

Research on multi-beam line survey problem based on geometric analysis and particle swarm optimization

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Abstract. Multi-beam bathymetry uses acoustic reflection to measure the depth of seawater. In this paper, through geometric analysis, a model is established to consider the influence of slope, transducer opening angle and distance from the center point of the survey line on the coverage width and overlap rate. In this paper, the particle swarm optimization is also used to determine the optimal survey line. Calculate the depth, coverage width, and overlap rate based on the distance of the survey line from the center point. The direction of the survey line is determined by the angle between the normal direction of the seabed slope and the horizontal plane projection, and the relationship between the coverage width and the seabed depth is studied. The coverage efficiency of straight lines and curves is the same, and the shortest measurement length is the survey line along the short side of the rectangular sea. The optimization model is established by the overlap rate constraint, and the number of measuring lines and the shortest measurement length are calculated. According to the single-beam bathymetry data, the area is delineated, the length and overlap rate of the survey line are calculated, and the length of the entire survey line is accumulated. The shortest survey line length is obtained by constraining the overlap rate, and the total length of the overlap area that exceeds the limit is calculated. Combined with the number of nautical miles in the direction of the sea area, the percentage of the missed sea area is calculated as the percentage of the sea area to be measured.

Keywords: Multi-beam bathymetry, Geometric analysis, Particle swarm algorithm.

1. Introduction

The multi-beam sounding system has developed rapidly in technology, from single beam to multi-beam, realizing the transformation from "point-line" to "line-surface" measurement mode. Acoustic signal processing techniques such as linear and non-linear frequency modulation also enhance the accuracy and stability of detection. Multi-beam bathymetry systems are used in a variety of applications, including topographic surveys of seabeds, rivers, reservoirs, etc. It is very useful in determining the situation of stranded and sunken ships in emergency scans, improving efficiency. There are relevant R&D institutions and products at home and abroad, such as the high-end shallow water multi-beam sounding sonar technology of Harbin Engineering University, which has reached the international leading level.

2. The relationship between the seawater depth, overlap rate and the distance between the center of the survey line in the two-dimensional state

2.1. Establishment of the equation of the relationship between the parameters of the plane

According to the index parameters in the literature, this paper assumes that the coverage width of the multi-beam bathymetry strip is W , while the depth of the sea is D . The angle between the submarine slope and the plane perpendicular to the survey line is the slope α , and the transducer opening angle is θ , as is shown in Fig 1.

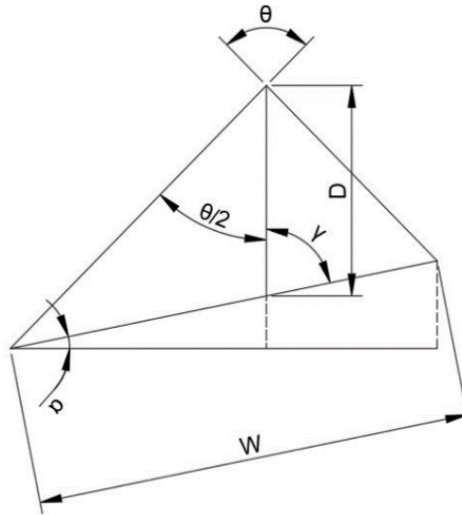


Fig. 1 Schematic diagram of multi-beam bathymetry

It can be seen from the knowledge of triangles: $\alpha + \gamma = \frac{\pi}{2}$. Accordingly, combined with the sinusoidal theorem, it can be obtained:

$$W = D \left[\frac{\sin \frac{\theta}{2}}{\sin(\pi - \frac{\theta}{2} - \gamma)} + \frac{\sin \frac{\theta}{2}}{\sin(\gamma - \frac{\theta}{2})} \right] \quad (1)$$

Considering special circumstances, when the distance between the survey line and the center is 0, it satisfies $D=70\text{m}$. At the same time, when the position of the survey line changes at the center point of the sea area, the expression of the sea depth D will also change accordingly, as shown in Fig 2.

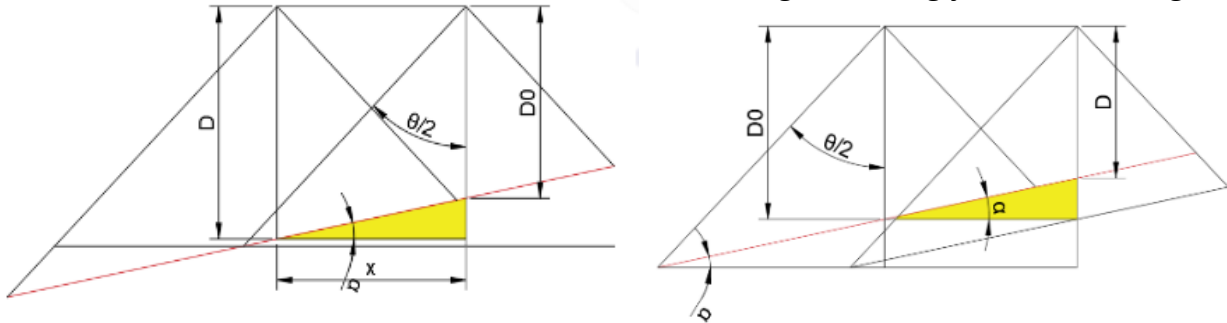


Fig. 2 Schematic diagram of the relative position of the survey line

When the survey line is to the left of the center point of the sea area:

$$D = D_0 + |x| \tan \alpha \quad (2)$$

When the survey line is to the right of the center point of the sea area:

$$D = D_0 - |x| \tan \alpha \quad (3)$$

In the above equation, x is the distance between the survey line and the center point. At the same time, the overlap rate of adjacent bands is introduced η this indicator. Fig 3 illustrates the definition of the overlap rate in this article.

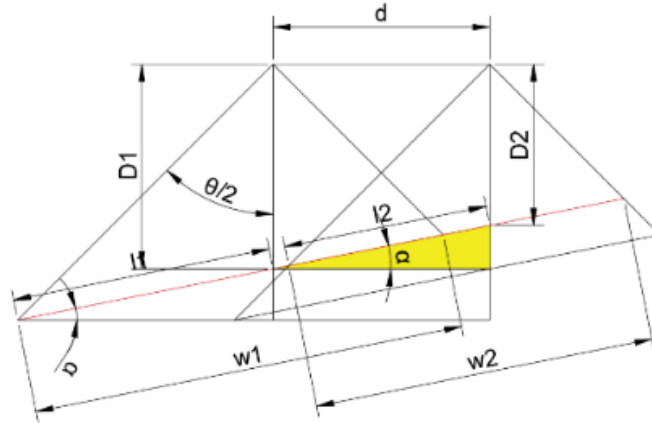


Fig. 3 Illustrative diagram of the overlap rate of adjacent bands

$$\eta = 1 - \frac{d}{(w_1 - l_1) + l_2} \quad (4)$$

The formula for calculating the overlap rate is also given. In the above formula, l_1 and l_2 are the left half of the two lines. From the definition of the overlap rate, the overlap rate of adjacent bands i and j can be obtained:

$$\eta_{ij} = 1 - \frac{|D_i - D_j| \cot \alpha}{D_i \frac{\sin \frac{\theta}{2}}{\sin(\gamma - \frac{\theta}{2})} + D_j \frac{\sin \frac{\theta}{2}}{\sin(\pi - \frac{\theta}{2} - \gamma)}} \quad (5)$$

2.2. Interpretation of two-dimensional relationships based on detailed data

Based on the data given on the official website of mathematical modeling, the following information can be obtained according to the known conditions and corresponding parameters:

$$\begin{cases} \alpha = 1.5^\circ \\ \theta = 120^\circ \\ x = 800, 600, \dots, -800 \end{cases} \quad (6)$$

The above known conditions are only for the multi-beam line survey problem at the two-dimensional level, and the line survey problem in space will be discussed below. According to the above data, the substitution formula can calculate the values of the overlap η rate of the seawater depth D , the coverage width W and the overlap rate of the previous survey line in the case of different distances between the survey line and the center point x , as shown in Table 1:

Table 1. The calculation conclusion of each parameter is carried out

x/m	Depth of the sea D/m	Coverage width W/m	Overlap rate η /%
-800	90.948737	315.705107	-
-600	85.711553	297.525571	34.881600
-400	80.474369	279.346035	30.783175
-200	75.237184	261.166499	26.134206
0	70	242.986963	20.815772
200	64.762816	224.807427	14.672049
400	59.525631	206.627892	7.494781
600	54.288447	188.448356	-1.000792
800	49.051263	170.268820	-11.214614

3. The relationship between the seawater depth, overlap rate and the center distance of the survey line in the three-dimensional state

3.1. Establishment of the equation of the relationship between the parameters of space

Unlike the two-dimensional planar state above, the depth of the sea in space and the distance from the center of the survey line will change, as shown in Fig 4.

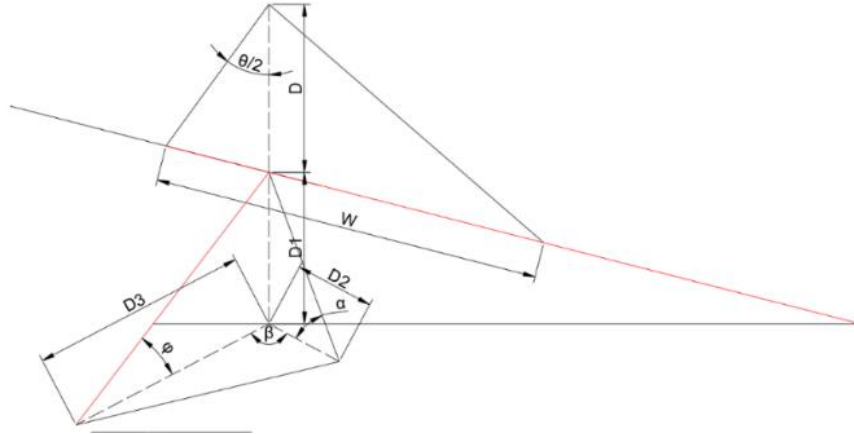


Fig. 4 Schematic diagram of relative positions in space

From Fig 4, the relationship can be obtained:

$$D_2 = D_1 \cot \alpha \quad (7)$$

$$D_3 = D_2 \sec \beta = D_1 \cot \alpha \sec \beta \quad (8)$$

$$\varphi = \arctan\left(\frac{D_1}{D_3}\right) = \arctan\left(\frac{D_1}{D_1 \cot \alpha \sec \beta}\right) = \arctan(\tan \alpha \cos \beta) \quad (9)$$

Combined with the knowledge of trigonometric functions, it is possible to calculate and simplify the coverage width W of multi-beam bathymetry as:

$$W = D \left[\frac{\sin \frac{\theta}{2}}{\cos(\frac{\theta}{2} - \arctan(\tan \alpha \cos \beta))} + \frac{\sin \frac{\theta}{2}}{\cos(\frac{\theta}{2} + \arctan(\tan \alpha \cos \beta))} \right] \quad (10)$$

At the same time, as shown in Fig 5, the seawater depth D can be expressed as:

$$D = D_0 + x \tan \varphi = D_0 + \tan \alpha \cos \beta \quad (11)$$

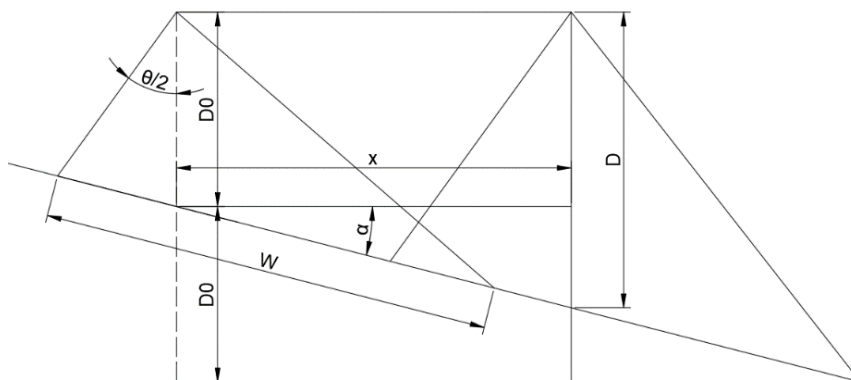


Fig. 5 Schematic diagram of the sea depth in space and the location of the center of the sea area

3.2. Interpretation of 3D relationships based on detailed data

According to the known conditions given in the literature, the following parameter values can be obtained:

$$\begin{cases} \alpha = 1.5^\circ \\ \theta = 120^\circ \\ D_0 = 120 \end{cases} \quad (12)$$

Substituting the above data, we can calculate the value of the multi-beam bathymetric coverage width when the distance between the measurement ship and the center point of the sea area and the angle between the direction of the survey line changes, as shown in Table 2:

Table 2. Coverage width of multi-beam bathymetry when the direction angle changes

W/m		x/ nautical mile							
		0	0.3	0.6	0.9	1.2	1.5	1.8	2.1
β /°	0	416.692	467.212	517.732	568.252	618.772	669.292	719.812	770.332
	45	416.192	451.872	487.552	523.232	558.912	594.592	630.273	665.953
	90	415.692	415.692	415.692	415.692	415.692	415.692	415.692	415.692
	135	416.192	380.511	344.831	309.151	273.471	237.791	202.110	166.430
	180	416.692	366.172	315.652	265.132	214.612	164.092	113.571	63.051
	225	416.192	380.511	344.831	309.151	273.471	237.791	202.110	166.430
	270	415.692	415.692	415.692	415.692	415.692	415.692	415.692	415.692
	315	416.192	451.872	487.552	523.232	558.912	594.592	630.273	665.953

The relationship between the coverage width and distance and the direction of the survey line and the angle formed by the projection of the normal direction of the submarine slope on the horizontal plane is established, as shown in Fig 6:

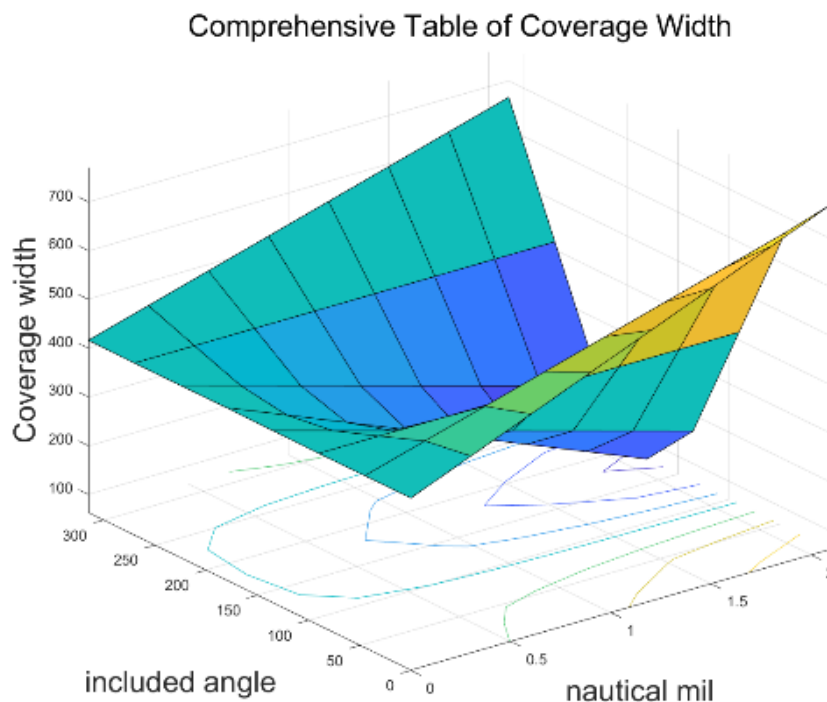


Fig. 6 Relationship between coverage width, distance and projection angle

4. Example of a rectangular sea area survey

4.1. The shortest conditions for the survey line are described in detail

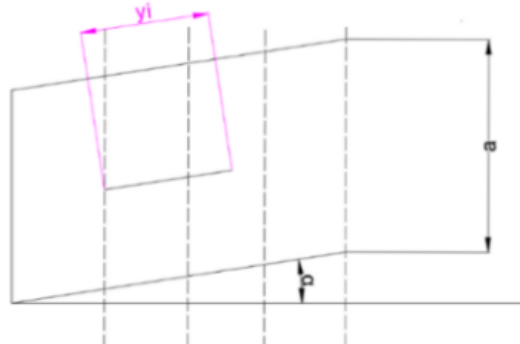


Fig. 7 Schematic diagram of the sea area to be measured

As can be seen from Fig 7, hypothetical y_i is the relative length of the distance from the measurement line to the center, k is the number of lines. For ease of understanding, y_i counts from left to right, it means that:

$$y_1 < y_2 < y_3 < \dots < y_i < y_{i+1} < \dots < y_k \quad (13)$$

Therefore, the overlap rate of adjacent bands can be solved as follows:

$$\eta_j = 1 - \frac{|D_{i+1} - D_i| \cot(\arctan(\tan \alpha \cos \beta))}{D_i \frac{\sin \frac{\theta}{2}}{\sin(\frac{\pi}{2} - \frac{\theta}{2} - \arctan(\tan \alpha \cos \beta))} + D_{i+1} \frac{\sin \frac{\theta}{2}}{\sin(\frac{\pi}{2} - \frac{\theta}{2} + \arctan(\tan \alpha \cos \beta))}} \quad (14)$$

In order to facilitate the solution of the overlap rate between the shortest distance and the adjacent band, the β angle is set to a right angle, and the relevant proof process is as follows: Defines the coverage efficiency of a line or curve as the ratio of the area of the graph to the length of a trajectory to a point with a fixed distance from a curve or straight line. On the premise of ensuring the coverage efficiency, the direction of the measurement line is regarded as a straight line. In the context of this problem, the following is a process that uses the micro-element method to prove that the coverage efficiency of straight lines and curves is the same. This article assumes that the curve equation is $y = f(x)$. The equation for a straight line is $y = kx + b$. The distance from the point to the curve and the straight line is d_p , as is shown in Fig 8.

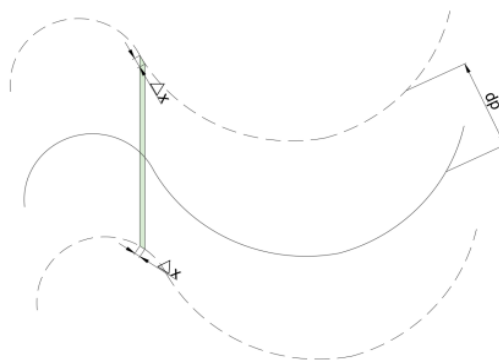


Fig. 8 Schematic diagram of curve coverage efficiency

In the curve area $[x, x + \Delta x]$, according to Fig 8, it is easy to see that the vertical height Δh is:

$$\Delta h = f(x + \Delta x) - f(x) \quad (15)$$

Construct a very small region with d_p as the microelement, The area dS of the Fig can be obtained:

$$dS = d_p \sqrt{(\Delta x)^2 + [f(x + \Delta x) - f(x)]^2} \quad (16)$$

From the geometric relationship, it is easy to know that the curve length dL is:

$$dL = \sqrt{(\Delta x)^2 + [f(x + \Delta x) - f(x)]^2} \quad (17)$$

According to this, the coverage efficiency can be concluded as:

$$d\alpha_1 = \frac{dS}{dL} = d \quad (18)$$

For straight lines, a similar method is used to obtain the coverage efficiency of a small segment. Since the slope of a straight line is constant, the efficiency of the segment is $d\alpha_2 = d$, no matter how small the segment is chosen. Therefore, straight lines and curves per unit length are covered with the same efficiency. For convenience, the line is measured in a straight line instead of a curve. The following proves that the survey line is vertically cut along the short side of the rectangular area to be measured as the optimal survey line direction, and then it can be seen that the angle between the direction of the survey line and the normal direction of the submarine slope is 90° .

Assuming that the length of the short side of the rectangle is m , the length of the long side is n , and the angle between the survey line and the long side of the rectangle is μ , no measurement line is made during the analysis process, and only the first and last two measurement lines are displayed, which are separated from DK. In order to find the optimal direction of the survey line, that is, when the distance between the two survey lines d is determined, the number of survey lines k is as small as possible, that is, the dk is the smallest.

(1) The survey line is parallel to the long side of the rectangle and perpendicular to the short side, as shown in Fig 9. In this case, $\mu = 0$ and the line spacing $dk = m$.

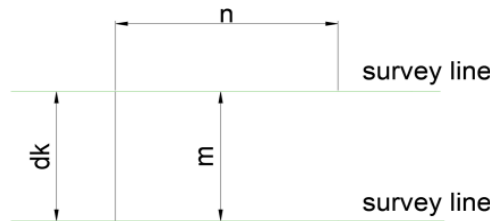


Fig. 9 The survey line is paralleled and perpendicular

(2) The survey line intersects with the rectangular short-side extension line, as shown in Fig 10, and dk is satisfied $0 < \mu \leq \arctan \frac{m}{n}$. In this case, the survey line spacing dk satisfies: $d_k > d_{k2} > m$.

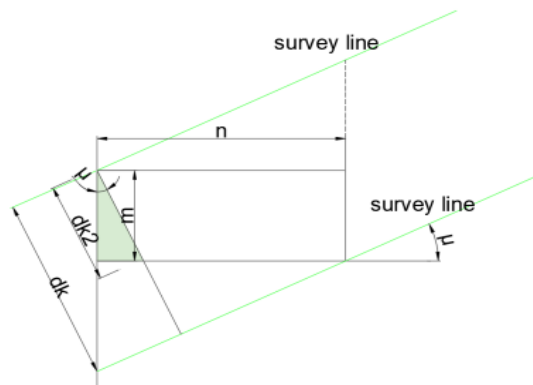


Fig. 10 Intersection of the survey line and the short-side extension line of the rectangle

(3) The survey line intersects with the extension line on the long side of the rectangle, as shown in Fig 11, and satisfies $\arctan \frac{m}{n} < \mu \leq \frac{\pi}{2}$. In this case, the survey dk satisfies: $d_k > d_{k3} > n > m$.

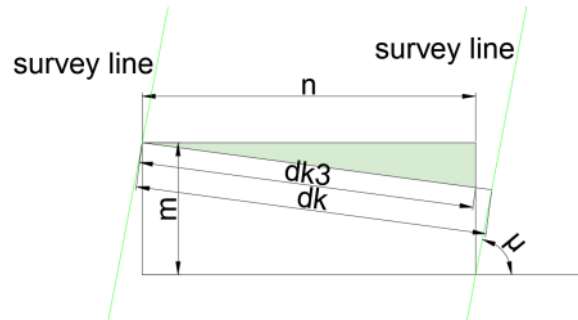


Fig. 11 The intersection of the survey line and the long extension line of the rectangle

The above is the case in a rectangle for a square or in $\mu > 90^\circ$. Therefore, $d_{k\min} = m$ holds when the direction of the line is perpendicular to the direction of the short side. Combined with the above contents, the survey line is the shortest when the angle formed by the projection of the direction of the survey line and the normal direction of the submarine slope on the horizontal plane is $\beta = 90^\circ$.

4.2. Example calculations based on given conditions and proof processes

In this paper, under the above proof conditions, under the given conditions, the number and length of the measured lines are calculated by the PSO algorithm. The data source of this article is the official website of the National Mathematical Modeling Competition. Through the proof process in part 4.1, it is not difficult to obtain the expression of line L in this paper as follows:

$$\min L = ka \quad (19)$$

At the same time, the constraints of this article are as follows:

$$\begin{cases} 0.1 \leq \eta_j = 1 - \frac{|D_{i+1} - D_i| \cot \alpha}{D_i \frac{\sin \frac{\theta}{2}}{\cos(\frac{\theta}{2} + \alpha)} + D_{i+1} \frac{\sin \frac{\theta}{2}}{\cos(\frac{\theta}{2} - \alpha)}} \leq 0.2 \\ j = 1, 2, 3, \dots, k-2, k-1 \end{cases} \quad (20)$$

Based on the PSO algorithm, this paper initializes the measured distance from the center nautical mile into random particles, and adjusts the adaptation value and penalty function of the individual according to the numerical value and constraint function given in this question, and finds the optimal solution of individuals and races through iteration in the solution space. Through MATLAB programming, it can be solved: the number of measuring lines is 38; The length of the survey line is about 76 nautical miles.

5. Conclusion

The geometric analysis, the differential element method and the classification discussion method adopted in this paper provide a basis for solving the relationship between seawater depth, overlap rate and survey line center distance, but the traditional two-dimensional plane model algorithm to solve the three-dimensional problem requires a lot of time and computing resources, and will affect the fitting accuracy and calculation accuracy. Based on the characteristics of the above three models, this paper combines the particle swarm optimization algorithm, uses the powerful iteration ability and data processing ability of the algorithm, fits the data on the three-dimensional plane on the basis of

the relationship between the two-dimensional plane, and uses the comprehensive construction of the model by using the four algorithms. The calculation results show that the proposed algorithm is in line with the actual situation, has good fitting accuracy, and has certain application value.

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