

## Research on the annual average thermal power of heliostat field in tower solar energy

Tian Luan<sup>#</sup>, Hongbo Wang<sup>#</sup>, Jinxia Ren<sup>#</sup>, Yujie Ren<sup>\*,#</sup>, Hao Su<sup>#</sup>

Academy of Reading, Nanjing University of Information Science & Technology, Nanjing, China, 210044

\* Corresponding author: 202183100045@nuist.edu.cn

<sup>#</sup>These authors contributed equally.

**Abstract.** The concentrating mirror field is a concentrating and heat collecting subsystem in the tower solar thermal power station, and its optical performance directly affects the solar energy utilization efficiency of the power station. In this paper, the coordinate system of heliostat field and single heliostat is established, and the grid division technology is introduced to describe its reflection effect on sunlight and its occlusion. In this paper, the shadow occlusion loss that affects the optical efficiency is divided into three parts, including incident light occlusion loss, reflected light occlusion loss and absorption tower shadow occlusion loss. For any time, the output power of the heliostat near the solar direction is small, while the output power of the heliostat far away from the solar direction is large. Compared with other influencing factors, the cosine loss has the greatest influence on the heliostat field, and the center of the absorption tower installed in the circular heliostat field is obtained. The size of the heliostat is a square with a side length of 6m, the installation height is 4m, and the annual average optical efficiency is 0.6216 when the position of the heliostat is determined. The annual average output thermal power is 36.3958 MW, and the annual average output thermal power per unit mirror area is 0.5794Kw/m<sup>2</sup>.

**Keywords:** Heliostat Field, Mesh Subdivision, Specular Reflection, Ray Tracing.

### 1. Introduction

As the global demand for renewable energy and clean energy continues to grow, solar thermal power stations are gradually becoming an indispensable part of the power system with their unique advantages. Among them, tower solar thermal power plants, in particular, have shown high annual average power generation efficiency, which makes it comparable to traditional fossil fuel power plants in terms of sustainability and stability. For the study of the optimization design of the heliostat field, Liu Jianxing used the radioactive grid method to establish a simulated mirror field. Through the iterative optimization process of adaptive gravitational search, the distribution of cosine efficiency, shadow occlusion efficiency, atmospheric attenuation efficiency and comprehensive efficiency at different time points in the mirror field was obtained [1]. Yang Hongtao's team used the ray tracing method to establish the heliostat focusing model, and simulated each beam of light in the reflected light cone [2]. Therefore, in this paper, the grid division method is used to analyze the solar reflection and shading, and the ray-tracing method is used to solve the incident light shading loss, calculate the optical parameters at a specific point in time, and derive the monthly average optical efficiency. Finally, the annual average output thermal power and the annual average output thermal power per unit mirror area are calculated. In this paper, the above results are obtained under the premise of determining the position, size and installation height of the heliostat mirror, and the correlation coefficient can be changed according to the actual situation so as to meet the requirements and find the most suitable arrangement for the heliostat mirror field design.

## 2. Establishment of Heliostat Thermal Power Model

### 2.1. the System of Coordinates

In order to describe the coordinates of the sun and the heliostat, this paper establishes a heliostat field coordinate system  $Oxyz$  and a single heliostat mirror coordinate system  $O_i x_i y_i z_i$ . The intersection point of the central axis of the absorber tower and the circular heliostat field is the origin of the heliostat field coordinate system. The east direction is the positive direction of the x-axis, the north direction is the positive direction of the y-axis, the direction perpendicular to the circular heliostat field is the z-axis, and the vertical direction is the positive direction of the z-axis. The three-dimensional space rectangular coordinate system is established. The coordinate system of a single heliostat is based on the center of the heliostat as the origin, the normal direction of the heliostat is the z-axis direction, the x-axis is parallel to the ground, the y-axis is perpendicular to the z-axis and the x-axis, and the upward direction is the positive direction to establish the coordinate  $O_i x_i y_i z_i$ . As shown in Fig.1.

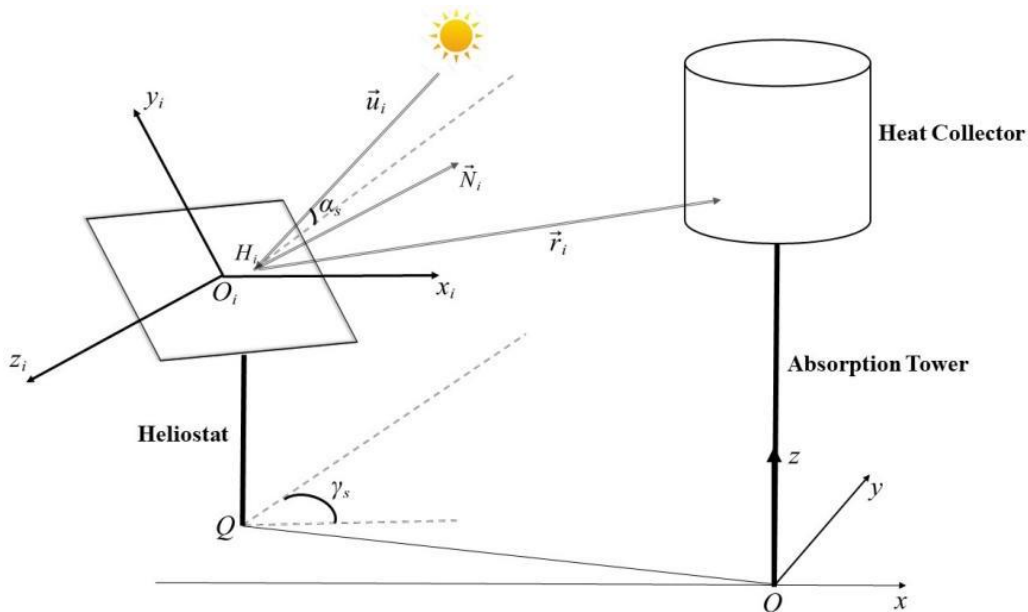


**Figure 1.** Diagram of Coordinate System

It can be seen from Fig.1 that  $\alpha_s$  is the solar elevation angle and  $\gamma_s$  is the solar azimuth angle.

### 2.2. Two Vector Representations

#### (1) Direction vector of central axis of incident light cone

The central axis direction of the solar incident light is closely related to the solar altitude angle and the solar azimuth angle, and the solar altitude angle and the solar azimuth angle are related to the solar declination angle, the solar hour angle and the local latitude. The solution method of known solar declination angle [3].

$$\sin \delta = \sin \frac{2\pi D}{365} \sin \left( \frac{2\pi}{360} 23.45 \right) \quad (1)$$

Among them,  $\delta$  is the solar declination angle, and  $D$  is the number of days starting from the vernal equinox as the 0th day. The formula of solar hour angle [4]:

$$\omega = \frac{\pi}{12} (ST - 12) \quad (2)$$

Where  $ST$  is the local time and  $\omega$  is the solar time angle.

The solar elevation angle [5] and the solar azimuth angle can be calculated by the solar declination angle, the solar hour angle and the local latitude. The calculation formula of the solar elevation angle and the solar azimuth angle [6]:

$$\sin \alpha_s = \cos \delta \cos \varphi \cos \omega + \sin \delta \sin \varphi \quad (3)$$

$$\cos \gamma_s = \frac{\sin \delta - \sin \alpha_s \sin \varphi}{\cos \alpha_s \cos \varphi} \quad (4)$$

Among them,  $\varphi$  is the local latitude (the north latitude is set to be positive in the title),  $\alpha_s$  is the solar altitude angle, and  $\gamma_s$  is the solar azimuth angle. The above solar elevation angle and solar azimuth angle can be used to calculate the unit vector of the central axis of the incident light cone (the direction is from the sun to the mirror reflection point) in the heliostat field coordinate system can be expressed as:

$$\vec{u}_i = (-\sin \gamma_s \cos \alpha_s, \cos \gamma_s \cos \alpha_s, \sin \alpha_s) \quad (5)$$

## (2) Heliostat Normal Vector

The reflected light vector is set as  $\vec{OB}$ , where  $O(a_i, b_i, c_i)$  is the central position of the heliostat, and  $B(a_0, b_0, c_0)$  is the central position of the collector receiving light. Then  $\vec{OB} = (a_0 - a_i, b_0 - b_i, c_0 - c_i)$ . Then the reflected unit vector can be obtained:

$$\vec{r}_i = \frac{\vec{OB}}{|\vec{OB}|} \quad (6)$$

From the reflection theorem, the anti-direction unit vector of the incident light and the reflection unit vector, the unit vector of the heliostat normal can be obtained:

$$\vec{N}_i = \frac{-\vec{u}_i + \vec{r}_i}{|-\vec{u}_i + \vec{r}_i|} \quad (7)$$

## 2.3. Grid Division of Heliostat Mirror Surface

For any heliostat, in order to describe its reflection effect on sunlight and its occlusion, the grid is divided into  $10 \times 10$  grids, and the point in the centre of the grid is selected as the representative point of the grid. The performance of the representative point is the performance of the grid. When the representative point is occluded and cannot reflect the light effectively, it is considered that the grid cannot reflect the light effectively.

## 2.4. Two Vector Representations

Based on the grid division of the heliostat, it is judged whether the grid is occluded by judging whether the center representative point of each grid is occluded by shadow. Considering that the sunlight is a conical ray, and it is difficult to require the reflection of each ray, this paper analyses the central ray in the conical ray. Suppose that the vector of a central ray in the heliostat field coordinate system is:

$$\vec{V}_0 = (a, b, c) \quad (8)$$

Shadow occlusion efficiency can be calculated by shadow occlusion loss:

$$\eta_{sb} = 1 - \text{Shadow blocking loss} \quad (9)$$

Among them,  $\eta_{sb}$  is the shadow occlusion efficiency. Shadow occlusion loss is divided into three parts to study. The first part is the loss caused by the occlusion of the front heliostat when the light emitted by the sun reaches each mirror, which is called the incident light occlusion loss. The second part is that when the mirror can receive the incident light, the reflected light is blocked by the front heliostat, resulting in the heat loss that cannot be absorbed by the collector, which becomes the reflected light occlusion loss; the third part is the loss of the heliostat field caused by the shadow formed by the absorption tower, which is called the absorption tower occlusion loss.

### (1) Occlusion Loss of Incident Light

Requiring the occlusion loss of the incident light [7], it is necessary to project the remaining heliostats to the plane where the heliostat is located. In order to facilitate the projection operation of the heliostat, it is necessary to establish the coordinate transformation relationship between the coordinates. According to the relationship between the heliostat mirror surface coordinates and the heliostat field coordinate system, it can be seen that the coordinate transformation includes the rotation of the coordinate system and the movement of the coordinate system.

According to the principle of coordinate rotation, the coordinate transformation from the rotating coordinate system to the pre-rotating coordinate system is a matrix T, as shown in formula (10).

$$T = \begin{pmatrix} d_x & d_y & d_z \\ e_x & e_y & e_z \\ f_x & f_y & f_z \end{pmatrix} \quad (10)$$

Where,  $(d_x, e_x, f_x), (d_y, e_y, f_y), (d_z, e_z, f_z)$  are the unit vectors of each coordinate axis in the rotating coordinate system in the pre-rotating coordinate system.

Assuming that the occlusion between two heliostats is calculated, the two heliostats are named A and B below. In order to calculate the coordinates of a point in the A heliostat falling into the B mirror through the light. Firstly, a certain point  $H_1$  in the A heliostat is converted to the heliostat field coordinate system by using the conversion relationship T, and it is expressed as  $H'_1$ . Secondly, the point  $H'_1$  in the ground coordinate system is converted to the B heliostat coordinate system, and the obtained point is  $H''_1$ . Finally, the incident light vector in the heliostat field coordinate system is converted to the B heliostat mirror coordinate system to obtain the light vector  $\vec{V}_a$ . Through the above conversion relationship, the coordinate of the A heliostat mirror coordinate can be converted to the B heliostat mirror coordinate system, and the direction vector of the incident light in the B heliostat mirror coordinate system can be obtained. According to the principle of two points and one line, the intersection points  $H_2(x_2, y_2, 0)$  of the incident light and the B heliostat satisfies the following formula:

$$\frac{x_2 - x'_1}{a} = \frac{y_2 - y'_1}{b} = -\frac{z'_1}{c} \quad (11)$$

By solving this equation, we obtain:

$$\begin{cases} x_2 = \frac{cx'_1 - az'_1}{c} \\ y_2 = \frac{cy'_1 - az'_1}{c} \end{cases} \quad (12)$$

Through the above method, the four vertices of the heliostat A can be projected onto the plane where the heliostat B is located. By judging whether all the grid representative points are in the projection, the occlusion loss due to the incident light can be calculated.

### (2) Reflected Light Occlusion Loss

According to the principle of reversible optical path, to determine whether the light reflected by the mirror can reach the collector, it can be achieved by judging whether the reverse light emitted from the collector can reach the reflection point of the mirror.

Through the relationship between incident light, normal line and reflected light, the following formula can be obtained.

$$\vec{V}_2 = -\vec{V}_1 - 2 \left( -\vec{V}_1 * \vec{n}^T \right) \cdot \vec{n} \quad (13)$$

Where,  $\vec{V}_2$  is the reflection vector,  $\vec{V}_1$  is the incident vector, and  $\vec{n}$  is the unit vector.

At this time, the direction of the reflected light can be regarded as a new incident light vector after the line is reversed, and the incident light is calculated according to the algorithm of the incident light occlusion, and the loss of the reflected light occlusion is obtained.

### (3) Absorption Tower Occlusion Loss

Aiming at the shielding loss of the absorption tower, the method similar to the representative point of the heliostat grid centre can be used to calculate, and the position of the support rod of the heliostat is the representative centre; by projecting the absorption tower and judging whether the position of the heliostat is in it, it is judged whether the heliostat is occluded. If the centre position of the heliostat is occluded, it is considered that the heliostat is completely occluded. If its centre position is not occluded, it is considered that the heliostat is not occluded.

The occlusion range of the heliostat is required, and the projection of its top on the ground can be obtained by line. Then the projection is connected with the bottom of the absorption tower to obtain the projection of the absorption tower on the ground. It is assumed that there is a point  $A(x_3, y_3, z_3)$  at the top, and the direction vector of the sunlight is  $\vec{V}_0(a, b, c)$ . When the intersection point with the ground is  $A'(x', y', 0)$ , it can be obtained from the principle of two points and one line, which conforms to the following relationship:

$$\frac{x' - x_3}{a} = \frac{y' - y_3}{b} = -\frac{z_3}{c} \quad (14)$$

By solving this equation, we obtain:

$$\begin{cases} x' = \frac{cx_3 - az_3}{c} \\ y' = \frac{cy_3 - az_3}{c} \end{cases} \quad (15)$$

Through the above method, the projection of a point at the top of the absorption tower on the ground can be obtained. By selecting multiple points at the top of the absorption tower, the projection of the top of the absorption tower on the ground can be calculated. By connecting it with the bottom of the absorption tower, the projection of the absorption tower on the ground can be obtained. By judging whether the centre of the heliostat is in its shadow, the occluded heliostat can be judged.

## 2.5. Cosine Efficiency

Because the position of the sun is different at different times, it is impossible for the heliostat to be facing the sun at every moment. This tilt of the heliostat will reduce the area where the heliostat can reflect the sun's light, resulting in cosine efficiency. The cosine efficiency is the cosine value of the incident angle of the sun on the heliostat surface. The smaller the incident angle, the larger the effective receiving area of the heliostat, and the higher the cosine efficiency [8]. In this paper, the calculation method of cosine efficiency is obtained, as shown in Formula (16):

$$\eta_{cos} = \cos\theta \quad (16)$$

## 2.6. Atmospheric Transmissivity

As the light is focused by the heliostat and then reflected to the collector tower, the impurities and dust in the air will partially block the light and cause losses. The atmospheric projection rate  $\eta_{at}$  is affected by water vapor saturation, clouds, atmospheric thickness, dust, polluting gases, air pressure, temperature, wind and other factors. Therefore, the value of atmospheric transmittance has a nonlinear relationship with distance. The atmospheric transmittance formula (17) below 1000 m above sea level shows that:

$$\eta_{at} = 0.99321 - 0.0001176d_{HR} + 1.97 \times 10^{-8} \times d_{HR}^2 \quad (d_{HR} \leq 1000) \quad (17)$$

Where  $d_{HR}$  is the distance from the centre of the mirror to the centre of the collector (unit: m).

## 2.7. Truncation Efficiency of Collector

The stage efficiency of the collector is the ratio of the number of rays absorbed by the collector to the total number of rays reflected by the heliostat, as shown in Formula (18):

$$\eta_{trunc} = \frac{\text{Collector absorbs energy}}{\text{Mirror total reflection} - \text{Shadow occlusion loses}} \quad (18)$$

Considering that the sunlight is a cone-shaped light, the reflected light is also a cone-shaped light, so some of the light cannot be irradiated on the collector, resulting in a truncation efficiency. To calculate the truncation efficiency, it is necessary to perform ray tracing and determine whether it will illuminate the collector. In order to facilitate the calculation, the method of heliostat grid division is still used to calculate the proportion of the reflected light absorbed by the centre representative point as the proportion of the reflected light absorbed by the grid. For a grid heliostat, the light emitted by its centre representative point is a cone light. It is necessary to divide the cone light into several rays, and calculate the absorption of these rays to reflect on the absorption of the entire cone light.

#### (1) Segmentation of Cone Light

For a bunch of cone-shaped light, it can be segmented according to certain rules, and the cone-shaped light can be converted into multiple rays. The half-angle of the sun's rays is 4.65mrad, so it can be divided at a uniform angle in the direction of its half-angle broadening, and divided in four directions perpendicular to it. With its central light, 21 rays can be obtained. The profile of the light cone is shown in Fig 2.

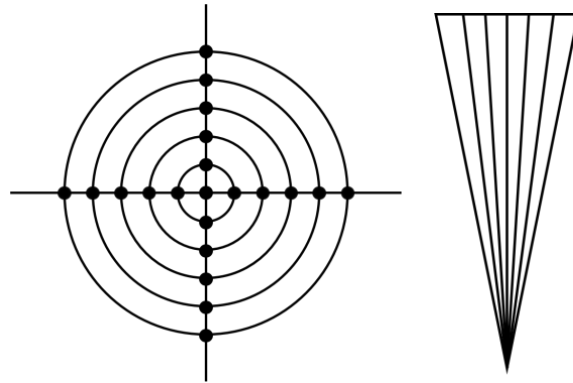


**Figure 2.** Light Cone Profile Diagram

#### (2) Absorption Judgment of Cone Light

By dividing the cone light, 21 rays can be obtained. For any one of the rays, when it can be illuminated on the collector, it must intersect with the judgment plane that passes through the central axis of the collector and is directly opposite to it.

Let the light direction vector be  $(l, m, n)$ , the coordinates of the representative point of the heliostat grid center are  $(x'', y'', z'')$ , and the intersection point with the judgment plane is  $(x, y, z)$ , then it satisfies Formula (19).

$$\begin{cases} \frac{x-x''}{l} = \frac{y-y''}{m} = \frac{z-z''}{n} \\ lx + my = 0 \end{cases} \quad (19)$$

After solving this equation, the intersection points of the other judgment planes are obtained, and whether they can be absorbed by judging whether they are within the range of the collector.

### 2.8. Annual Average Optical Efficiency of Heliostat Field

The optical efficiency of the heliostat field [9] consists of five parts: shadow shading efficiency  $\eta_{sb}$ , cosine efficiency  $\eta_{cos}$ , atmospheric transmittance  $\eta_{at}$ , collector truncation efficiency  $\eta_{trunc}$ , and specular reflectance  $\eta_{ref}$ , which is taken as a constant value 0.92. It can be obtained that the instantaneous optical efficiency of the heliostat at any time can be expressed by the product of the above five parts, as shown in Formula (20):

$$\eta = \eta_{sb} \times \eta_{cos} \times \eta_{at} \times \eta_{trunc} \times \eta_{ref} \quad (20)$$

For the daily average optical efficiency, this paper takes 9: 00, 10: 30, 12: 00, 13: 30 and 15: 00 as the representative, and the average optical efficiency at each time represents the average optical efficiency of the day. For the annual average optical efficiency, this paper uses the monthly average method to select one day as the representative day of the month in the 12 months of the year. The daily average optical efficiency of the representative day is averaged again to approximately represent the annual average optical efficiency of the heliostat field.

## 2.9. Annual Average Output Thermal Power of the Heliostat Field

To calculate the annual average output thermal power of the heliostat field, the output thermal power of the heliostat field must be obtained first to obtain the annual average output thermal power of the heliostat field. The output thermal power of the heliostat field is required to know the normal direct radiation irradiance [10]. See formula (21) for its solution.

$$\begin{cases} \text{DNI} = G_0 \left[ a + b \exp \left( -\frac{c}{\sin \alpha_s} \right) \right] \\ a = 0.4237 - 0.00821(6 - H)^2 \\ b = 0.5055 - 0.00595(6.5 - H)^2 \\ c = 0.2711 - 0.01858(2.5 - H)^2 \end{cases} \quad (21)$$

Where, DNI is the normal direct radiation irradiance,  $G_0$  is the solar constant with a value of  $1.36\text{Kw}/\text{m}^2$ , and H is the altitude. From this, the output thermal power of the heliostat field can be obtained, as shown in Formula (22):

$$E_{field} = \text{DNI} \cdot \sum_i^N A_i \eta_i \quad (22)$$

Where,  $E_{field}$  is the output thermal power of the heliostat field, N is the total number of heliostats,  $A_i$  is the daylighting area of the  $i$ th heliostat, and  $\eta_i$  is the optical efficiency of the  $i$ th mirror. For the daily average output thermal power, this paper takes 9: 00, 10: 30, 12: 00, 13: 30, 15: 00 in the day as the representative, and the average value of the output thermal power at each time represents the average thermal power of the day. For the annual average output heat power, this paper uses the monthly average method to select one day as the representative day of the month in the 12 months of the year. The daily average output thermal power of the representative day is averaged again to approximately represent the annual average output thermal power of the heliostat field.

## 2.10. Model Solution

Through the above model, the calculation of the output thermal power of the heliostat field at this time can be realized. By calculating the output thermal power of each time on different dates, the daily average output thermal power and annual average output thermal power of the heliostat field can be calculated.

In this paper, the distributions of the output power of the heliostat field at 9:00, 10:30, 12:00, 13:30 and 15:00 on the equinoxes are obtained by calculation. In Fig 3, only the distribution of the output power of the heliostat field at 9:00 and 15:00 are shown.

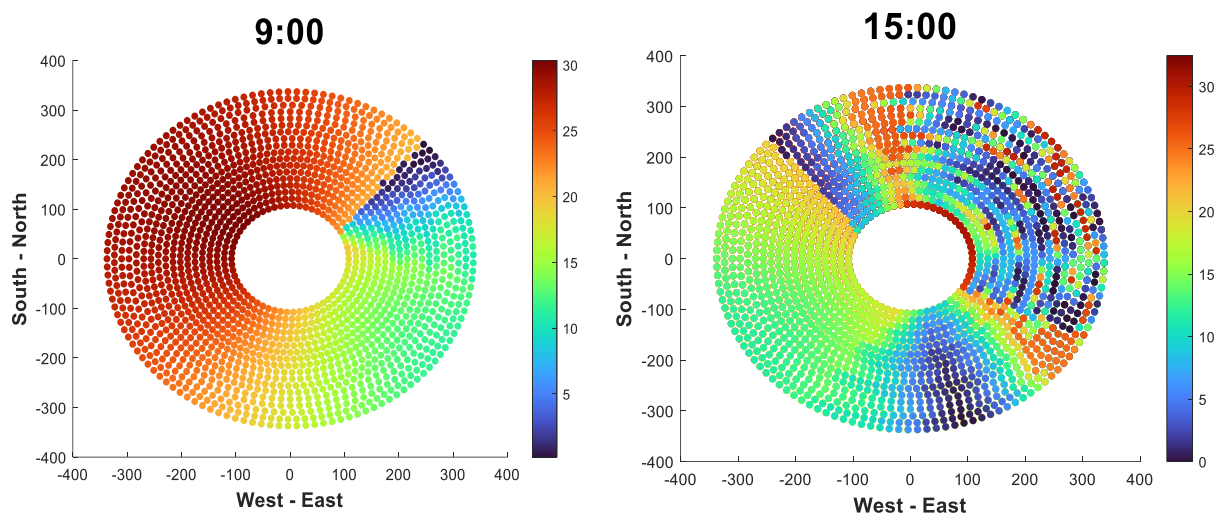


**Figure 3.** The distribution of output power of heliostat field at 9:00 and 15:00.

The average output power of the heliostat field at 9:00 and 15:00 shows that the output power of the heliostat close to the solar azimuth in the heliostat field is very small at any given time, while the output power of the heliostat far from the solar azimuth is large. The main reason is the existence of cosine loss. The heliostat near the solar direction in the heliostat field has a larger incident angle of sunlight, and its corresponding cosine loss is larger. The distribution of the annual average output power of the heliostat is shown in Fig 4(a), and the average output thermal power on the day of the 21st of each month varies with the month is shown in Fig 4(b).

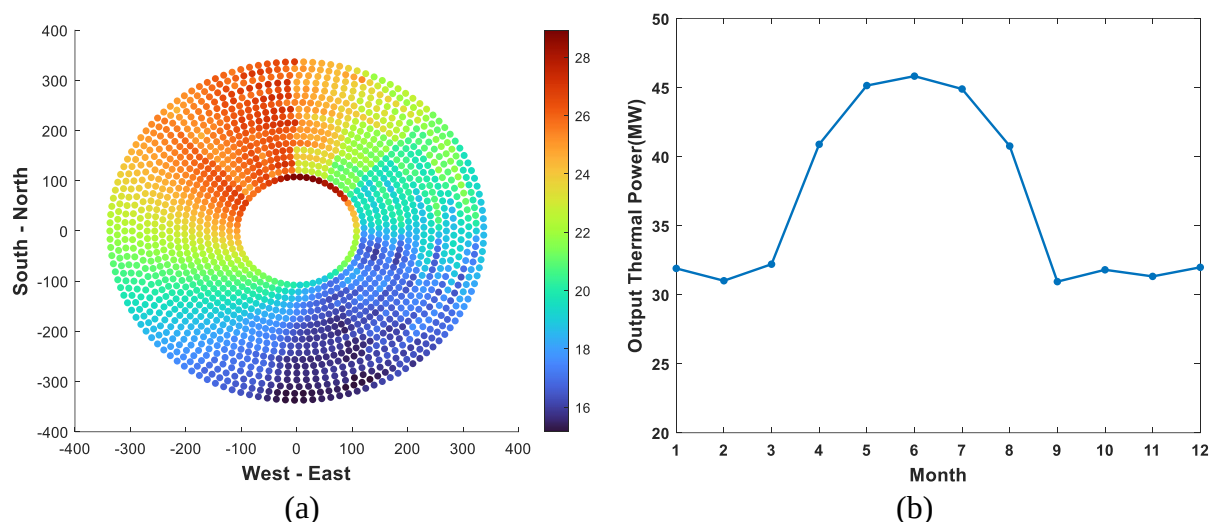


**Figure 4.** The distribution of the annual average output power of the heliostat and the average output thermal power of the heliostat on the day of the 21st of each month change with the month.

Through Fig.4, it can be seen that the larger value of the annual average output thermal power of the heliostat is mainly concentrated in the north-west position, and the reason for this distribution is the cosine loss. Because the sun rises from the east, turns to the south, and finally falls in the west, the incidence angle of the south heliostat is larger and the cosine loss is larger. According to the average output thermal power of the day on the 21st of each month, it can be found that the average output thermal power is larger in summer, which is due to the change of solar elevation angle and cosine loss. In summer, the solar elevation angle is larger, which makes the incidence angle of the heliostat smaller and the cosine loss smaller. The annual average optical efficiency and output power are shown in Table 1.

**Table 1.** Annual average optical efficiency and output power

| Interrelating Parameter                                                  | Value   |
|--------------------------------------------------------------------------|---------|
| Annual average optical efficiency                                        | 0.6216  |
| Annual average cosine efficiency                                         | 0.7533  |
| Annual average shadow occlusion efficiency                               | 0.9474  |
| Annual average truncation efficiency                                     | 0.9853  |
| Average annual output thermal power (MW)                                 | 36.3958 |
| Average annual output thermal power per unit area of mirror ( $Kw/m^2$ ) | 0.5794  |

### 3. Conclusion

In this paper, the thermal power of the heliostat is calculated by studying the grid division of the heliostat and the calculation of the shadow shading efficiency. After calculation, when the absorber tower is installed in the center of the circular heliostat, the size of the heliostat is a square with a side length of 6m, the installation height is 4m, and the location of the heliostat is determined, the annual average thermal output power is 36.3958 MW. Through this study, we get the annual average output power of the heliostat field in the conventional situation, which will play a role in determining the design problems of the subsequent heliostat field.

### References

- [1] Liu Jianxing. Modeling and simulation of optical efficiency of tower photothermal power station and optimal arrangement of heliostat field [D]. Lanzhou Jiaotong University, 2022.
- [2] Yang Hongtao. Research on heliostat concentrating model and optimal scheduling system [D]. Xi 'a University of Technology, 2023.
- [3] Cai Zhijie. Sun shadow positioning [J]. Mathematical Modeling and Its Applications, 2015, 4 (04): 25 – 33.
- [4] Yu Bingwen, Lou Guangguo, Cai Hongwei and so on. Several calculation methods and accuracy analysis of solar angle in optical observation [J]. Computer Measurement and Control, 2022, 30 (12): 306 - 318.
- [5] Zhang Xingqing, Lu Weiping, Li Yuhong, et al. Characteristics of maximum deviation of regional solar elevation angle [J]. Solar Journal, 2020, 41 (07): 196 - 201.
- [6] Zhuang Yanhao. Derivation of solar elevation angle and solar azimuth angle formula based on discipline integration——Taking the geography test of senior three in Jinhua ten schools in November 2020 as an example [J]. Geography teaching, 2022, (02): 38 - 40.
- [7] Zhang Ping, Xi Zhengwen, Hua Wenhan, etc. Calculation method of optical efficiency of solar tower photothermal mirror field [J]. Technology and market, 2021, 28 (06): 5 - 8.
- [8] Fang Miaosen, Lu Jing, Shen Zhigang. Modeling and application of front-end efficiency of tower solar heliostat [J]. Journal of Changzhou College of Information Technology, 2021, 20 (03): 20 - 24.
- [9] Gao Bo, Liu Jianxing, Sun Hao et al. Optimal arrangement of heliostat field based on adaptive gravitational search algorithm [J]. Journal of Solar Energy, 2022, 43 (10): 119 - 125.
- [10] Du Yuhang, Liu Xiangmin, Wang Xingping. Analysis of the influence of different focusing strategies of heliostat in tower photothermal power station [J]. Journal of Power Engineering, 2020, 40 (05): 426 - 432.