

Research On Underwater Search Strategy for Wrecked Submarine Based on Monte Carlo Simulation Algorithm

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Abstract. In recent years, visiting undersea relics by submarine as a new type of tourism program has a high risk. In order to improve the safety of the submarine, an effective simulation is carried out for the submarine that loses power, and a reasonable method to optimize the search mode is proposed. In this paper, this paper consider a submarine that has lost power, which is subject to its own gravity by the buoyancy of the water and the resistance generated by the relative motion of the ocean current and the submarine, so this paper establishes and solves the differential equations of motion of the wrecked submarine in three-dimensional space according to Newton's second law. Also considering that the state of the submarine at the time of failure is unknown, Monte Carlo simulation is used to randomly sample 1000 sets of initial velocities and positions around a number of famous undersea sites and simulate their trajectories, so as to obtain the probability density distribution of the submarine's position at any given moment, and to plot the 3D and 2D images of the probability density distribution of the submarine's position at the end of each day. In the process of searching for submarines against multiple warships, this paper adopts a greedy search algorithm, which integrates the traveling speed of the searching ships, the search speed and the success rate of searching unit area. At the beginning of each day, the warships select the most probable submarine location based on Monte Carlo simulation and travel to that location for search. The search program has a high probability of finding the lost submarine within 10 days.

Keywords: Ocean Current, Newton's Second Law, Differential Equation, Runge-Kutta Algorithm, Monte Carlo Simulation.

1. Introduction

With the development of modern technology, the way people travel is no longer confined to one form. Travel methods have diversified, sometimes even flying into space or diving into the deep sea. The company wants to capitalize on this opportunity by developing a manned submarine that can travel to the deepest parts of the ocean. This submarine can be launched without being connected to a host ship. In this way, the purpose of taking travelers to explore the Ionian shipwrecks at the bottom of the sea is realized. However, due to the many dangers associated with deep sea travel, such as loss of communication or loss of power of the submarine[1]. Therefore, in order to improve the safety of the submarine, this paper want to develop a model. This model can predict the exact location of the submarine over time to ensure the safety of the travelers. At present, scholars have done less research on how to effectively search for submarines that have failed on the seafloor, so in this paper, according to Newton's second law, this paper use the Runge-Kutta algorithm to establish and solve the differential equations in three-dimensional space[2]. Then this paper uses Monte Carlo simulation [3] to get the probability density distribution of the submarine position at any moment. Finally, three warships are sent to select the most probable submarine location based on the Monte Carlo simulation and go to that location to search for it[4].

2. Data

By analyzing the problem, it is clear that the oceanographic data of the Ionian Sea as well as the basic data of the submarine are what this paper need to know. Table 1 shows the main data and their sources

Table 1 Sources of Data

Database Names	Database Website
Ocean current	https://earth.nullschool.net/
Seafloor topography	https://download.gebeo.net
Longitude & Latitude	www.google.com.hk/maps
Height above sea level	www.google.com.hk/maps

3. Monte Carlo-based search program for unpowered submarines

3.1. Preparation of the model

3.1.1 Seafloor topography

The Ionian Sea lies between 34-40 degrees north latitude and 15-22 degrees east longitude and covers an area of approximately 160,000 square kilometers. It is part of the Mediterranean Sea and is located at the intersection of the African, Asia-Europe and Asia Minor plates. This place is due to the subsidence of the earth's crust, which has resulted in the formation of sea basins. It is also characterized by the formation of topography such as trenches and ridge seabed, which results in large differences in elevation.

In order to visualize the topography of the Ionian Sea, this paper use Python to visualize the elevation and draw a topographic map as shown in Figure 1.

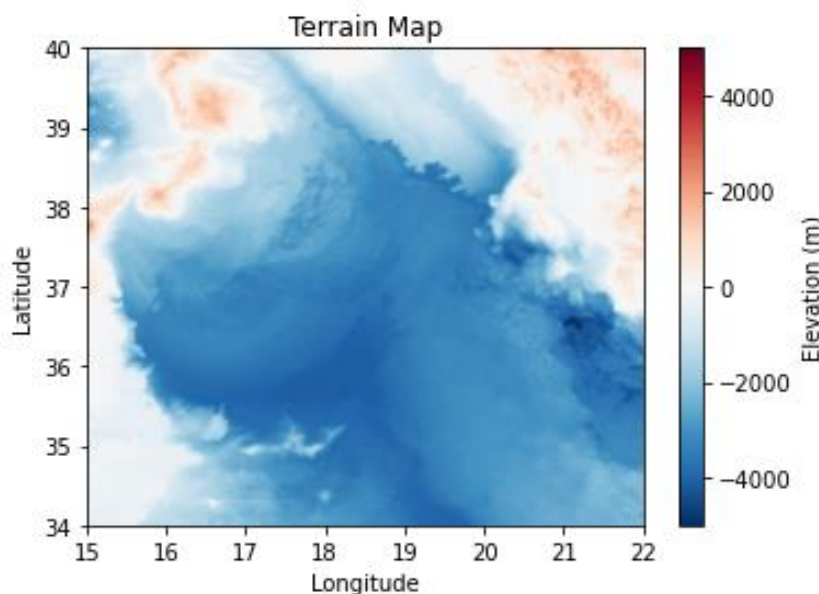


Figure 1 Ionian Sea terrain

3.1.2 Submarine modeling

Firstly, in order to calculate the magnitude of the four forces on the submarine, this paper need to process the required data computationally. Since the submarine under study is used as a small manned tourist. Thus this paper can approximate this submarine as an ellipsoid of diameter(as shown in Figure 2 below) with each of the three axis lengths as follows:

$$\begin{cases} a(\text{Axis 1 length of the ellipsoid}) = 3 \text{ m} \\ b(\text{Axis 2 length of the ellipsoid}) = 3 \text{ m} \\ c(\text{Axis 3 length of the ellipsoid}) = 6 \text{ m} \end{cases} \quad (1)$$

At this point the volume of the ellipsoid is $V = \frac{4}{3} \pi abc$.

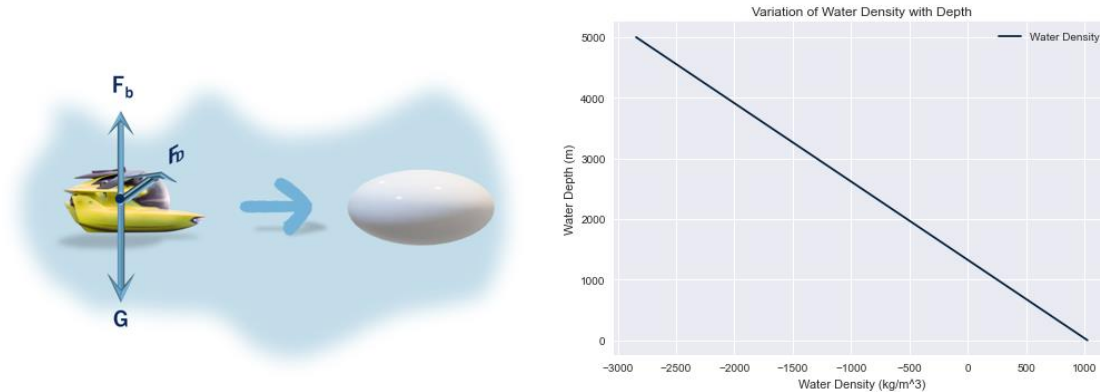


Figure 2 Submarine approximated as an ellipsoid.

3.1.3 Buoyancy

The main working principle of a submarine is to control its up and down movement by utilizing the buoyancy force it receives in seawater. One of the main determinants of buoyancy is the density of the liquid. So this paper need to find out the exact density of the water in the Ionian Sea.

This paper found that the density of seawater is affected by a number of factors, such as salinity, temperature, depth, etc., by reviewing the literature [5]. The equation for the variation of density is shown below:

$$\rho(h) = \rho_o(1 - \beta \cdot t \cdot h) \quad (2)$$

Where:

- ρ_o denotes the density of surface seawater, which is affected by salinity and temperature
- t denotes the seawater temperature
- h denotes the absolute value of seawater depth
- β denotes the sensitivity of temperature to density change

However, since this company operates in the Ionian Sea, the basic characteristics of seawater in this area are more or less the same. By looking up the data, it is found that the density of surface seawater ρ_0 in this sea area is 1025 kg/m^3 , βt is taken as 0.0002 km^{-1} , and the temperature of seawater is about 10 degrees Celsius on average. After bringing in the data, it can be found that the density of seawater at this time is only affected by the diving depth of the submarine. After further exploring the relationship between seawater density and water depth, it can be seen that the deeper the water, the greater the density.

At this point, this paper know the volume of the submarine and the density of the sea water. Next, this paper bring the data into $F_b = \rho g V$ and calculate the amount of buoyant force on the submarine.

3.1.4 Forces of ocean currents on submarines

Finally, the resistance of the current is more pronounced due to the fluidity of seawater, especially in the presence of ocean currents. So this paper have to take into account this force that has a pronounced effect on the submarine. The force given to the submarine by the ocean current is essentially a frictional force, but unlike other frictional forces, the force of the ocean current has a propulsive effect on the submarine. A review of the literature shows that the formulas are as follows:

$$F_D = \frac{1}{2} \rho C_D A (\vec{u} - \vec{v})^2 \vec{e} \quad (3)$$

where:

- C_D denotes the drag coefficient
- A denotes surface area of an ellipsoid
- \vec{u} denotes water velocity

- \vec{v} denotes vessel velocity
- \vec{e} denotes unit vector

Superposition of the current circle

If a submarine malfunctions, it is highly likely that the defective submarine will be located on the sea floor or at some point of neutral buoyancy. Its position may be affected by ocean currents, or by the topography of the seafloor. In order to more accurately identify changes in the position of the submarine over time, this paper specifically analyze the seafloor topography as well as the ocean currents separately. This paper quantify the effect of both on the submarine's position to obtain the trajectory of the submarine.

3.1.5 Ocean current

Currents are the movement of bodies of water in the ocean in the horizontal direction, and they are formed by a combination of factors. For example, the main driving force of ocean currents is wind, but the geostrophic force due to the rotation of the Earth also affects the speed of ocean currents. Therefore, this paper cannot only consider the magnitude of the current speed in a certain period of time, or simply regard the current speed as a fixed value.

Through the above analysis, this paper believe that if this paper want to determine the velocity of the ocean currents, this paper need to randomly generate a reasonably complete map of the ocean current field. Since the submarine's exposure to ocean currents at the time of the problem is uncertain, there are many ocean current circles superimposed. The force F_D of the current on the submarine at this point is a weighted average of the forces acting on the submarine from each current circle.

(1) From a horizontal viewpoint, a submarine is affected by multiple current circles, each of which has a different magnitude of influence on the submarine. The farther away from the center of the circle weighted average weight is smaller, that is, the weight is inversely proportional to the radius.

This paper consider each current circle as a current model with an approximate circular shape. The center of each model is determined by latitude and longitude, defining r_{ref} as the reference radius and v_{ref} as the reference velocity. Next, the absolute velocity of the ocean current is calculated. Since the velocity increases first linearly and then exponentially with radius, this paper can derive the formula for the absolute velocity of the ocean current based on the radius r of the center of the circle:

$$v_a = v_o \cdot \left(\frac{r}{r_o}\right) \cdot e^{\left(1-\frac{r}{r_o}\right)} \quad (4)$$

Next, this paper calculates the normal vector of the current cell based on its displacement difference (D_x, D_y) with respect to the center of the circle. Finally, the final velocity is computed from the normal and radial velocities of the currents for a given latitude and longitude.

(2) From a vertical perspective, friction exists on the seafloor due to the presence of friction, which decreases the closer you get to the water surface. The coefficient of friction of the seabed is:

$$v_f = \frac{(z - z_0)^2}{(z - z_0)^2 + h^2} \cdot v \quad (5)$$

From the formula above this paper can conclude that as the depth of dive increases, the speed of the current gradually slows down from its normal speed to almost stationary. Similarly, this paper can generalize to the velocity of currents affected by distance from shore. This paper therefore assume that the velocity of the current in the sea at vertical orientation, i.e., at vertical depth, is a sinusoidal function. With the latitude, longitude and elevation coordinates of the starting position of the sine wave, the magnitude of the period of the sine wave and the maximum velocity of the current known. This paper can calculate the velocity of the current and this velocity has a component only in the vertical direction, while the horizontal component is zero.

(3) Taken together, this paper can combine the horizontal and vertical perspectives. This paper was able to simulate a complex system of currents with horizontal and vertical flow to accurately predict the force of the currents on the submarine.

Since the submarine's route is not pre-specified, any of the velocities and positions in a given area are possible. Therefore this paper treat the initial position of the submarine as randomly distributed in the selected region. The following simulated submarines randomized between 34-39 degrees north latitude and 15-22 degrees east longitude are subjected to the forces of the ocean currents.

3.2. Building and solving the model.

In order to determine the motion of the submarine, this paper analyze the forces on the submarine to see that the submarine is not only affected by conventional forces, but also by forces due to the flow of ocean currents. Therefore, the submarine is affected by three main forces. These are gravity, buoyancy, and the force of ocean currents on the submarine. These four forces make up the combined force on the submarine F . From this, this paper make the following calculations.

(1) First, this paper defines the basic submarine condition. Define P is the initial position of the submarine and V as the submarine rate. And assume that (x, y, z) is the position of the submarine, where x denotes longitude, y denotes latitude, and z denotes absolute altitude. This paper derives the following expression:

$$\begin{cases} \vec{P}(x, y, z) \\ \vec{V}(v_x, v_y, v_z) \end{cases} \quad (6)$$

(2) After getting the above basic information, this paper use Newton's second law, $F = ma$, to solve the equations of motion of the submarine. And use the "scipy" library "odeint" function to solve the differential equations. This paper has established the following equation, where \dot{x} , \dot{y} , and \dot{z} are the results obtained by taking the derivatives of x , y , and z , which are the submarine's partial velocities in the three directions. \ddot{x} , \ddot{y} and \ddot{z} which are the results of taking the second-order derivatives, give the magnitude of the submarine's acceleration in the three directions.

$$\begin{cases} F(x, y, z, \dot{x}, \dot{y}, \dot{z}) = m(\ddot{x} + \ddot{y} + \ddot{z}) & \text{bottom} < z < 0 \\ \dot{x} = \dot{y} = \dot{z} = 0 & z \leq \text{bottom} \\ \dot{z} = 0, F(x, y, \dot{x}, \dot{y}) = m(\ddot{x} + \ddot{y}) & z \geq 0 \end{cases} \quad (7)$$

Since the object of study is a submarine, there are special characteristics of its working place. Therefore, this paper define **bottom** to denote the spatial extent below the sea surface and above the seabed. Then denotes that the submarine is in the given area at the current moment, while diving outside the given area is the velocity defined as 0.

(3) Next, this paper substitutes the four forces on the submarine calculated above:

$$\begin{cases} G(m) = mg\vec{e}_z \\ F_{up}(z) = \rho(z)gV\vec{e}_z \\ F_{cor}(x, y) = F_x, F_y, F_z \\ F_D(x, y, z, v_x, v_y, v_z) = \frac{1}{2}\rho(z)\left[(u_x - v_x) + (u_y - v_y) + (u_z - v_z)\right]C_D A\vec{e}_{u-v} \end{cases} \quad (8)$$

(4) Finally, by bringing in the submarine when it is in a given region **bottom**, this paper arrives at the following equation:

$$-mg\vec{e}_z + \rho gV\vec{e}_z + \frac{1}{2}\rho\left[\vec{x}-u(\vec{x})\right]\cdot\left|\vec{x}-u(\vec{x})\right|\cdot C_D A = m\ddot{\vec{x}} \quad (9)$$

Substituting the position and velocity of the submarine into the equation, the combined force F is calculated. $a = F/m$ is then used to find the acceleration, and the equation of motion of the submarine is finally solved [6-8].

After calculating all the above data, this paper substitute the data into the differential equation. The differential equations are used to simulate the trajectory of the submarine in the ocean due to different forces. And this paper visualize it by showing graphically the trend of the trajectory over time. This paper start from the moment 0, in minutes, for a total time of 10 hours. Here is an example (as shown in Figure 3) of solving a differential equation: the geographic trajectory of a submarine’s initial position, located at latitude 35 degrees and longitude 22 degrees, and at a depth of -3000 meters, changes with time. From the z-axis of depth, it can be seen that the submarine is at some point of neutral buoyancy underwater.

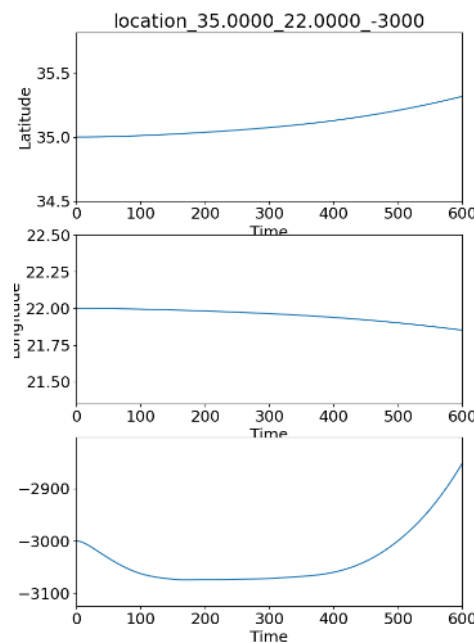


Figure 3 Submarine's geo-trajectory trend

Then this paper collected the undersea wreck locations. This paper selected the locations of 4 arbitrary wreck sites. After that, using the method of normal distribution, this paper take the points with velocity range of (-10~10, -10~10, -1~1) so as to get the trajectory of submarine's latitude, longitude and depth over time. The 2D (Figure 4) and 3D (Figure 5) plots of the probability density distribution of submarines over time are shown below.

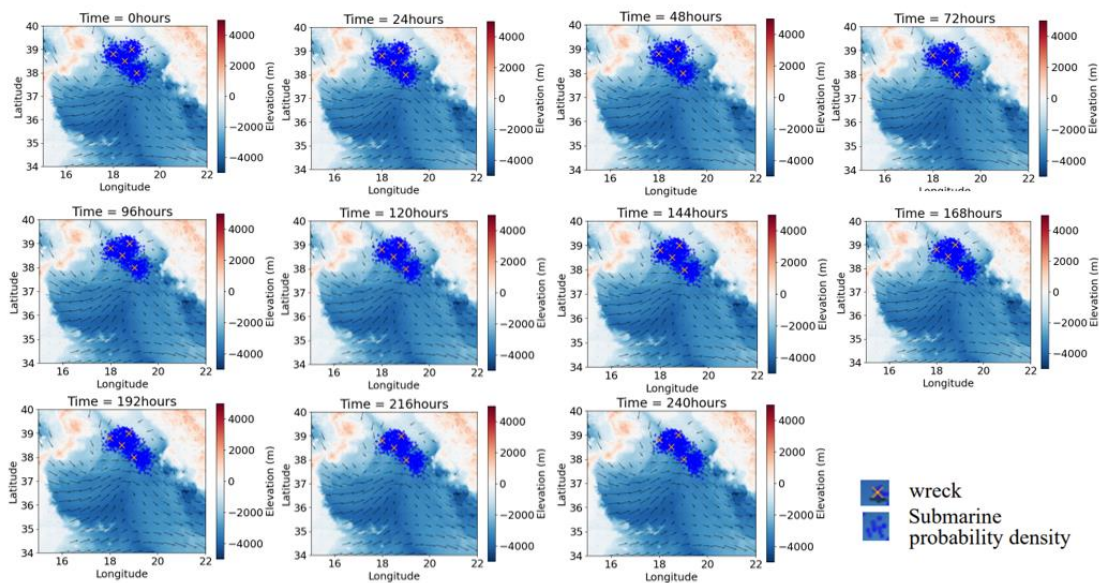


Figure 4 Probability density distribution of submarines over time (2D)

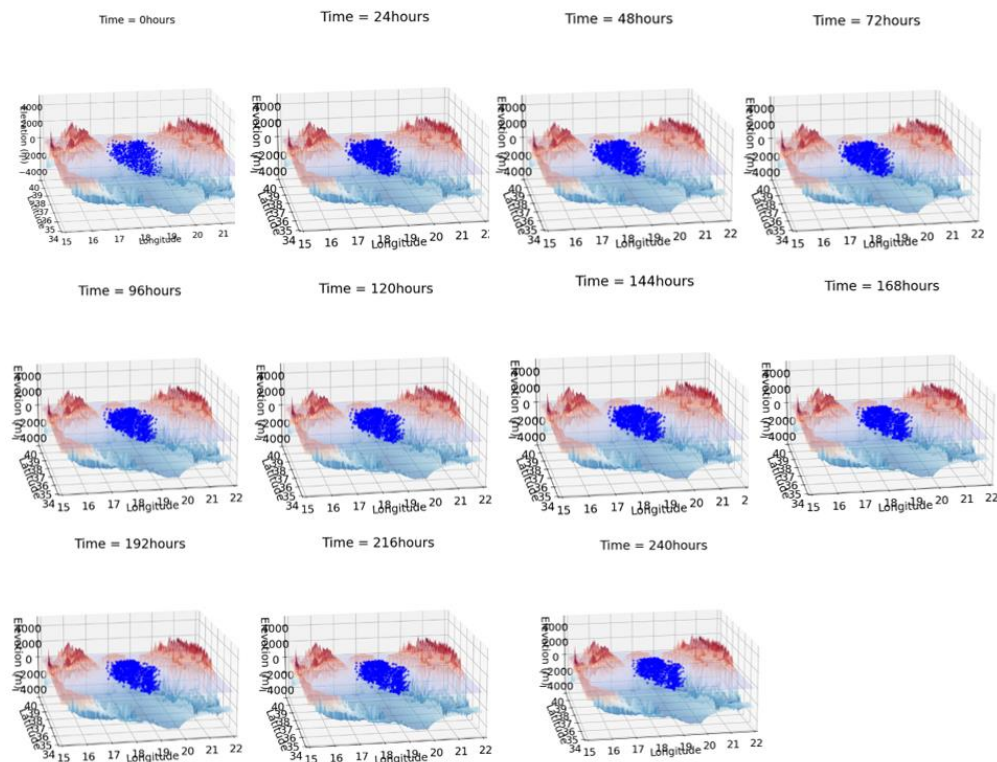


Figure 5 Probability density distribution of submarines over time (3D)

3.3. Greedy search based on Monte Carlo simulation methods.

In order to determine the position of the submarine since its position is randomly distributed in the sea water, this paper use the Monte Carlo simulation method to randomly generate 1000 samples to produce simulations based on the ordinary differential equation model obtained in the previous section to form the probability distributions of submarine. The frequencies with which the 1000 data points generated by the simulation appear in the lattice can be regarded as their probability densities.

It is known that the distance between each degree of longitude or latitude is about 111,000 meters, and considering that search and rescue missions are often undertaken by national navies, warships are often able to travel at speeds of up to 0.5 degrees of latitude and longitude per hour. And the

radius of the search waters is often no more than 2 degrees of latitude and longitude, so each ship can travel about 12 degrees of latitude and longitude per day.

Therefore, using the example of 3 warships involved in the search, this paper assume that these three ships work simultaneously, with each ship searching a 0.25 x 0.25 grid area per day. The grid is chosen based on the probability density, i.e., the frequency of occurrence of data points, derived from the simulation. Once a grid has been searched by a ship, this paper remove the points that have been searched in it and reorder them in the rest of the grid. This ensures that each round of searching starts with the grid with the highest probability density, reducing rescue time [9-10]. Simulating the search and rescue process is shown in Figure 6

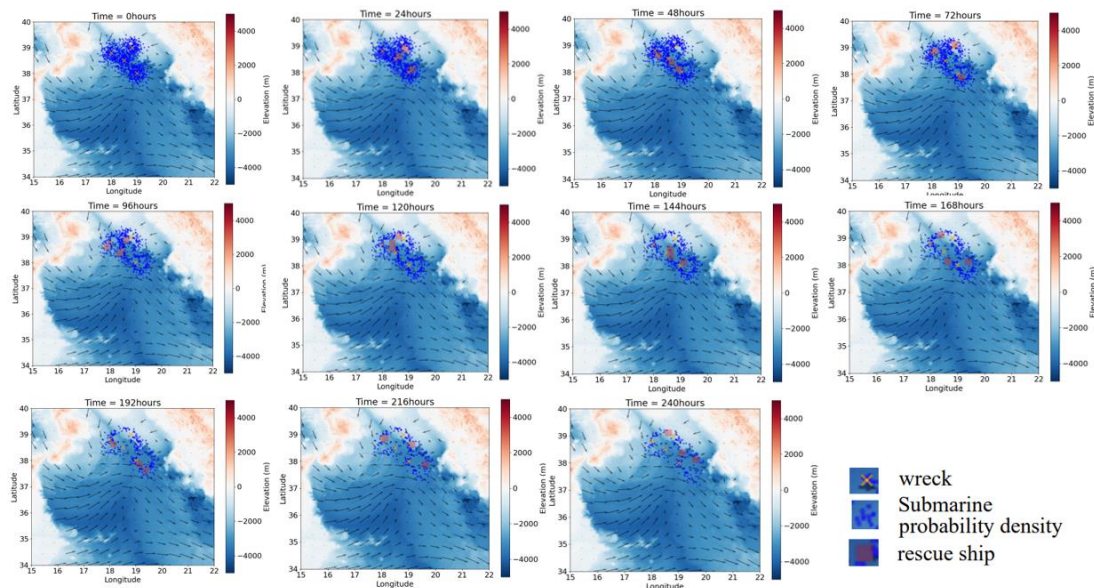


Figure 6 Simulating the search and rescue process.

It was calculated that in each 24-hour period, the three warships used 4 hours to arrive at the 3 positions in the positive search waters where submarines were most likely to be found and used 20 hours to search in those waters. At the beginning of each day, the 3 positions most likely to have submarines were selected based on Monte Carlo simulations, and the three warships were allowed to complete the search at those 3 positions. After the search was completed, if no target was found, the frequency points indicating the probability density in the corresponding waters were removed. The deletion of frequency points for searched areas is probabilistic. This is because this paper considered the possibility that the submarine was located in the search area of the warships, but the warships did not succeed in finding the submarine.

According to the simulation, it will take us about 10 days to complete the search. And with time, the unexplored grid area becomes less and less. Thus with increasing time, the probability that the ship is not found is decreasing, i.e., the probability of searching for the target ship minus increases. The remaining unexplored points are inversely proportional to the search time, as shown in Figure 7.

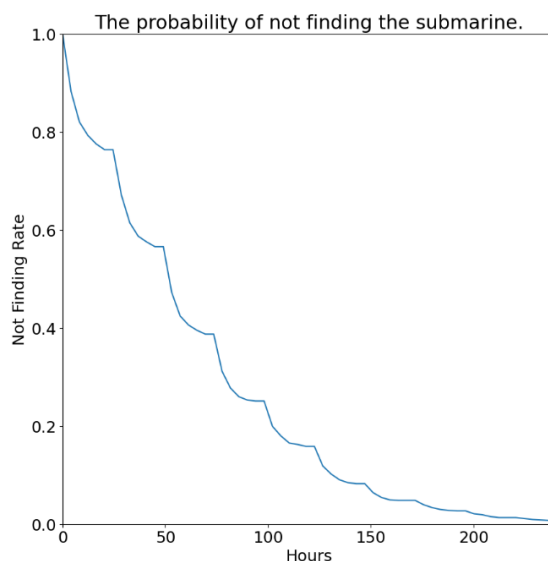


Figure 7 Unsearched points versus time

With this integrated search strategy, this paper can efficiently deal with dynamically changing currents and conduct the search in a systematic way. This approach not only improves the efficiency of the search, but also makes the search process more explicit and predictable. As a result, the rescue time can be greatly shortened and the survival rate of the passengers can be increased.

4. Conclusions

In this paper, for the problem of predicting and searching for a submarine that has lost power on the seafloor, differential equations based on Newton's second equation are developed and solved to determine the change in the position of the submarine on the seafloor over time. And the probability density distribution of the submarine's position at any moment is obtained using the Monte Carlo algorithm. Finally, at the beginning of each day, three warships were dispatched to select the most probable submarine location based on the Monte Carlo simulation and traveled to that location to conduct the search.

It is assumed that the ocean currents are relatively stable over short periods of time and that the amount of action scale is extremely large. However, many other factors (geostrophic forces, magnetic forces, etc.) are involved in the actual prediction of the position of a de-powered submarine on the seabed. Therefore, the company should take into account a number of factors to predict the position of the lost power submarine, which will help the company to get accurate data and make a reasonable search and rescue plan.

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