Optimization of Multibeam Survey Measurement Based on Greedy Algorithm

Ziye Zhang*

Computer college, Sichuan university, Chengdu, China, 610041
* Corresponding Author Email: 2021141460069@stu.scu.edu.cn

Abstract. The effectiveness of multibeam bathymetric technology is widely recognized in marine depth measurements, especially in areas with complex and variable seafloor topography. This study aims to design a multibeam bathymetric survey plan for a specific maritime region, with the objectives of minimizing the total survey line length, ensuring comprehensive coverage of the target area, controlling the overlap between adjacent survey lines, and fully considering variations in seafloor topography. An optimization model was developed to determine the optimal route for comprehensive seafloor depth measurements within the specified maritime area. The model initially considers the influence of seafloor slope, establishing a single optimization objective model with the goal of minimizing the total survey line length. The decision variables primarily include the spacing between adjacent survey lines. The objective function of the model aims to reduce the number of survey lines, thereby achieving the minimization of the total survey line length. Constraints of the model encompass limitations on coverage range, calculations of coverage width, and relationships between survey lines. The model is solved using a greedy algorithm-based optimization approach, iteratively updating the positions of survey lines while determining the spacing based on the relationship between depth and coverage width. Ultimately, a layout of survey lines with the shortest total length, totaling 64 nautical miles with a configuration of 32 survey lines, is obtained for the assumed maritime area. Multibeam bathymetric technology holds broad prospects in the field of marine measurements, particularly in areas characterized by complex and variable seafloor topography. This study, by designing a multibeam bathymetric survey plan for a specific maritime region, not only contributes to improving measurement efficiency and accuracy but also reduces measurement costs and resource consumption. The outcomes of this research have significant practical implications for marine resource development, seafloor engineering construction, environmental protection, providing professionals in related fields with an effective measurement method and optimization strategy.

Keywords: Greedy Algorithm, Multibeam Bathymetry, Optimization Algorithm

1. Introduction

In recent years, due to the demands of ocean resource development, seafloor engineering construction, and environmental protection, there has been an increasing focus on the precise measurement of seafloor topography and depth [1]. Traditional single-beam bathymetric methods, due to their inherent technological limitations, face challenges such as low measurement efficiency, discontinuous data, and susceptibility to missing measurements in the measurement of large-scale and complex seafloor topography. To overcome these drawbacks, multibeam bathymetric systems are gradually being promoted and applied. The multibeam bathymetric system adopts a swath measurement pattern and is currently the primary technical means for obtaining seafloor topography measurement data [2,3]. The multibeam bathymetric system simultaneously emits dozens or even hundreds of beams, receiving the seafloor-returned acoustic signals through transducers, thereby achieving full-coverage measurements of the swath width.[4] Utilizing transducers, the multibeam bathymetric system emits a fan-shaped array of signals. After reflection from the seafloor, the transducers receive scattered signals within a narrow beam. When the transducers receive the return signals, the receiving array synchronously records time and echo angles. During this process, the acoustic signals form a rectangular projection on the seafloor. Depth calculations are then performed based on acoustic signal projection and other parameters. Through depth values, the underwater
topography is inversely deduced, constructing a seafloor model to obtain underwater terrain features [5].

This study aims to design a set of survey routes to achieve the objectives of minimizing the total route length, fully covering the entire designated area, and minimizing the overlap between adjacent routes, assuming a rectangular maritime region.

2. Research Methodology

Due to the higher measurement accuracy of the central beam in multibeam bathymetry compared to the edge beams [6,7,8], it is necessary to maintain the minimum overlap between adjacent survey lines while minimizing the total survey line length to achieve complete coverage of the target maritime area. The core objective of the problem is to optimize for the shortest total length while adhering to pre-defined constraints.

The greedy algorithm [9], serving as a local search algorithm, plays a crucial role in this study. The fundamental idea of the greedy algorithm is to choose the locally optimal solution at each step, with the expectation of ultimately obtaining a globally optimal solution. In this study, the greedy algorithm is applied to determine the optimal spacing between survey lines to meet the specified overlap requirements.

The greedy algorithm, originally proposed by Edsger W. Dijkstra, is a commonly used algorithmic paradigm for solving combinatorial optimization problems. It incrementally constructs the solution in problem-solving, selecting the currently best local solution at each step. Although the greedy algorithm does not necessarily guarantee obtaining a globally optimal solution, in specific situations, it can effectively address problems.

In this study, depth information is initially obtained from the east, west, south, and north boundaries of the target maritime area. Subsequently, using this information, the required detection coverage width is calculated to ensure coverage of the entire target maritime area. Then, through the application of the greedy algorithm, the optimal spacing between survey lines is computed to meet the specified overlap requirements.

Finally, based on the calculated spacing, the number of survey lines required to cover the entire maritime area is determined. This research provides a method for efficiently optimizing marine measurement tasks, considering both resource utilization efficiency and measurement accuracy.

3. Model Establishment and Solution

3.1. Premises for Model Establishment

This study aims to find an optimal survey route within a rectangular maritime area assumed to have dimensions of 3704m in north-south length and 7408m in east-west length, with an overall slope of 1.5° in the east-west direction. The objective is to conduct seafloor measurements and cover the entire uncharted area. Under the premise of optimizing for the shortest overall route length, with a constraint on coverage ranging from 10% to 20%, a single-objective optimization model is established. To ensure the optimization of routes, literature review suggests that the main survey lines within the measurement area should be parallel and aligned with the direction of contour lines [10].

The overlap rate between two survey lines is lower in shallower waters than in deeper waters, as illustrated in Figure 1. Thus, it is inferred that the overlap rate gradually increases with the deepening of the water.
In Figure 1, the distance between survey lines 1 and 2 is greater than the distance between survey lines 2 and 3, but the overlap rate between survey lines 1 and 2 is lower than the overlap rate between survey lines 2 and 3. Therefore, reducing the distance between survey lines can increase the overlap rate.

To meet the constraint on overlap rate, survey lines are denser in shallower waters and sparser in deeper waters. Assuming the angle between the direction of survey lines ($\beta$) is constant, the model is established under the premise that the angle $\beta$ is 0, as this minimizes the number of survey lines and thus the total length of survey lines.

### 3.1.1 Determination of Decision Variables

Under the assumption that survey lines are parallel and aligned with contour lines, the distance between adjacent survey lines $d$ is the main factor influencing the number of survey lines. Therefore, the decision variable used in this study is the distance between adjacent survey lines $d_i$.

### 3.1.2 Determination of Constraint Conditions

The constraint condition $\eta$ is set to a range of 10% to 20%

$$0.1 \leq \eta \leq 0.2$$  \hfill (1)

$$\eta = 1 - \frac{d}{W}$$  \hfill (2)

The coverage width $W$ can be derived as follows:

$$W = \frac{D \cdot \tan \frac{\theta}{2}}{1 - \tan \frac{\theta}{2} \cdot \tan \alpha \cdot \sin \beta} + \frac{D \cdot \tan \frac{\theta}{2}}{1 + \tan \frac{\theta}{2} \cdot \tan \alpha \cdot \sin \beta}$$  \hfill (3)

Where $\theta$ is 120°, $\alpha$ is 1.5°, $\beta$ is 90°.

Assuming the depth $D$ is only influenced by the east-west movement of the vessel, a two-dimensional Cartesian coordinate system is established as shown in Figure 2:

![Figure 2. Schematic Diagram of East-West Profile of Seabed Slope](image-url)
Where point A is (-3704, 0), point B is (3704, 0), and the coordinates of the vessel are \((x, 110 + 3704 \times \tan 1.5^\circ)\). The expression for slope BC is given by:

\[ y = \tan 1.5^\circ x + 3704 \times \tan 1.5^\circ \]  

(4)

From the diagram, it can be derived that:

\[ D = 110 + 3704 \times \tan 1.5^\circ - y \]  

(5)

Thus,

\[ D = 110 - \tan 1.5^\circ x \]  

(6)

The coverage width \(W\) for this problem can be obtained from equations (3) and (6), as follows:

\[ W = \frac{(110 - \tan 1.5^\circ x) \times \tan 60^\circ}{1 - \tan 60^\circ \times \tan 1.5^\circ \times \sin 90^\circ} + \frac{(110 - \tan 1.5^\circ x) \times \tan 60^\circ}{1 + \tan 60^\circ \times \tan 1.5^\circ \times \sin 90^\circ} \]  

(7)

Simplifying further:

\[ W = \frac{(110 - \tan 1.5^\circ x)\sqrt 3}{1 - \sqrt 3 \tan 1.5^\circ} + \frac{(110 - \tan 1.5^\circ x)\sqrt 3}{1 + \sqrt 3 \tan 1.5^\circ} \]  

(8)

Initialization of the starting point is depicted in Figure 3:

**Figure 3.** Schematic Diagram of East-West Width

Let’s assume that the starting point coordinates are \(x_1\), the coverage width for the \(i - \)th survey line is \(W_i\), the \(x\)-coordinate of the \(i - \)th survey line is \(x_i\), the spacing between the \(i - 1 - \)th and \(i - \)th survey lines is \(d_i\), and the total distance on the \(x\)-axis is \(X\).

From the above diagram, it can be inferred that to ensure complete coverage of the entire maritime area, the left side of the initial coverage width on the vessel’s path must precisely coincide with the slope. The expression for the left side coverage width of the first vertical line is:

\[ \frac{(110 - \tan 1.5^\circ x_1) \times \tan 60^\circ}{1 - \tan 60^\circ \times \tan 1.5^\circ \times \sin 90^\circ} \]  

(9)

We can derive the value of \(x_1\) as:

\[ x_1 = 110\sqrt 3 - (1 - \sqrt 3 \tan 1.5^\circ) \times 3704 \]  

(10)

In the \(x\)-axis direction, the total distance is obtained from the geometric relationship:

\[ x_i = x_{i-1} + d_{i-1} (i \geq 2) \]  

(11)
According to the assuming condition, the entire maritime area needs to be completely covered, leading to the equation:

\[ X = x_{n-1} + \frac{(110 - \tan 1.5^\circ x_{n-1}) \cdot \tan \frac{\theta}{2}}{1 + \tan \frac{\theta}{2} \cdot \tan \alpha \cdot \sin \beta} \]  

(12)

In summary, the optimization model for this problem is given by:

\[
\begin{align*}
\{ & 0.1 \leq \eta \leq 0.2 \\
W & = \frac{(110 - \tan 1.5^\circ x)\sqrt{3}}{1 - \sqrt{3}\tan 1.5^\circ} + \frac{(110 - \tan 1.5^\circ x)\sqrt{3}}{1 + \sqrt{3}\tan 1.5^\circ} \\
\text{s. t.} & \quad x_{n-1} + \frac{(110 - \tan 1.5^\circ x_{n-1}) \cdot \tan \frac{\theta}{2}}{1 + \tan \frac{\theta}{2} \cdot \tan \alpha \cdot \sin \beta} = 4 \text{ nautical miles} \\
& \quad x_1 = 110\sqrt{3} - (1 - \sqrt{3}\tan 1.5^\circ) \times 3704. \\
& \quad x_i = x_{i-1} + d_{i-1} (i \geq 2) \\
& \quad Z = \min 3704n
\end{align*}
\]  

(15)

3.2. Optimization Model Solution Based on Greedy Algorithm

![Figure 4. Schematic Flowchart of the Algorithm](image)

From Figure 4, we can understand the algorithmic process of this model.

Initially, an appropriate initial position is iteratively sought to ensure that the coverage requirements are met at that position. Considering the seafloor slope and the opening angle of the multibeam transducer, the strip coverage width and overlap rate are calculated at a given depth. Iteratively calculating the strips at different depths while meeting the coverage requirements of 10%-20%, the positions, strip coverage widths, and overlap rates of the survey lines are collected. By continuously trying different depth positions and survey line spacings in a loop, the shortest survey line layout satisfying the overlap rate requirement is collected. In each initialization of x, the value of d needs to be updated. Since shallower depths result in lower coverage widths, the updated d values will remain constant.

3.3. Presentation of Optimization Model Results

According to the above algorithm, the corresponding shortest route \( Z \) is 64 nautical miles, and the number of survey lines is 32.

The survey results and coverage area are presented as follows:
From Figures 5 and 6, it can be observed that the measurement area covered the entire uncharted maritime region. Additionally, as depth increases, coverage width also increases, consistent with the model assumptions.

4. Discussion

Possible issues in practical applications: In practical applications, factors such as vessel speed, navigation time, and measurement equipment performance need to be considered, which may affect the optimization of measurement routes. The seafloor topography in actual maritime areas may differ from the assumed topography in the model, leading to a potential decrease in model performance in practical applications. In actual operations, attitude sensors and transducers are generally installed separately, and there may be offsets between the attitudes of the survey vessel and the transducer.

Limitations of the model: This model assumes a simple slope for the seafloor topography, which may differ from more complex topographies encountered in real-world applications. Additionally, the model does not consider the performance limitations of measurement equipment, such as the transducer's beamwidth and measurement range. These limitations may result in decreased model performance in practical applications. It should be noted that attitude angles and motion residuals can
also affect the planar position of beam points, assuming a relatively accurate planar position, leading to some errors in the matching model presented in this study [11].

Future research directions: Future research can consider incorporating more complex seafloor topographies into the model while also considering the performance limitations of measurement equipment to improve the model's practicality. Additionally, research could explore how to incorporate factors such as vessel speed and navigation time into the model for a more accurate prediction of the optimization effects of measurement routes in practical applications.

5. Conclusion

This study provides a method for effectively optimizing marine measurement tasks, considering both resource utilization efficiency and measurement accuracy. Through the optimization model based on the greedy algorithm, a multibeam bathymetry survey route was successfully designed for a specific maritime area, achieving minimization of the total survey line length, comprehensive coverage of the target maritime area, control of the overlap rate between adjacent survey lines, and thorough consideration of seafloor topographical variations. However, the model still has limitations, such as the simplification assumption of seafloor topography and equipment performance restrictions. Future research should consider incorporating more complex seafloor topography and equipment performance limitations into the model to enhance practicality and prediction accuracy.

References