Comparison of the Different Celestial Length Measurements Approaches: Trigonometric Parallax, Cepheids and Type Ia Supernovae

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Abstract. As a matter of fact, astronomical measurements are one of the important topics in astronomy especially in recent years, with significant implications for spacecraft navigation within the solar system, stellar modeling, cosmic scales as well as the evolution. On this basis, this study presents several existing measurement methods and provides detailed explanations of three commonly used methods, i.e., trigonometric parallax, cepheid, and type Ia supernovae. To be specific, the principles as well as instruments associated with these methods are detailly discussed and analyzed. According to the analysis, the typical observations as well as current state-of-art results are also presented. At the same time, the current limitations such as equipment resolution, model calibration, and differentiation of background galaxies are compared and clarified. Furthermore, the prospects for future updates and developments of new approaches are also explored in the meantime. Overall, these results shed light on guiding further exploration of length measurement for celestial.

Keywords: Celestial length measurements, trigonometric parallax, cepheid, type Ia supernovae.

1. Introduction

Measuring the distance of celestial bodies has a long history and is known as astrometry, which is a fundamental branch of astronomy [1-3]. Accurate distance measurements are crucial for spacecraft navigation within the solar system, allowing to send probes to other planets and gather useful data [4].

By obtaining apparent brightness to predict actual luminosity and modeling stellar structure and evolution, human beings can estimate the age of celestial bodies [5]. Galaxy distance measurement also provides insight into the scale and evolutionary history of the universe [6, 7]. Additionally, by measuring the Hubble constant (H_0), one can verify the validity of the Big Bang model and study large-scale peculiar velocity fields to infer distribution and properties of dark matter and dark energy.

As a matter of fact, there are several different methods to realize the length measurement in recent years. With this in mind, this study will compare three common methods to illustrate the current results, limitations as well as the further improvements strategy. The rest part of the paper is organized as follows. The Sec. 2 will discuss typical methods for astronomical distance measurement; subsequently, the Sec. 3-5 will introduce three methods including trigonometric parallax, variable stars, Ia supernovae, respectively; afterwards, this study will offer a summary to compare limitations among these methods while looking forward to future developments in Sec. 6.

2. Basic Decsriptions

There are many feasible methods for measuring celestial distance [4-11]. Here are some typical measurement methods briefly introduced as presented in Fig. 1. Radar ranging is mainly used in the solar system, which is limited by the maximum power of the radar transmitter. By measuring the time difference between the transmitted pulse and the echo pulse, the distance between the target celestial body can be directly obtained. The accuracy is generally above 10^8, and is limited by the measurement time. Typical results include the first measurement of the distance to Venus in 1961, and the distances of Mercury and Mars to the Earth were subsequently measured.

Moving star cluster refers to the dispersed star cluster whose member stars have similar self-motion speed and direction. Because they are close to the sun, the convergence point can be determined. The actual distance of the star cluster can be obtained by measuring the geometry of the convergence point, the self-motion speed, and the apparent velocity. This method has a high accuracy in the determination of the near-star, which can be confirmed by the triangular parallax method. The main measurement range is within 500pc. This is a method to identify the distance of Pleiades and Perseus clusters, providing some ideas for the subsequent provision of many types of stars as standard candles.

Main sequence fitting, which is based on the understanding that all main sequence stars have the same properties and the main sequence stars of the same spectral type have the same absolute magnitude, compares the actual observed luminosity with the theoretical value in order to obtain the target distance. The accuracy is limited by the accuracy of the main sequence in the H-R diagram of the standard cluster used for reference, and the measurement range is mainly between 100pc and 100kpc. Using this method requires the observation of enough stars' brightness and color. The stars in the same cluster or galaxy have similar ages and formation environments, and are distributed along a zero-age main sequence line (ZAMS), so it is often used to the open cluster.

In addition, there are spectroscopic parallax, statistical parallax, Baade-Wesselink method, galaxy surface brightness functions, planetary nebula luminosity, $D_n - \sigma$ relation, lensed quasar variability phase shift and other methods. Next, this study will introduce trigonometric parallax, cepheid and type Ia supernova [1].

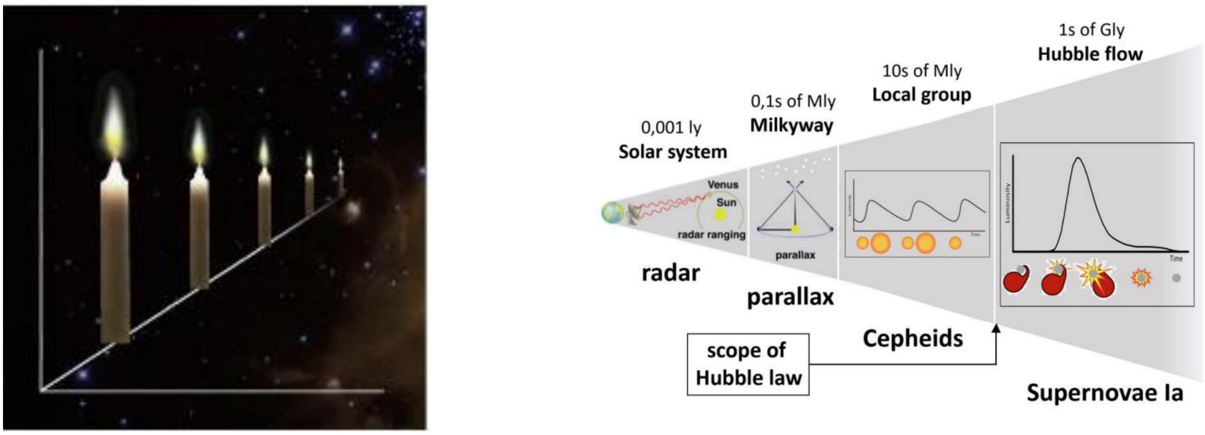

Figure 1. The application scope of each method is illustrated.

3. Trigonometric Parallax

The difference in the observed direction of a celestial body by observers at different locations is referred to as parallax. When measuring the parallax of celestial bodies within the solar system, the baseline used is Earth's radius, and this measured parallax is known as diurnal parallax. For stars, the baseline used is the average distance between the Sun and Earth, resulting in annual parallax measurements. The distance can be calculated using the trigonometric relationship d=1AU/tanπ. The measurable range for trigonometric parallax remains limited to less than 100 pc. It should be noted that proper motion of celestial objects needs to be subtracted during measurement. Currently, optical measurements are primarily relied upon for stars while radio observations are utilized for pulsars when estimating distances through trigonometric parallax measurements.

The Hipparcos satellite mission successfully employed this method to measure distances for over 100,000 stars with a nominal visual magnitude ranging from V = 7.3-9.0 mag across its full range (V = 12.4 mag). Positional accuracy reached up to 0.002 arcseconds (B=9 mag), while parallax precision also achieved 0.002 arcseconds (B=9 mag). Proper motion precision remained below 0.002 arcseconds per year (B=9 mag), with systematic errors smaller than 0.001 arcseconds.

The Gaia mission precisely measures the positions of approximately one billion stars in the Milky Way and other galaxies within Local Group Galaxy Cluster with a remarkable level of accuracy, up to 24 microarcseconds. This represents a significant advancement compared to Hipparcos. For example, Gaia's main mirror has a larger collecting area, allowing it to capture over 30 times lighter than its predecessor. As a result, Gaia enables more sensitive and precise measurements. Additionally, Gaia surpasses Hipparcos by achieving an exceptional level of accuracy in determining star positions that is 200 times superior. These changes in stellar positions and motions are recorded as minute angles. To capture images effectively, Gaia utilizes CCD technology, i.e., a highly efficient camera capable of simultaneously imaging multiple celestial objects across wide angle, unlike the Hipparcos which was limited to recording information from only one celestial object at a time.

4. Cepheids

Cepheids are a type of high-luminosity periodic pulsating variable stars, whose luminosity varies between F-type stars and G-K-type stars and light cycle is between 1 and 135 days. The light cycle of Cepheids is linear with its absolute magnitude. As one of the standard candles, its brightness is inversely proportional to the square of the distance. Through the perigee relationship, the distance to the galaxy or cluster can be calculated. The measurement range is roughly 100pc-100kpc. Early typical results include the discovery of Cepheids M33, M31 and NGC 6822 by Edwin Hubble in 1923, and the estimation of the distance. Wisniewski and Johnson (1968) made UBV RIJKL photoelectric observations of Cepheids in the Milky Way.

The Kepler space telescope launched in 2009 has a diameter of 0.95 meters and a field of view of about 100 square degrees. The data revealed the period doubling and the additional frequency components below the millimagnitude level in fundamental-mode stars. The Hubble Space Telescope (HST) can observe Cepheids at 80 Mpc, with a limit of $m_v \approx 26$. The goal of the SH0ES project is to calibrate the host galaxies of H_0 and type Ia supernova (SNe Ia) by using the Cepheid variable to calibrate the extragalactic distance scale, using HST F555W, F814W and F160W, and NGC 4285 as the main observation targets. The measured H_0=73.04±1.04$kms^{-1}Mpc^{-1}$, with an uncertainty of 1% [8].

As a new generation of space telescopes, James Webb Space Telescope (JWST) has an angular resolution comparable to WFC3/UVIS in the visible band. In the infrared band, the observation is less affected by interstellar dust, which can reduce the impact of dust effect on the observation results. It can be used to verify the HST measurement data. The current JWST observation results show that the HST measurement is affected by the dense background star, and the error may be relatively large, and the system deviation at 20 Mpc may exceed 10%. A typical result is given in Fig. 2 [9].

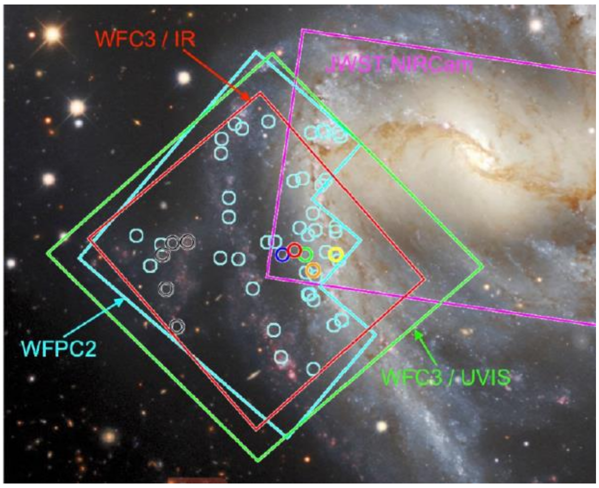

Figure 2. Observation footprints of NGC 1365 by JWST NIRCam (magenta), HST WFPC2 (cyan), WFC3/UVIS (green), and WFC3/IR (red). The circles indicate the positions of Cepheid variables [8].

5. Type Ia supernovae

Type Ia supernovae spectra exhibit strong absorption lines of SiIIwithout hydrogen lines (see Fig. 3). The progenitor star models can be broadly categorized into the accretion model involving carbon-oxygen white dwarfs and the merger model comprising carbon-oxygen white dwarfs, although a consensus has not yet been reached [10]. Due to the similar masses of white dwarfs producing SNe Ia, their luminosities are also comparable (with peak brightness related to the shape of their light curves). By comparing apparent magnitudes with absolute magnitudes (typically ranging from -18.5 to -19.5 MB), distances can be estimated with an accuracy of approximately 3% using this method. The primary measurement range extends within 200 Mpc.

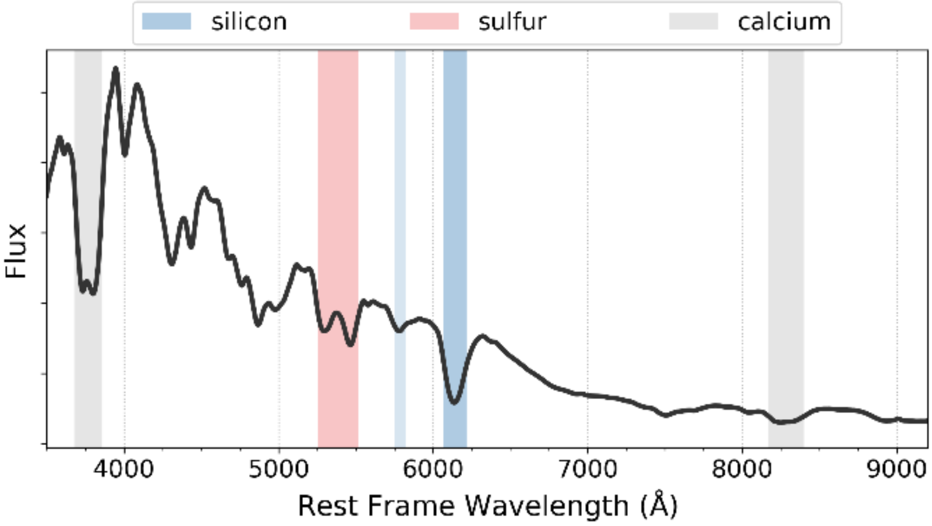

Figure 3. Type Ia supernova spectrum diagram.

Due to the significant impact of interstellar dust on the blue end of the spectrum, supernova spectra are primarily observed using near-infrared wavelengths. Observations can be conducted utilizing advanced instruments such as HIRES at Keck Observatory, UVES and FORS at Very Large Telescope, FOCAS and HDS at Subaru Telescope, DBSP at Palomar Observatory, EFOSC2 at La Silla Observatory, STIS and COS on HST, NIRSpec and MIRI on JWST. NIRSpec offers a range of observation modes including micro-shutter spectroscopy, integral field spectroscopy, and slitless spectroscopy. It enables simultaneous acquisition of spectra for over 100 objects within a field of view spanning 9 square arcminutes. Moreover, it provides medium-resolution spectra in the wavelength range from 1 to 5 micrometers as well as low-resolution spectra in the wavelength range from 0.6 to 5 micrometers. Consequently, it exhibits immense potential in facilitating supernova detection.

Owing to the scarcity of supernovae in most galaxies within a given timeframe, it is not feasible to randomly select a galaxy and determine its distance solely through supernova observations. Spectroscopic surveys play a crucial role in the quest for supernovae. The Sloan Digital Sky Survey (SDSS) Supernova Survey employed the SDSS 2.5-meter telescope to conduct an array of searches for these cosmic explosions. The primary objective of the SDSS supernova project was to identify and measure a substantial number of Type Ia supernovae spectra, which serve as standard candles aiding in unraveling the nature of cosmic acceleration. Following in the footsteps of PTF and iPTF, ZTF represents an advanced transient survey initiative that leverages an enhanced 48-inch (1.2-meter) Schmidt telescope situated at Palomar Mountain. ZTF swiftly scans for transients like supernovae and spectroscopically classifies them accordingly. The Large Synoptic Survey Telescope (LSST), now referred to as Vera C. Rubin Observatory's principal optical telescope, is an ongoing construction endeavor anticipated to commence operations during the mid-2020s period. It will undertake a decade-long survey with its highly automated observing program enabling it to unearth numerous supernovae while acquiring their spectra and light curves [10, 11].

6. Conclusion

To sum up, trigonometric Parallax is primarily employed for measuring the distances to stars near Earth. Due to Earth's annual revolution around the Sun, a minimum of six months is required to complete a parallax measurement, thereby limiting prompt results acquisition. The precision of telescopes and instruments restricts the accuracy of these measurements. In cases where the distance is substantial, it leads to diminished accuracy in parallax angles (typically at milliarcsecond levels). Ground-based observations are susceptible to atmospheric turbulence-induced refraction of starlight, resulting in less precise measurements compared to space telescopes. To ensure accurate outcomes, caution must be exercised against excessive motion or proximity of background stars while comparing their positions with target stars. Proper motion should also be accounted for during measurements.

The Cepheid variable star method is mainly used for measuring nearby star clusters and galaxies outside the Milky Way galaxy. When stellar pulsations have significant amplitudes, period-luminosity relationships may exhibit non-linear characteristics that require complex models for correction. The brightness of Cepheid variables depends on metallicity levels. Higher metallicity affects their internal structure: higher opacity leads to changes in temperature, radius, and luminosity of these stars. When using this method, it is crucial to distinguish between Cepheid variables and overall backgrounds within host galaxies since crowded regions like galactic cores or globular clusters may introduce additional sources mixing with Cepheids' light and causing measurement errors. Additionally, corrections for interstellar dust reddening effects are necessary as Cepheid variables are often found within dusty disks predominantly seen in spiral or irregular galaxies.

SNe Ia are primarily utilized for distance measurements in extragalactic astronomy. Distance measurements on a cosmic scale can introduce a range of potential errors, including systematic differences observed in SNe Ia at different redshifts. If the characteristics of SNe Ia in the early universe differ from those observed today, it may introduce bias into distance inferences based on them. The accuracy of redshift measurements and assumptions about cosmological models, such as the expansion rate of the universe, can also impact distance accuracy for very distant SNe Ia. Additionally, there is a sample bias known as Malmquist bias due to easier detection of brighter supernovae, leading to underestimated distance estimates. To ensure accurate calibration of their absolute luminosity and maintain their effectiveness as standard candles, precise calibration relying on distance scales within the Milky Way galaxy or other external indicators like Cepheid variables is necessary despite uncertainties associated with these indicators themselves. Furthermore, variations in peak brightness and diversity in spectra and light curves need to be corrected for due to unclear progenitor star models for SNe Ia. Moreover, host galaxies and interstellar dust like Cepheid variables also influence this method of measuring distances.

Future astronomical distance measurement techniques will benefit from advanced observational facilities and more sophisticated theoretical models. The next generation of telescopes, including the Extremely Large Telescope (ELT), the Giant Magellan Telescope (GMT), the Thirty Meter Telescope (TMT) on the ground, and the James Webb Space Telescope (JWST) as well as future space-based telescopes such as the Nancy Grace Roman Space Telescope, will provide higher resolution and sensitivity observation data. The standard sirens method for inferring distances to gravitational wave sources based on their amplitude has been developed through gravitational wave astronomy; this method can be further refined with data from detectors such as LIGO, VIRGO, and LISA. Phenomena like active galactic nuclei (AGN), red supergiant, Type II supernovae, acoustic oscillations in cosmic microwave background radiation, time delays in gravitational lensing events, and planetary transient events may serve as new standard candles after calibration. Studies on dark matter-dark energy interactions, redshift space distortions (RSD), and baryon acoustic oscillations (BAO) will advance the understanding of cosmic expansion while also improving accuracy in astronomical distance measurement methods. Observations using multiple messengers across different wavelengths such as neutrinos and gravitational waves can provide a more comprehensive view of the universe that helps calibrate distance indicators while eliminating systematic errors.

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