Analysis of the Hubble Tension and Modifications to the Standard Cosmological Model

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Abstract. The Hubble constant is crucial to the knowledge of the universe's nature and expansion, remaining as an active area for research in the recent decades. Cosmological microwave background observations revealed a lower value of 67–69 $km^{-1}s^{-1}$Mpc for the Hubble constant, but local data showed a higher value of 73–75 $km^{-1}s^{-1}$Mpc. This study focuses on the Hubble constant and its measurement, systematic uncertainties, observed tension, and potential modifications to the standard cosmological model. To be specific, this paper aims to address the tension between different measurement methods, which are Cepheid variables and supernovae, Baryon Acoustic Oscillations, and Gravitational waves, and understand systematic uncertainties. Results show discrepancies in Hubble constant values, indicating potential modifications to the standard cosmological model. Prospects include exploring alternative cosmological components and enhanced detection and computation methods. The study's implications lie in advancing the understanding of the expanding universe and guiding future research directions regarding to Hubble tension measurement.

Keywords: Hubble constant, cepheid variables, baryon acoustic oscillations, gravitational waves, standard cosmological model.

1. Introduction

The Hubble constant as a fundamental cosmological parameter has been the cornerstone in understandings of the expanding universe, represents the relentless pursuit of humanity to unravel the mysteries of the cosmos. Over the past century, visionary cosmologists and astronomers have embarked on a captivating journey of making iterative refinements, traversing a landscape of rich historical significance. In his seminal work, Georges Lemaître introduces the notion of a homogeneous universe with a growing radius, establishing a relationship between the distances of galaxies and their radial velocities [1]. His equations introduce the expansion of the universe, as shown in the following equation:

$$\frac{v}{c} = \frac{R_2 - R_1}{R_1} = \frac{R'}{R}dt = \frac{dR}{R} = \frac{R'}{R}r \tag{1}$$

with v/c representing the ratio of the recessional velocity (v) of an object to light speed (c), quantifying the redshift of light due to the universe's expansion. $(R_2 - R_1)/R_1$ is the change in the scale factor (R) between two different epochs, providing insights into the universe's expansion rate. By utilizing the period-luminosity relation of Cepheid variable stars to establish a linear relationship between the distances and radial velocities of receding galaxies, Edwin Hubble's groundbreaking work provided observational evidence in favor of the expanding universe theory [2].

Subsequent advancements were made by cosmologists and astronomers such as Allan Sandage, who dedicated themselves to refining Hubble's initial measurement by employing various distant indicators to enhance the accuracy. The launch of the Hubble Space Telescope (HST) created new measurement opportunities. Under the direction of Wendy Freedman and her colleagues, the HST Key Project started an amazing journey to determine the Hubble parameter with never-before-seen accuracy. The experiment used a variety of observations and distance markers during a ten-year period. Numerous investigations conducted after the HST Key Project have attempted to improve measurements of the Hubble constant acquired using different methods. There has been an increasing amount of disagreement between measurements made using various approaches, which has sparked discussions and motivated researchers to look further into the nature of the universe's expansion. In cosmology, figuring out the Hubble constant has been a top priority. Using the HST to examine Cepheid variables in distant galaxies, measurements in the early 1990s showed substantial inconsistencies, but by 2000 [3], a precise finding of H_0=72 ± 8 km sec^{-1} Mpc^{-1} was reached. A later recalibration of the measurement resulted in H_0=74 ± 2 km sec^{-1} Mpc^{-1}. These results point to a young expansion age of the universe relative to some of the oldest stars, which creates a Hubble tension coupled with the theoretical expectation of a high matter density with Ωm ∼ 1. Whereasr the identification of cosmic acceleration and an improved comprehension of the composition and recent expansion history of the led to a rise in the universe's estimated age and resolved.

By examining the temperature and polarization fluctuations in the cosmic microwave background (CMB), the lower value of the measured Hubble constant H_0=72 ± 8 km $sec^{-1}Mpc^{-1}$ is found, which aids in the calibration of parameters in the Lambda Cold Dark Matter (ΛCDM) model. Calibrated using CMB data, the lower measurements of H_0 are acquired by studying special variations and distributions of galaxies (inverse distance ladder). The higher value of H_0 measured, ranging from 70 to 75 km $sec^{-1}Mpc^{-1}$ involving significant investment of the HST, encompasses the precise (within 5%) and recent measurements in the "late" universe (seen from Fig. 1) [4, 5].

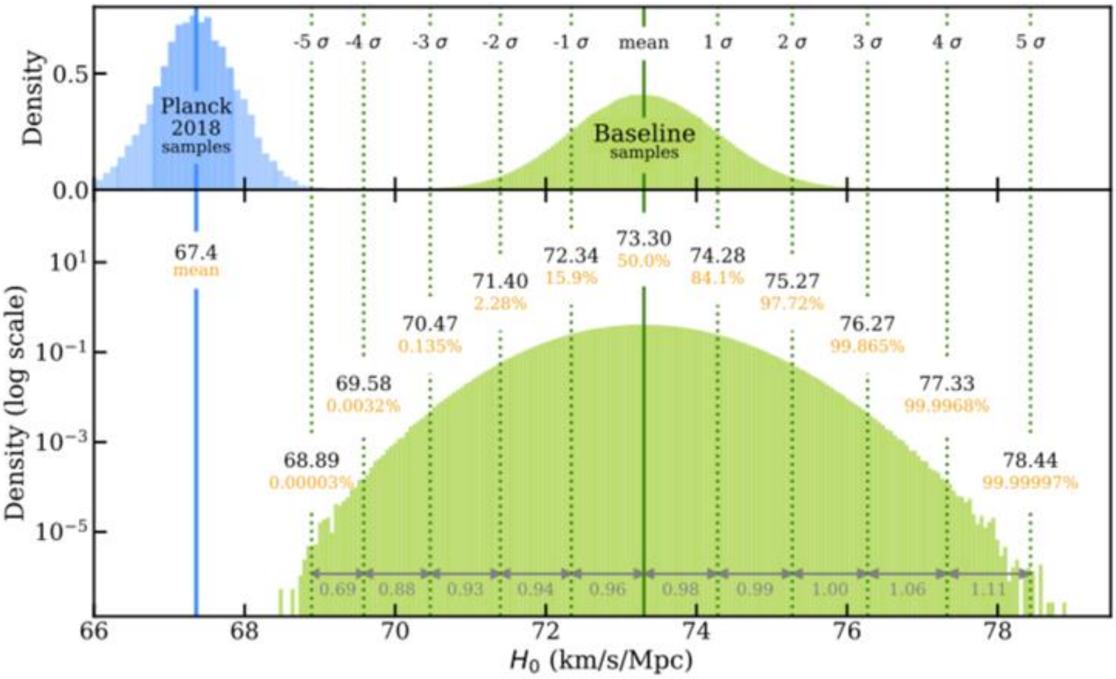

Figure 1. To get data up to the 5σ confidence level, the posterior was sampled using an extended MCMC method for H_0 [4].

Significant progress has been made in the past 24 years in the realm of cosmology, resembling high-energy physics experiments in terms of advanced analyses, the volume of data, careful consideration of systematic errors, and the establishment of a successful standard cosmological model with precise parameters. However, human beings are currently facing another discrepancy regarding the Hubble constant, which is smaller in magnitude compared to previous tensions but highly significant, with an 8% deviation between H_0 measurements and a confidence level >∼ 5σ. Measuring the Hubble constant is significant for various reasons. Firstly, it is paramount for the exploration of the foundations of cosmology and helps determine if modifications or extensions to existing theoretical frameworks are necessary. Secondly, the discrepancies in the values of the Hubble parameter suggests the presence of new physics, allowing more explorations and potential discoveries of novel theories of gravity, modifications to the Standard Model, or alternative explanations for dark matter and dark energy. Furthermore, resolving the Hubble tension is essential for addressing the limitations of the standard model and advancing humanity's understanding of the governing laws of the universe. Moreover, precise measurements of the Hubble parameter enhance the estimations of the universe's age, size, geometry, structure, evolution, and fate. Additionally, the significance of measuring the Hubble tension extends beyond cosmology, with profound consequences for fundamental physics, impact understandings of gravity and particle physics [6-8].

The motivation for this study lies in the aspiration to gain a deeper understanding of the underlying physics and potential modifications to the standard cosmological model. By thoroughly investigating the systematic uncertainties and identifying the factors contributing to the Hubble tension, scientists aim to unearth potential biases, limitations, or unknown phenomena that may be influencing the measurements. Resolving the Hubble tension could lead to new insights into the fundamental physics governing the universe, potentially necessitating modifications to the standard cosmological model.

2. The Hubble Constant and its Measurement

2.1. Defining the Hubble Constant

The cosmos's expansion rate, or the pace at which galaxies and other things in the universe move apart as a result of the universe's expansion, is quantified by the Hubble constant (H_0). In the equation $cz=H_0D$, where D is distance and z are redshift in the condition $z\geq 0$, it is the constant of proportionality [6]. The following equation is the generalized version of the equation for $z > 0$:

$$D = \frac{cz}{H_0}\left[1 - \left(1 + \frac{q_0}{2}\right)z + \left(1 + q_0 + \frac{q_0^2}{2} - \frac{j_0}{6}\right)z^2 + O(z^3)\right] \tag{2}$$

where c is the speed of light (3×10^8m/s), q_0 represents the deceleration parameter, j_0 is the jerk parameter, which is the third derivative of the scale factor describing the changing rate of the deceleration of the universe's expansion. $O(z^3)$ represents the higher order terms involving powers of $z \geq 3$, indicating existing extra terms beyond the quadratic term z^2, often becoming more significant at larger redshifts.

2.2. Overview of Different Measurement Methods

2.2.1. Cepheid variables and supernovae

Cepheid variables and Type Ia Supernovae serve as distance indicators due to their consistent luminosity. By comparing the apparent brightness to their known brightness, distances can be inferred [7]. Then, by connecting the distances to the redshift, the Hubble constant can be calculated. These measurements rely on telescopes and instruments capable of observing and characterizing the properties of Cepheid variables and supernovae. Cepheids arise when the hydrogen cores of massive stars are exhausted and evolve rapidly across the Hertzsprung-Russel (HR) diagram. Because the Leavitt law [8], which describes the link between the pulsation period, intrinsic luminosity, and color, was so easily discovered, cepheids are easily recognized and are powerful tools for determining the distances to galaxies. The following set of equations,

$$Q = P\rho^{\frac{1}{2}} \tag{3}$$

$$\rho = \frac{3X}{R^3 4\pi} \tag{4}$$

Where P is the pulsation period, ρ is the mean star density, Q represents the pulsation constant, X represents the mass of the star, and R is the radius of the star. The Stefan-Boltzmann law establishes the connection between the luminosity L and its effective temperature T where σ is the Stefan-Boltzmann constant:

$$L = R^2 4\pi\sigma T^4 \tag{5}$$

A star's brightness can be expressed in terms of its magnitude or absolute magnitude. The star's apparent magnitude, as if it were ten parsecs away, is represented by its absolute magnitude:

$$m = A - 2.5\,log\left(\frac{L}{d^2 4\pi}\right) \tag{6}$$

$$M = A - 2.5\,log\left(\frac{L}{[10pc]^2 4\pi}\right) \tag{7}$$

The following relations can also be established over the mass range of Cepheids [9]:

$$log\,X = M_{bol}a_1 + c \tag{8}$$

$$M_{bol} = M + a_2C + c \tag{9}$$

$$log\,T = a_3C + c \tag{10}$$

Where a_i are constants that need to be found either theoretically or empirically, M_{bol} is the bolometric magnitude, M is the magnitude, C is the color, and c stands for constant. The following form of the Leavitt law is derived from the previous equations:

$$M = a_1 + a_2\,log\,P + a_3C \tag{11}$$

2.2.2. Baryon acoustic oscillations (BAO)

Acoustic waves from the early universe left behind imprints on the large-scale structure of the universe, known as baryon acoustic oscillations. The analysis of galaxy clustering and the observed excess of clustering at a given size sheds light on measurements of the Hubble constant and the universe's rate of expansion [10]. As the sound horizon is widely known, BAO data in an absolute Alcock-Paczynski test can be used to determine the Hubble rate and the angular diameter distance. The typical scale's line-of-sight and tangential models (s_{II} and $s_{\perp}$) are connected to the angular diameter and Hubble rate, respectively (seen from Fig. 2) as follows:

$$H(z) = \frac{c\Delta z}{s_{\parallel}(z)} \tag{12}$$

$$d_A(z) = \frac{s_{\perp}}{(1+z)\Delta\theta} \tag{13}$$

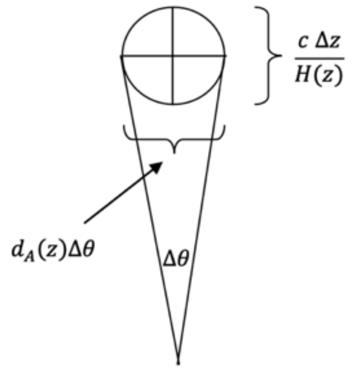

Figure 2. The Alcock-Paczynski test, a technique for determining an object's radial and transverse diameters, is depicted in the schematic picture. Examining the redshift difference between the object's front and back is part of the test. Equations x-y can be used to calculate the angular diameter and the Hubble parameter independently with the aid of this knowledge.

2.2.3. Gravitational waves

Gravitational waves are curvatures and ripples in the fabric of spacetime caused by accelerating masses. The detection of gravitational waves relies on advanced interferometric detectors such as LIGO, Virgo, and future detectors like the Einstein Telescope and LISA. Gravitational wave measurements involve observing the signals from merging dense objects and combining them with electromagnetic observations of related events. Scientists can establish the relationship between the luminosity distance and redshift through combining gravitational wave and electromagnetic data, thus facilitating the estimation of the Hubble constant [11]. As described in [8], the method for using gravitational waves to compute the Hubble constant utilizes a circular orbit of a coalescing binary system. Using General Relativity's basic quadrupole formula, one may determine the gravitational wave's amplitude and frequency change time scale. The amplitude denoted as Eq. (14) represents the root-mean-square average over detector and source orientations:

$$A = 1 \cdot 10^{-23} m_T^{\frac{2}{3}} \mu f_{100}^{2/3} r_{100}^{-1} \tag{14}$$

The rate at which gravitational wave frequency varies over time is expressed in seconds on the time scale by Eq. (15).

$$\tau = \frac{f}{\dot{f}} = 7.8 m_T^{-\frac{2}{3}} \mu^{-1} f_{100}^{-\frac{8}{3}} \tag{15}$$

3. Systematic Uncertainties in the Hubble Constant Measurements

3.1. Cepheid Variables and Supernovae

Two tests that investigate the effect of metallicity on the period-luminosity relation (PLR) of Cepheid variables are presented by Freedman and Madore in their publication "Two New Tests of the Metallicity Sensitivity of the Cepheid Period-Luminosity Relation (the Leavitt Law)". In the first test, Cepheids in the Small Magellanic Cloud (SMC) and the Large Magellanic Cloud (LMC), which have distinct metallicities, are compared with PLR. By examining the observed magnitudes and periods in the individual galaxies, the authors hope to examine whether there is a detectable variation in the PLR slopes. The second test examines the PLR for Cepheids in the nucleus of a barred spiral galaxy NGC 3551, which is a high metallicity environment. By comparing the PLR in this metal-rich region with that of Cepheids in lower metallicity environments, the authors aim to assess the impact of metallicity on the PLR. As shown in Fig. 3, the metallicity has a minimal effect on the PLR, reinforcing of using Cepheids as “standard candles” for distance estimation, hence the Hubble constant estimation [12].

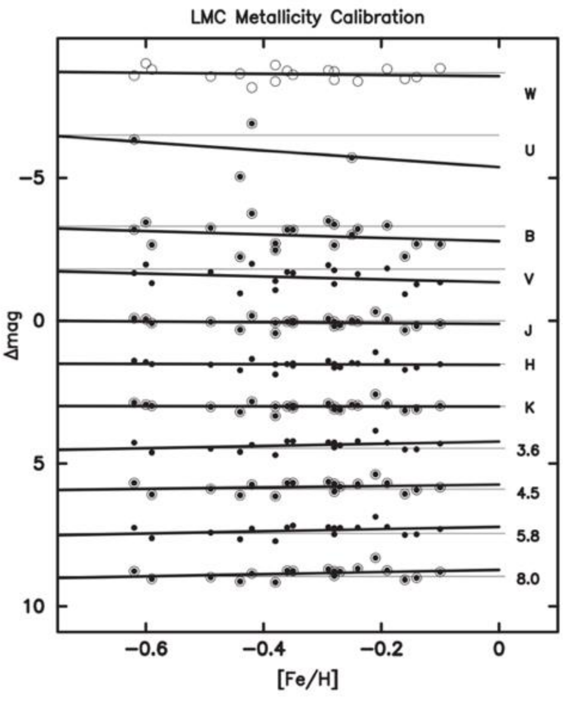

Figure 3. Plotting magnitude residuals versus spectroscopic metallicity values ([Fe/H]) of Cepheid stars is shown in [12].

The measurement of the Hubble constant, which expresses the rate of expansion of the universe, depends critically on the calibration of the absolute magnitude for Supernova Ia. Nevertheless, there are a number of unknowns associated with this calibration procedure, which affects how accurate the Hubble constant readings are. Firstly, SNe Ia exhibit intrinsic variations in their peak luminosities, even after factoring in light curve shape and color. The extent of this dispersion, referred to as the "intrinsic scatter," implies uncertainties in calibrating the absolute magnitude [13]. Secondly, calibrating the absolute magnitude requires accurate distance measurements to the SNe Ia, in which lies myriad uncertainties. These uncertainties in distance measurements propagate into the calibration process, making the Hubble constant determinations uncertain [13].

$$D_L = cH_0^{-1}(1+z)|\Omega_k|^{-\frac{1}{2}} sin \cdot n\left\{|\Omega_k|^{-\frac{1}{2}} \cdot \int_0^z dz[(1+z)^2(1+\Omega_M z) - z(2+z)\Omega_\Lambda\right]^{-\frac{1}{2}} \quad (16)$$

As shown in the following equation that calculates the luminosity distance (DL) based on cosmological parameters and redshift (z). Ωk is the curvature of the universe. Accurate distance measurements are crucial for calibrating the absolute magnitude of SNe Ia. The equation quantifies and accounts for these uncertainties as they propagate into the calibration process and contribute to the overall uncertainties in measuring the Hubble constant. Furthermore, there may be systematic uncertainties in the measurement of the light curves of SNe Ia, including assumptions about dust extinction, time dilation, or the physics of the explosions. These systematic uncertainties need to be carefully evaluated and quantified to ensure accurate calibration of the absolute magnitude.

As shown in Fig. 4, the spectral comparison between supernovae at different redshift values focuses on specific spectral features observed in the light emitted by these supernovae at different wavelengths. The spectral characteristics of a Type Ia supernova at a high redshift are very similar to those of supernovae at a lower redshift corresponds to less systematic uncertainties present. One of the main sources of inaccuracy in Cepheid distance measurements is statistical (random) errors, which show up as the scatter in the period-luminosity relation Eq. (1). The linearized Lemaitre rule is not followed empirically by Cepheid data, with individual variances reaching up to 30% in distance error. Thankfully, the distance error can be reduced to 10% with a comparatively small sample size because statistical mistakes vary inversely with the square root of the sample size [14-16].

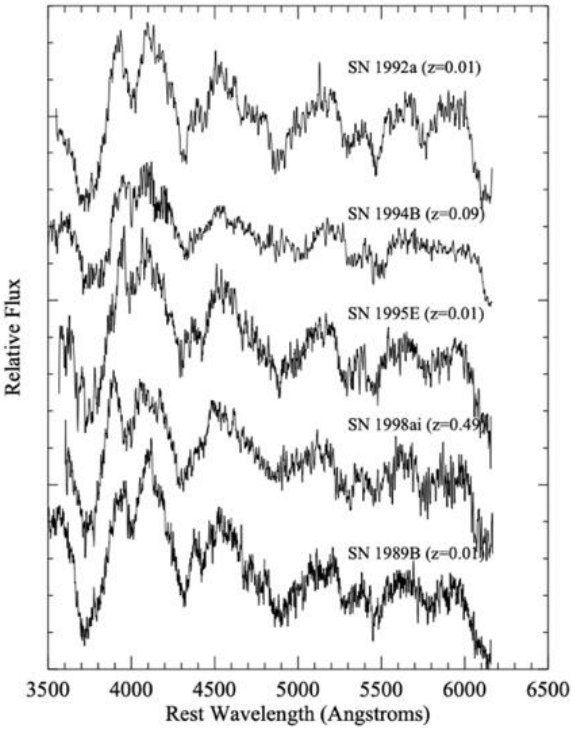

Figure 4. A Type Ia supernova (SN 1998ai) at a greater redshift (z≈0.49) and a set of Type Ia supernovae at a lower redshift (z≈0.1) that are at similar ages are compared spectrally. Displays comparison data from five days before attaining maximum brightness for the following supernovae: SN 1989B, SN 1992A, SN 1994B, SN 1995E, and SN 1998ai [14].

The reddening effect (interstellar extinction) is also a key source of systematic errors, since observed objects will appear fainter and farther. To free the computations from reddening, one can represent AV as the magnitude difference caused by reddening, described by the following equation:

$$A_V = R_{VI} \cdot E(V - I) \tag{17}$$

Where V and I are two wavelength bands, RVI is the total-to-selective absorption ratio, and E (V - I) is the color excess. This equation yields the Wesenheit magnitude, a corrected magnitude that takes the effects of reddening into account:

$$W \equiv V = R_{VI} \cdot (V - I) = V_o - R_{VI} \cdot (V - I)_o \tag{18}$$

3.2. Baryon Acoustic Oscillations (BAO)

The formula for the sound horizon at the photon decoupling redshift in cosmology is given below,

$$r_s(z_d) = \frac{1}{\sqrt{3}} \int_0^{\frac{1}{1+z_d}} \frac{da}{a^2 H(a) \sqrt{1 + \frac{3\Omega_b}{4\Omega_\gamma} \cdot a}} \tag{19}$$

Which links the horizon distance at z_d to a number of cosmological parameters. The sound horizon distance at the redshift of photon decoupling is expressed as $r_s(z_d)$, the Hubble parameter as a function of the scale factor is expressed as H(a), the baryon density parameter is represented by $\boldsymbol{\Omega}_b$, and the radiation density parameter is indicated by $\boldsymbol{\Omega}_\gamma$. Essentially, by integrating a function that depends on the rate of expansion of the universe, the density of baryonic matter, and radiation, the equation determines the sound horizon distance at the redshift of photon decoupling. The BAO and the sound horizon distance are intimately correlated. The acoustic waves freeze in when the universe turns transparent to photons at z_d, leaving a peak in the matter power spectrum that corresponds to the magnitude of the sound horizon at z_d. The expansion history of the universe, the nature of dark energy, and other cosmological aspects can be deduced by comparing the recorded BAO scale with theoretical predictions based on the sound horizon distance and other cosmological factors.

Nevertheless, there are still uncertainties in using BAO to measure the Hubble parameter. Firstly, **Ω**b and **Ω**γ involves measurements and observational data, which inherently carry uncertainties. This further propagates through the calculation of sound horizon and the inference of the Hubble constant. Secondly, the equation is derived within the linear perturbation theory, hence it assumes that the density fluctuations in the early universe are small and are describable using linear equations. However, as cosmological structures form and grow, the density fluctuations become nonlinear, and neglecting the nonlinearity introduces uncertainties to the calculated sound horizon distance. Scale-dependent bias and non-linear clustering are the sources of non-linearity effects in BAO measurements, as explained by:

$$P_{gal}(k, z) = P_{DM}(k, z) b^2(k, z) \tag{20}$$

Where P_{gal} and P_{DM} define the dark matter power spectrum and the galaxy power spectrum, respectively, and b denotes the bias at different scales, k. The peaks of the BAO in the standard ruler are shifted by moderate scale-dependent bias, and high bias can produce oscillations that are absent from the real D_M distribution. The power spectrum can be characterized in the redshift space where anisotropy due to the influence of galaxy unusual velocity along the line of sight occurs, thereby mitigating the scale-dependent bias. By applying Legendre polynomials to broaden the power spectrum, important information about the standard ruler can be extracted. Peak shifting and broadening are caused by non-linear clustering, which is related to the mode-to-mode connection. Because of the non-linear evolution of over-densities following recombination, the peak shift results in slight alterations on the standard ruler, which adds to the systematic inaccuracy. The power spectrum's small-scale oscillations are damped because of peak broadening (as illustrated in Fig. 5 [8]). As a result, the computed cosmological parameters and standard ruler have reduced accuracy.

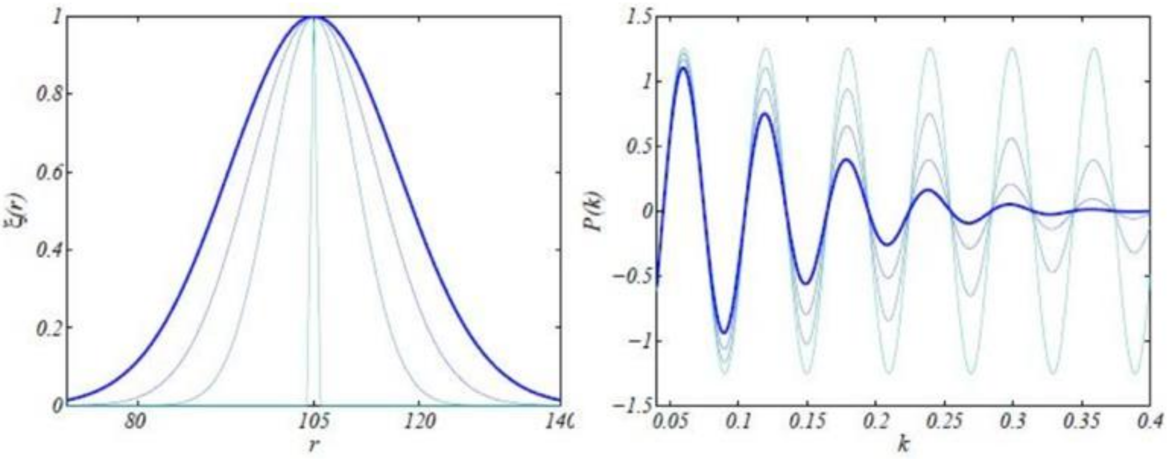

Figure 5. Non-linear clustering causes peak broadening in the correlation function and damping in the power spectrum [8].

Moreover, the formula assumes that the universe's matter and radiation contents are perfectly fluidly described. Baryons and radiation have more complex behavior, including particle interactions and hydrodynamic effects. Hence, assuming baryons and radiation as smooth fluids introduces uncertainties in the measurements of the sound horizon distance [17]. Furthermore, another source of mistake is the redshift at which observations are performed. Since the Hubble constant measurements are obtained from the radial BAO scale, even a modest redshift error of 1% will result in a considerable uncertainty in the BAO observations. This will also have a strong impact on the measurement of the Hubble constant. A larger area of the universe must be examined in order to increase the photometric redshift accuracy.

3.3. Gravitational Waves

Gravitational wave detectors detect and measure extremely faint ripples in spacetime propagating through the universe. These measurements are subject to statistical uncertainties arising due to several factors. These uncertainties can arise due to the noise in the detector, the sensitivity of the instrument, and the statistical properties of the gravitational wave signal. The noises can arise from various sources that limits the sensitivity and accuracy of the detectors. Thermal noise or Brownian noise arises from the motion of random molecules and atoms in the components of the detector, introducing fluctuations in the detector's response. Quantum noise, which is associated with the nature of light and the measurement of photons, introduces quantum fluctuations and contributes to the overall noise in the measurements. As in gravitational wave detectors, laser is used to measure the tiny displacements of baryonic matter caused by gravitational waves. Seismic noise from the Earth can propagate through the detector's structure and affect the precision of the measurements. Environmental noise, such as electromagnetic interference, acoustic noise, and magnetic field fluctuations can contribute to the noise in gravitational wave detectors (as depicted in Fig. 6) [18]:

$$L(f) = 2 \cdot \frac{N_{grav}(f)}{(2\pi f)^2} \tag{21}$$

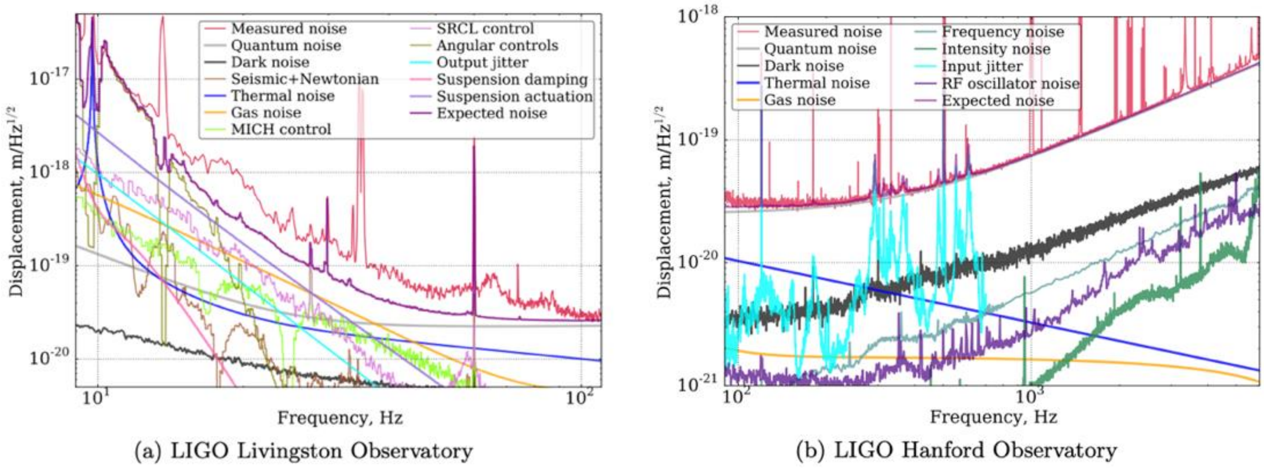

Figure 6. The noise budget for the two LIGO detectors, L1 and H1,'s gravitational wave. Low frequency curves for the L1 detector are the focus of Plot (a), whereas high frequency curves for the H1 detector are displayed in Plot (b) [18].

The gravitational wave signal at frequency f is represented by its amplitude, which is the gravitational wave strain, L(f). It is an indicator of how much spacetime is being stretched and compressed by the gravitational wave. The term $N_{grav}(f)$ describes the noise or disturbances that can interfere with the measuring of gravitational waves. The equation implies that the observed gravitational wave strain at a given frequency is determined by the gravitational wave noise at that frequency. This implies that to achieve a more precise computation of the gravitational wave strain, it is crucial to reduce the sources of noise and improve the signal-to-noise ratio when detecting or measuring gravitational waves. The following is an expression for $N_{grav}(f)$:

$$N_{grav}(f) = \beta \rho G N_{sei}(f) \tag{22}$$

Where G is the gravitational constant, ρ is the medium density, β is a constant or coefficient, and $N_{sei}(f)$ is a parameter related to the system's noise source.

Gravitational waves are generated by various astrophysical phenomena, such as binary BH and neutron star mergers, supernovae, and spinning compact object. It is extremely challenging to model these sources accurately given the complex physics involved. Theoretical models used to predict gravitational waves from those sources often rely on simplifying assumptions and numerical simulations, introducing uncertainties in its measurements. The uncertainties can also arise from unknown physics and thus the limitations in the computational methods employed.

4. Observed Tension in the Hubble Constant

The observed Hubble tension refers to the discrepancies between different measurements of the Hubble parameter. Various methods, such as using Cepheid variables and supernovae as standard candles, studying the BAO, and detecting gravitational waves, have yielded somewhat disparate results, leading to the observed tension. Observing the values measured, considerable tension exists as shown in Fig. 7 and Fig. 8 [19-22].

The implications of the Hubble tension for cosmological models are significant and have sparked considerable debate and interest within the scientific community. The presence of new physics is one of the major aspects of discussion. The Hubble tension suggests the possibility of unknown or unaccounted physical phenomena that impacted the expansion rate of the universe and thus the Hubble parameter measurements, leading to the exploration and discussion of alternative cosmological frameworks and introducing modifications to the cosmological equations. Alternative models for dark energy, such as dynamical or interacting dark energy scenarios have been proposed, introducing new fields or interactions that potentially affects the expansion rate, resolving the discrepancy [23]. Another area of exploration is modifications to the theory of gravity, motivating investigations into gravity theories and proposing changes to the equations that describe gravity's behavior on cosmological scales [24].

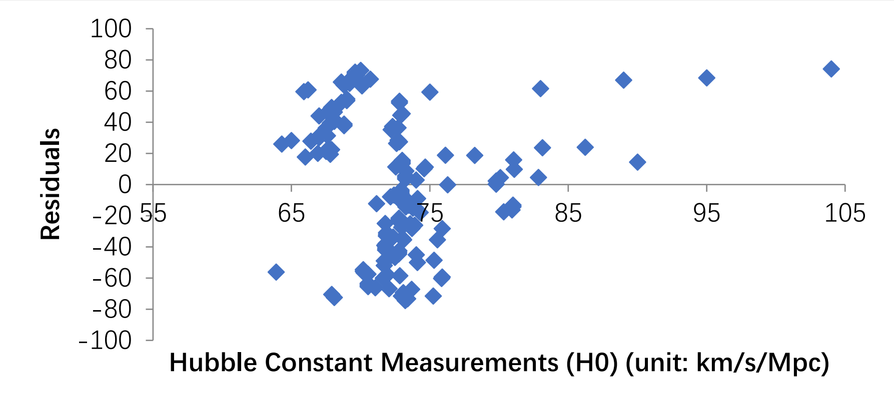

Figure 7. Residual plot of 150 Hubble constant measurements data from [19-22].

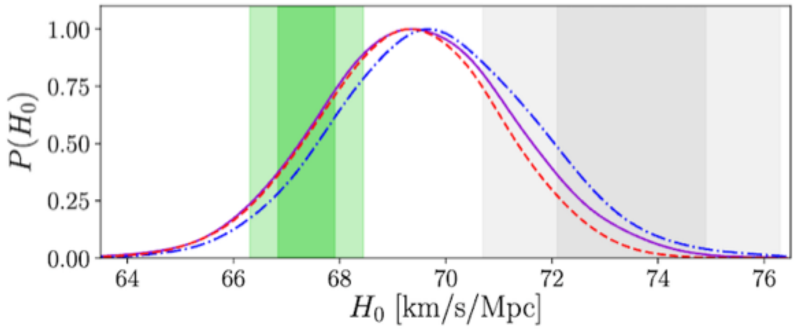

Figure 8. Constraints on H0 derived from BAO data. The vertical green bars line up with the Planck mission's best-fit ΛCDM. The measurement of H0 by the SH0ES (Supernovae, H0, for the Equation of State) study is indicated by the grey bands [22].

5. Shedding Light on Underlying Physics and Potential Modifications

There is interest in investigating possible changes to the mainstream cosmological model because of the Hubble tension. Parnovsky proposes in his paper "Possible Modification to Standard Cosmological model to resolve tension with Hubble Constant Values" [25] that the flat realivistic isotropic homogeneous model be taken into consideration, where A(z) is a parameter that describes the divergence from the ΛCDM model. The following equation describes A(z):

$$A(z) = \sqrt{\frac{\frac{\rho_m}{\rho_{m0}}\Omega_{m0}+\frac{\rho_{dc}}{\rho_{de0}}\Omega_{de0}}{\Omega_{\Lambda 0}+\Omega_{m0}(1+z)^3}} \tag{23}$$

Where ρ_{dc} is the dark energy density, ρ_{de0} is the current dark energy density, Ω_{de0} is the current dark energy density parameter, and $\Omega_{\Lambda 0}$ is the current density parameter for the cosmological constant. The matter density is represented by ρ_m, its current value by ρm0, and its current density parameter by Ω_{m0}. A(z) could deviate from one, specifically in the early universe. The observed tension denotes that in different cosmological eras, A(z) can be different, and thus, by stating $A(z_r) \approx 0.92$, Parnovsky is suggesting that the modification should be such A deviates from its ΛCDM value of 1 and instead takes on a value close to 0.92 at the recombination era, resolving the tension.

Parnovsky also introduced the modified equation of state,

$$w(z) = B(1+z)^{\alpha}, B < 0, a = const. \tag{24}$$

Here, w(z) is the equation of state parameter as a function of z and B is constant. Since dark matter is usually believed to be pressure less in the traditional cosmological model, minor negative pressures associated with it can be taken into account.

6. Conclusion

To encapsulate, this study highlights the different methods and their systematic uncertainties in measuring the Hubble constant and discusses the underlying physics and potential modifications to the standard cosmological model. According to the analysis, various measurement methods, specifically Cepheid variables and supernovae, baryon acoustic oscillations, and gravitational waves have been used to determine the Hubble constant but yield somewhat disparate results. Systematic uncertainties play a crucial role in Hubble constant measurements, including factors like assumptions, calibration, and modeling uncertainties specific to each measurement method. The observed Hubble tension suggests the possibility of unaccounted or unknown physical phenomena and sparked interest in potential modifications to the standard cosmological model, including alternative models for dark energy and exploration of the properties of dark matter. Looking forward, developments in computational techniques and improvements to alleviate the systematic uncertainties in each measurement method should be made. The significance of this study is not merely limited to advancing the understanding of the universe, but also addressing implications for cosmological models or theories and guiding future research directions.

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