Research on Binary Black Hole Systems by Analysis of Gravitational Waves

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Abstract. The mass ratio, effective spin, and chirp mass are three important properties to describe the binary systems of black holes. The gravitational wave sources can be a breakthrough in the investigation of black holes. The formation of black holes is a significant problem to solve as the electromagnetic waves (light) cannot escape from the black hole, and the gravitational waves can be emitted when the binary systems of black holes or neutron stars merge. Based on the detection of gravitational waves, the process of the formation of black holes can be traced and studied in the binary black holes and primordial black holes formed in the early universe. The mass ratio distribution is plotted to investigate the difference between the binary black hole and neutron star black hole and the mass ratio which makes it easier for the merger of the black holes to happen. The investigation of the effective spin helps classify the model of the formation of black holes as there are five models of the black hole. The statistical model can be tested with the chosen data, and the cumulative standard Gaussian distribution is used in this paper. The result states that the model is not suitable for the chosen data and another statistical model should be introduced to analyze the effective spin. These parameters are all important for determining the stage and formation process of the binary systems.

Keywords: Gravitational waves, Binary systems, LIGO.

1. Introduction

The existence of black holes has been predicted by Einstein's general relativity and it is almost confirmed that the formation of black holes is due to the collapse of massive stars [1]. While based on the detection of gravitational waves and the posterior of primary mass, black holes are not expected to form due to the above reason [2]. The densest objects which are the neutron stars enable the exploration of the most extreme conditions in nature [3]. The development of gravitational waves astrophysics has played an important role in the research on black holes and neutron stars especially for binary systems [4], as there are no expected electromagnetic waves emitted in the process of binary black hole mergers [5]. The Gravitational-Wave Transient Catalog 3 (GWTC-3) combines the observations from O1, O2, and O3, which detect the gravitational wave signals of 90 mergers comprised of black holes and neutron stars, the neutron star black hole binary mergers were not detected until O3 [6, 7]. The astrophysical origin of the binary systems (BBH, BNS and NSBH) can be investigated by analyzing the distribution of binary black holes parameters like masses, spins, and redshifts which helps with testing the models of formation for these systems [8]. Gravitational physics had progressed on different routes by detecting gravitational waves which provide signatures of relativistic gravity, including spin [1]. The research on gravitational waves is worth highlighting as providing the understanding of the origin of the merging of the black hole population and the primordial black hole which is formed in the early universe provides an important property of dark matter [9]. Moreover, the increasing observations increase the accuracy of characterizing the properties of the underlying population of black holes and the relative contribution of various binary black hole formation channels [2]. It is of paramount importance that the study of gravitational waves was involved in the development of black holes and cosmology and astrophysics, which could help with a proof of general relativity these waves bring information about the early universe which helps with the study of the galaxy formation, black hole formation and the stellar evolution. Based on the detection of the signal is still not sensitive enough, the number of strong spin constraints. The problem that arises from reviewing the articles focused on researching gravitational waves is that there has
been no new data on gravitational waves since September 2020, the discussion about the GWs is out
of date and the plan of LIGO (Laser Interferometer Gravitational waves observatory) is to collect
the gravitational waves data in 2024 [1]. The gravitational waves study focused on the binary systems
of black hole-black hole or black hole-neutron star, the gravitational waves carry the energy and angular
momentum away from the system, leading to a gradual decay of the orbit and a collision and/or
merger of two components eventually. The signal of the gravitational waves is processed by using
the method of matched filter and the Fourier transform, the signal-to-noise ratio is also calculated to be a
standard to refine the data for further analysis. By combining the theoretical cosmological model and
the signal of the gravitational waves detected, fitting the parameters to match the signal to get the
properties of the system detected.

In this paper, the three parameters are discussed and analysed which are mass ratio, effective spin,
and chirp mass. the data is chosen from GWTC-2 and GWTC-3 events. The mass ratio analysis uses
all the data from these events and the mass ratio could define the difference between binary systems.
The effective spin analysis uses the data from GWTC-3 events refines the data with a \( p_{\text{astro}} \) larger than
0.99 and compare it with cumulative Gaussian distribution to see if the graph match, where \( p_{\text{astro}} \) is
the probability of astrophysical origin estimated based on the previous research on the focused large
binary black hole region [10]. The chirp mass distribution is analysed as it shows the frequency
evolution of the inspiral waveform. By analysing the parameters of these systems, the process of
formation of the black hole can be classified into dynamical formation channels or isolated formation
channels [8]. The distributions of the effective spin for these systems are classified using the mass
distribution and different formations give different effective spin distributions which will be stated in
the next analysis section.

2. Method and Results

2.1. Methodology

2.1.1. Data collection

The gravitational waves data is collected from the LIGO (laser interferometer gravitational waves
observatory) of GWTC-2 and GWTC-3 events.

2.1.2. Properties calculation

For the signal processing of gravitational waves, the match filter method is used to extract the
signals by creating a template and doing the Fourier transform on the signal:

\[
\tilde{x}(f) = \int_{-\infty}^{\infty} x(t) e^{-2\pi if t} dt
\]

\[
x(t) = \int_{-\infty}^{\infty} \tilde{x}(f) e^{2\pi if t} df
\]

Where equation (1) is the Fourier transform equation for \( x(t) \) and equation (2) is the inverse Fourier
transform equation. By creating the template and using the matched-filter method, the signal-to-noise
ratio can be calculated and reliable data sets can be chosen.

There are three stages of the merger of the binary black hole:

Inspiral: In the binary black hole system, the two black holes are separated and orbit the centre of
mass, the factor which characterizes the frequency evolution of the gravitational waveform in the
merger process of the binary system is chirp mass, which is calculated by [1]:

\[
\mathcal{M} = \frac{(M_1 M_2)^{3/5}}{(M_1 + M_2)^{1/5}}
\]

Coalescence: The gravitational waves are emitted in this process which will carry away the energy
and linear momentum. Then the signal of gravitational waves can be used to determine the formation
scenarios of the binary black hole system.
Ringdown In this process which is after the merger, the mass and the spin of the black hole determine the frequencies and decay rates of the oscillation modes.

The effective spin which is in the direction of angular momentum is calculated by [1]:

$$\chi_{\text{eff}} = \frac{M_1\vec{a}_1 + M_2\vec{a}_2}{M_1 + M_2} \cdot \hat{L}$$  \hspace{1cm} (4)

With $\vec{a}_1$ and $\vec{a}_2$ are the vector pointing in the direction of the first and second spin with the magnitude equal to the dimensionless spin parameter of the black holes and $\hat{L}$ is the unit vector pointing towards the direction of the angular momentum.

2.2. Results Analysis

The data is from the LIGO website with the signal is processed already using the matched filter method, the parameter is already fitted the gravitational waveform. The binary black hole system can be working with the mass ratio first. Figure 1 shows the mass ratio between mass1 and mass2 from the GWTC-2 and GWTC-3 events, the orange line is $y = x$ and the two red dots are the events which are classified as the NSBH system and other blue dots are the BBH systems.

**Figure 1.** The mass ratio between mass1 and mass2 from the GWTC-2 and GWTC-3 events [2].

**Figure 2.** The mass ratio distribution of these events and the green line and the orange line are the two events of NSBH (Photo/Picture credit: Original).
From Figure 1, the orange line represents the best fit line of the mass ration and the green line is the line with slope is 1, which means $\frac{m_2}{m_1} \leq 1$. The mass ratio of NSBH system of the two points is very small as the neutron star is the densest objects in the universe and the mass of the neutron star is the largest mass which can combine the black hole in the system, then the mass ratio is approximately 0. Figure 2 shows the mass ratio is concentrated between the region of 0.6-0.8, which may state that with value of the mass ratio in this region is more easily to merger which may due to the reason that the smaller mass ratio is too distant and the gravity could not bind the two objects together.

Figure 3. The effective spin distribution of the events of binary systems detected in the GWTC-2 and GWTC-3 (Photo/Picture credit: Original).

The effective spin influences the shape and properties of the gravitational wave signal during the process of inspiral and merger. From Figure 3, the distribution of the effective spin of the black hole has a peak at 0, which gives the information that the number of the binary systems with effective spin higher than 0 is more than the number of the binary systems with effective spin lower than zero, and this shows that the distribution of the effective spin is asymmetric. The dynamical formation of the black holes predict that the distribution of the effective spin should be symmetric.

For the effective spin is higher than 0, from equation 4, the effective spin has the same direction with the angular momentum which contributes positively to the total angular momentum. The waveform emitted from these events will have certain features which impacts the observed waveform morphology and parameters. With the effective spin is lower than 0, the effective spin contributes negatively to the total angular momentum of the system. The gravitational waveforms are affected by the anti-aligned spin and affecting the observed waveform characteristics.
**Figure 4.** The selected events from the $p_{\text{astro}} > 0.99$ and re-order based on the size of the value of effective spin and plot with the error bar provided in the data set and the cumulative standard Gaussian distribution is plotted [8].

**Figure 5.** The plot for the origin plot of the effective spin with the error bar [8].
3. Conclusion

This paper provides the analysis of the mass ratio, and effective spin of the binary black hole systems and two neutron star black hole systems identified in GWTC-2, and GWTC-3. For the relationship of the mass ratio investigated in this paper, it has been found that the system detected with primary mass is about 40 solar mass and the mass ratio of the NSBH system is approximately zero which indicates that the mass ratio could help define the type of the binary systems. The investigation of the effective spin helps define the formation types of binary black holes and it also affects the information of the gravitational signal. In this paper, I have estimated that all of the black holes from these events are formed in the dynamical process and found out that the events did not give results that match the prediction which should be the symmetric distribution about the point zero, while the results show the distribution is more concentrated on the positive side and less in the negative side. This research has an important effect on testing the statistical model of analysing the formation of binary black hole systems and it has been proved that the cumulative standard Gaussian distribution is not suitable for this model. The research provides the clue that the black holes are not formed dynamically as well with the effective spin and for the high redshift and large mass system, we still cannot detect and find the rule of how they form and work.

The detection of gravitational waves will be more important in future research on cosmology and astrophysics with the research on formation of the black holes and by investigating the primordial black hole to research about the early universe. With the accumulation of the data for the gravitational waves, more reliable data can be used to investigate the relationship between the formation of the binary systems and the mass ratio, effective spin, chirp mass, and redshift.
References


