carried out by MATLAB. The effects of Coriolis force and drag on the motion of the submersible were further discussed and the equations of motion of the submersible were established using differential equations. Finally, a MATLAB program was designed for the simulation of the horizontal trajectory of the submersible, and the simulation results show the trajectory of the submersible after the loss of power.

Keywords: Submarine submersible, motion modeling, differential equations.

1. Introduction

The prediction of the motion trajectory of submarine submersibles is crucial in ocean exploration and rescue missions. The aim of this paper is to analyze the motion modeling of a submarine submersible and its solution after loss of power [1-2]. We provide an in-depth analysis of the problem and propose seven key factors, including initial velocity, drift velocity, current speed and direction, water depth and topography, time factor, weather conditions, and other factors. By modeling parametric current data, Coriolis force considerations, and differential equation modeling, we are able to accurately simulate the trajectory of a submarine submersible. The models and solutions presented in this paper provide important theoretical support and practical value for ocean exploration missions [3].

2. Models and Solutions

2.1. Submersible exploration model

2.1.1. Analysis of model

The core of question1 is to find the approximate three-dimensional coordinates of the submarine within a certain error. In order to code with this problem, we conclude the following 7 factors [4-5].

1. Initial velocity of the submarine: The initial velocity of the submarine before losing power is an important factor in determining its path of motion, which must be measured through submarine recording data, instruments or other reliable sources.

2. Drift speed of the submarine: After losing power, the drift direction and speed are affected by many factors, such as sea currents, water temperature, wind speed and output, etc. These factors are observed and analyzed to determine the drift direction and speed values.

3. Ocean current speed and direction: Knowing the speed and direction of the ocean current, you can calculate the drift direction and speed and determine the direction of search and rescue.

4. Water depth and topography: The effects of water depth and topography can be identified and measured through seabed topographic maps, underwater suspension equipment, etc. to infer the drift range and determine the search range.

5. Time factor: The time factor is an important factor in calculating the drift distance, because the longer the drift time, the farther the drift distance. It is necessary to determine the drift distance in a specific time period.

6. Weather conditions: Changes in weather conditions such as temperature, humidity, and waves will also affect the drifting speed and direction, which is very important for calculating the accuracy of the submarine's drifting path.

7. Other factors: It is also necessary to consider factors such as human factors (such as whether a life raft is used) and path instability caused by the submarine's possible drift or spread after the hull ruptures or other damage caused by external environmental water pressure.

When calculating the movement path of a submarine that has lost power, it is also necessary to conduct analysis and simulation based on the data collected from the actual situation, so as to accurately calculate the position, direction, speed and time of the submarine drifting, and finally determine the search and rescue area, strengthen the search and rescue direction, and maximize the greatly improve the probability of search and rescue success [6-7].

2.1.2. Model Visualization

We need to find the approximate three-dimensional coordinates of the submarine within a certain error. Therefore, we need to obtain the motion equation of the submarine, and obtain the motion trajectory information of the submarine through mathematical models and Matlab calculations. Based on the above ideas, we need to consider the stress of the submarine in the ocean. According to the above assumptions, the initial position of the submarine should be between 200 meters and 5,000 meters underwater, so the submarine is less affected by sea wind and turbulence effects. Since the mass and volume of the submarine are relatively large, we took into account the Coriolis force, the friction of water, the thrust of ocean currents on the submarine, buoyancy, gravity, and the pressure difference caused by the difference in seawater flow velocity on the upper and lower surfaces of the submarine. The picture of the lonian sea is shown in figure 1.

Figure 1. The picture of the lonian sea [8]

In other words, problem one is transformed into the establishment of the submarine motion equation, taking into account the comprehensive effects of various factors, and then using the iterative results of Matlab as corrections.

2.1.3. Ocean current data grid parameterization model

Undoubtedly the most important of these factors is the power provided by ocean currents to the submarine. However, when we rummaged through a large amount of data, we found that the speed and direction of ocean currents vary greatly with location, so the volume of parameter data about ocean currents is very large. In order to better parameterize the ocean current data and establish the corresponding differential equations, we propose the following grid processing model. Gridded image of the Ionian Sea is shown in figure 2.

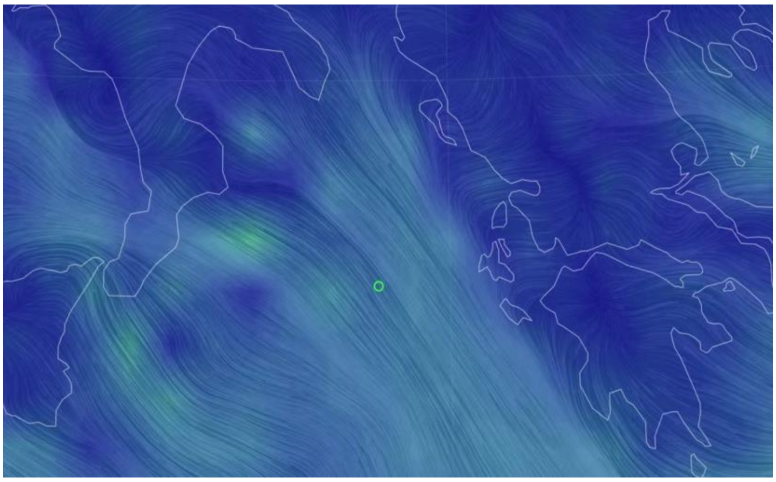

Figure 2. Gridded image of the Ionian Sea [9]

First, we consider the modeling of the water velocity field, assuming that the movement of water in a two-dimensional plane can be described by a two-dimensional vector field. Each point (x, y) can be expressed as v(x)=(υx(x),vy(x)), In this code, we use a piecewise constant vector field, that is, divide the plane into multiple rectangular grids, and assume that the water velocity is a constant vector on each grid. Therefore, we can divide the plane into $n_x \times n_y$rectangular grid, the water flow velocity of each grid is$v_{i,j} = (v_{x,i,j}, v_{y,i,j})$($i \in [0, n_x]$,$j \in [0, n_y]$).The velocity inside each grid can be regarded as a constant vector field, so we can pass the center point position (x_c, y_c) of the grid and grid size $\Delta_x$and Δ_y to calculate the water velocity at that point:

$$v_x(x_c, y_c) = \frac{v_{x,i,j}+v_{x,i+1,j}+v_{x,i,j+1}+v_{x,i+1,j+1}}{4}$$
$$v_y(x_c, y_c) = \frac{v_{y,i,j}+v_{y,i+1,j}+v_{y,i,j+1}+v_{y,i+1,j+1}}{4} \tag{1}$$

The greater the number of grids, the finer the sea area segmentation, and the closer the parameterized ocean current data is to the actual value. next, we consider the forces on the sunken ship. Assume that the mass of the submarine is m, the acceleration due to gravity is g, the location of the submarine is P= (x_p, y_p),the grid position of the submarine is(i, j), the horizontal and vertical velocities of the sunken ship are respectively $v_x$and v_y We need to calculate the total force on the submarine. F= (F_x, F_y).First, we consider the buoyancy force on the submarine F_b,which is related to the mass of the submarine and the water velocity, and can be expressed as:

$$F_b = mg - \rho V_0 v_p \cdot n \tag{2}$$

Among them,$V_0$is the volume of the submarine in the water, v_p is the speed of the submarine relative to the water, and n is the normal vector. In the code implementation, we can first calculate the buoyancy of the part within the grid where the sunken ship is located ($F_{b1}, F_{b2}F_{b3}F_{b4}$), and then Calculate the buoyancy of the grid where the sunken ship is located by weighted average:

$$F_b = \frac{(x_{p1}-x_p)(y_{p2}-y_p)F_{b1}+(x_p-x_{p0})(y_{p2}-y_p)F_{b2}+(x_{p1}-x_p)(y_p-y_{p0})F_{b3}+(x_{p1}-x_p)(y_p-y_{p0})F}{\Delta_x \Delta_y} \tag{3}$$

Next, we consider the thrust on the submarine (F_t). Thrust is related to the velocity of the water flow, and can be calculated by the thrust coefficients and the square of the water velocity. Specifically, it can be expressed as:

$$F_t = C_d A v_p^2 \tag{4}$$

Among them, v_p is the speed of the submarine relative to the current, and C_d is the thrust coefficient respectively (which can be calculated based on the shape of the submarine and hydrodynamic theory), and A is the reference area of the wreck.

2.1.4. The Coriolis force

In order to better solve this problem, here we have studied the relationship between Coriolis force and ocean current movement. First, we simply use the knowledge of linear algebra to derive the Coriolis force formula. First, we introduce the rotation matrix R:

$$R = \begin{pmatrix} \cos wt & \sin w\,t & 0 \\ -\sin wt & \cos w\,t & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R^{-1} = \begin{pmatrix} \cos wt & -\sin w\,t & 0 \\ \sin wt & \cos wt & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \dot{R} = \begin{pmatrix} -\sin wt & -\cos wt & 0 \\ \cos wt & -\sin wt & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \ddot{R} = \begin{pmatrix} -\cos wt & \sin wt & 0 \\ -\sin wt & -\cos wt & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{5}$$

Let the rotation angular velocity of the rotating reference frame be ω:

$$\vec{\omega}\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \tag{6}$$

Let the position vector of the stationary reference frame be r:

$$\vec{r} = \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \tag{7}$$

And the position vector of the rotating reference frame be r_1:

$$\vec{r_1} = \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix} \tag{8}$$

Then we can list the system of equations:

$$\begin{gathered} \begin{cases} \vec{r} = R \cdot \vec{r_1} \\ \vec{F} = R \cdot \vec{F_1} \end{cases} \\ \vec{F} = m \cdot \ddot{\vec{r}} \\ R \cdot \vec{F_1} = m \cdot \frac{d\left(\dot{R}\vec{r_1} + R\dot{\vec{r_1}}\right)}{dt} = m\left(\ddot{R}\vec{r_1} + 2\dot{R}\dot{\vec{r_1}} + R\ddot{\vec{r_1}}\right) \\ \vec{F_1} = mR^{-1}\ddot{R}\vec{r_1} + 2m\dot{R}R^{-1}\dot{\vec{r}} + m\vec{a_1} \\ m\vec{a_1} = -mR^{-1}\ddot{R}\vec{r_1} - 2m\dot{R}R^{-1}\dot{\vec{r}} + \vec{F_1} \\ m\vec{a} = m\omega^2\vec{r_1} + 2m\vec{\omega} \times \vec{v_1} + \vec{F_1} \end{gathered} \tag{9}$$

Among them, $m\omega^2\vec{r_1}$is the inertial centrifugal force, $2m\vec{\omega} \times \vec{v_1}$is the Coriolis force, $\vec{F_1}$ is the external force under the rotating reference frame, and $\vec{a}$is the acceleration under the rotating reference frame.

2.1.5. Establishment of submarine motion equations by differential equations

Overview: When simulating the movement of a submarine that loses power on the seabed, since the submarine is at a neutral buoyancy point at this time, the buoyancy and gravity are balanced, therefore, we do not consider the movement of the submarine in the vertical direction here, only Consider the motion state of the submarine in the horizontal direction. In order to facilitate the study of the problem, we now design a MATLAB program to simulate the movement of a submarine underwater.

Consideration and calculation of various factors:

1. Resistance: When a submarine moves in the water, it will be affected by resistance. The size of the resistance is related to the speed and shape of the submarine. Resistance force can be calculated by the following formula:

$$\overrightarrow{F_{drag}} = -\frac{1}{2} \cdot C_d \cdot A \cdot \rho_{water} \cdot |V_r| \cdot \widehat{V_r} \tag{10}$$

2. Coriolis force:

When a submarine moves at different latitudes, it will be affected by the Coriolis force. The Coriolis force is caused by the rotation of the Earth. Its size is related to the submarine's speed, ocean current speed, and latitude. The Coriolis force can be calculated by the following formula:

$$F_{coriolis} = 2 \cdot m \cdot |V_r| \cdot |V_c| \cdot sin\theta \cdot \hat{u} \tag{11}$$

3. Acceleration and velocity update: According to Newton’s second law, acceleration (a) is equal to the total force (F) divided by the mass (m):

$$a = \frac{F}{m} \tag{12}$$

The submarine's velocity (V) and position (P) can be updated by the following formula:

$$\begin{aligned} V_{new} &= V_{old} + a \cdot \Delta t \\ \mathrm{P_{new}} &= \mathrm{P_{old}} + \mathrm{V_{new}} \cdot \Delta \mathrm{t} \end{aligned} \tag{13}$$

2.2. Interpolation

(1) Point-taking process:

In the code, we have a set of longitude and latitude data points, as well as corresponding ocean current angle data (angleData) and ocean current speed data (speedData).

The current position of the submarine is represented by position, its longitude is position (1) and its latitude is position (2).

First, the code checks whether the submarine's position is within the range of the provided data. If it is not in range, an error will be raised.

Next, the code looks for the longitude and latitude data points closest to the submarine's position, finding four adjacent data points, namely (u_index, v_index), (u_index + 1, v_index), (u_index, v_index + 1) and (u_index + 1, v_index + 1).

(2) Interpolation process

Computes (u, v), which represent the relative position of the submarine's position relative to the closest longitude and latitude data points. (u, v) is calculated as follows:

$$\begin{aligned} \mathrm{u} &= \frac{\mathrm{position(1)-lon(u_{index})}}{\mathrm{lon(u_{index}+1)-lon(u_{index})}} \\ \mathrm{v} &= \frac{\mathrm{position(2)-lat(v_{index})}}{\mathrm{lat(v_{index}+1)-lat(v_{index})}} \end{aligned} \tag{14}$$

In this way, bilinear interpolation allows the angle and speed of the ocean current to be estimated based on the values of four adjacent data points and the position of the submarine, thus taking the effect of the ocean current into account in the simulation. The interpolation results are used to calculate the total force on the submarine, which in turn affects its trajectory [10].

In each time step, the above steps will be executed iteratively to simulate the horizontal motion trajectory of the submarine. Finally, by plotting the change in position over time, a map of the submarine's trajectory can be obtained. This simulation can help predict the trajectory of a submarine by understanding the effects of factors such as ocean currents, drag and the Coriolis force as it loses power in the ocean. Diagram of factors affecting submarine is shown in figure 3.

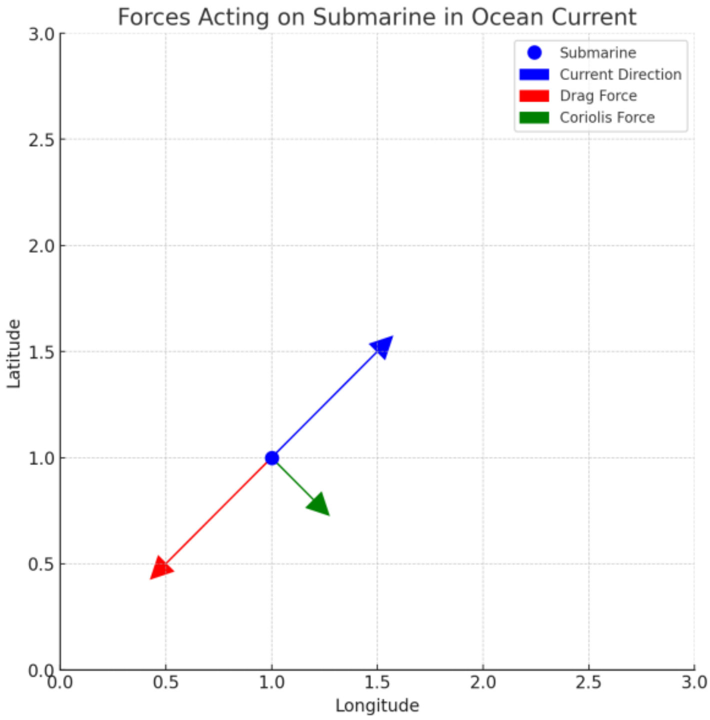

Figure 3. Diagram of factors affecting submarines

2.3. Programming

1. Initialize simulation parameters: set the total simulation time to 2 hours and the time step to 10 seconds.

2. Submarine initial data setting: In this code, we regard the shape of the submarine as a capsule, set the mass to 2500kg, and the volume to $20m^3$ (the data shows that the mass of a small submarine is about 2500kg, and the volume is about $20m^3$).

3. Set physical parameters: We defined some physical parameters, including seawater density (rho_water), submarine volume (volume), submarine mass (mass), drag coefficient (Cd), maximum diameter of the capsule (diameter), capsule transverse direction Cross-sectional area (A) and the angular velocity of the Earth's rotation (omega).

4. Ocean current angle and speed data: We provide 461 groups of longitude (longitude) and latitude (latitude) data points in the Ionian Sea, as well as the corresponding ocean current angle (current Angles) and ocean current speed (current Speeds) data.

5. Construction of query model: Since the submarine is in motion, the ocean current data moving to different locations will be different. Therefore, we need to build a model and let the submarine query the longitude and latitude corresponding to the current moment every time step. Ocean current speed and angle, and update the data. Since the ocean current data is an isolated point, considering the accuracy of the data when querying, we use the bilinear interpolation method, taking the four points closest to the current position as the basis for interpolation processing. In this way, the ocean current data at that point can be calculated more accurately.

6. Simulation process: Use loop iteration to simulate the movement trajectory of the submarine. At each time step, perform the following steps:

a. Query real ocean current data: According to the current position of the submarine, use the query Real Current Data function to query the real ocean current angle and speed.

b. Calculate the current ocean current velocity vector: Use the queried ocean current angle and speed to calculate the current ocean current velocity vector.

c. Calculate the total force: consider factors such as drag, Coriolis force, and ocean currents, but exclude gravity and buoyancy. The calculate Total Force function calculates the total force.

d. Calculate acceleration: Divide the total force by the mass to get the acceleration.

e. Update speed and position: Update speed and position based on acceleration.

f. Store the current position: Store the updated position in the positions array.

7. Draw the trajectory: After the simulation is completed, use the plot function to draw the movement trajectory of the submarine. The abscissa is longitude and the ordinate is latitude. Add axis labels and titles, and open the grid.

3. Conclusion

This paper presents a model and solution for the motion of a submarine submersible after loss of power. By analyzing seven key factors, including initial velocity, drift velocity, current speed and direction, water depth and topography, time factor, weather conditions and other factors, a parametric current data model is established, taking into account the Coriolis force and differential equations. These methods can accurately simulate the motion trajectory of submarine submersibles, providing important theoretical support and practical value for ocean exploration missions.

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