Research on survey line arrangement of a multibeam bathymetric system in ocean mapping

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Abstract. Submarine topography measurement technology is an important technology for exploring marine resources. Multi-beam bathymetry systems have high efficiency and important application value in underwater resource surveys, marine delimitation, and other fields. They are currently one of the most widely used measurement technologies. This study aims to optimize multi-beam bathymetry technology and apply it to seabed topography measurement, achieving a transition from point measurement to line measurement and minimizing measurement paths. A mathematical model was established for the coverage width and overlap rate between adjacent bands of multi-beam bathymetry at a given water depth based on planar geometric modeling and three-dimensional geometric modeling. The model was applied to a rectangular sea area with a length of 2 nautical miles (3704 meters) from north to south and a width of 4 nautical miles (7408 meters) from east to west. The optimal survey line planning was obtained as follows: all survey lines are in the north-south direction, with a total of 34 survey lines and a total length of 68 nautical miles. This model has good solution results for optimizing the layout of survey lines under different seabed conditions and can adjust important parameters of the system according to different conditions. The model has broad practical significance and can provide a reference for ocean-sounding work and other fields with high accuracy.

Keywords: Multi-beam sounding, geometric model, target planning model.

1. Introduction

With the gradual deepening of research on the structure of the Earth, ocean exploration has become an important research direction, and related technologies have developed rapidly in recent years. Among them, seabed topography measurement is a fundamental marine surveying work [1], which mainly constructs seabed topography and landforms by measuring the three-dimensional coordinates of seabed points. For underwater topography measurement, the measurement of water depth is particularly important. Currently, the main depth measurement technologies in the industry include single-beam depth measurement and multi-beam depth measurement. Compared to the limitations of traditional single-beam detectors, multi-beam detection systems can achieve full coverage and precise scanning of the seabed without dead angles, allowing users to obtain clear, comprehensive, and high-precision underwater microtopography and landforms [2]. In the process of multi-beam depth measurement, each measuring line will obtain a strip with a certain width, and the coverage width of the strip varies with the change of the transducer opening angle and water depth. To prevent missing measurements between bands, there is generally a certain degree of overlap between the bands [3]. To ensure the convenience of measurement and the integrity of data, the overlap rate between adjacent bands should be maintained between 10% and 20%. Therefore, setting up a reasonable route is a crucial aspect.

Ma et al. established a large-scale, high-resolution seabed DEM and implemented database management based on multi-beam sounding data using ArcGIS and GeoDatabas technologies [4]; Tian et al. obtained accurate water depth of the navigation channel using multi-beam depth measurement technology [5]; Xia et al. used a multi-beam depth measurement system to finely measure the bed terrain of a typical section of the Upper Jingjiang River for the first time. They used an improved sand wave morphology quantification algorithm to statistically analyze various sand wave morphology parameters and analyze the changing characteristics of sand wave morphology under different water flow intensities [6]; Zhao et al. combined the multi-beam system with unmanned ship technology and applied it in underwater terrain measurement, simplifying the installation and calibration process of sounding equipment and improving operational efficiency [7].

All of the above studies have investigated the application of multibeam bathymetry in different occasions, while the studies on the planning of survey lines when using multibeam bathymetry in different situations are relatively lacking. To analyze the relationship between the coverage of multibeam bathymetry and the routes in different situations, and to facilitate the application of multibeam bathymetry to more scenarios, this paper uses the data provided by the website https://cumcm.cnki.net to first analyze the coverage width of a single survey line along the direction of constant water depth at a certain moment for a non-flat seabed, and further study the overlap rate between equidistant routes. Then we take the center of a certain area size as the position of the survey vessel and study the coverage width of the survey vessel at different angles and distances by building a three-dimensional model. Finally, taking the length of the measuring line as the optimization objective, an optimization model is built, and the length and orientation of each measuring line under the shortest measuring line are finally obtained by analyzing the feasibility under different situations.

2. Two-dimensional geometric model

2.1. Width analysis

For a non-flat seabed, the coverage width of the strip is the horizontal distance between the intersection of the outermost beam and the seabed [8]. In this section, the construction process of the width model is analyzed using the example of the survey vessel on the left side of Figure 1.

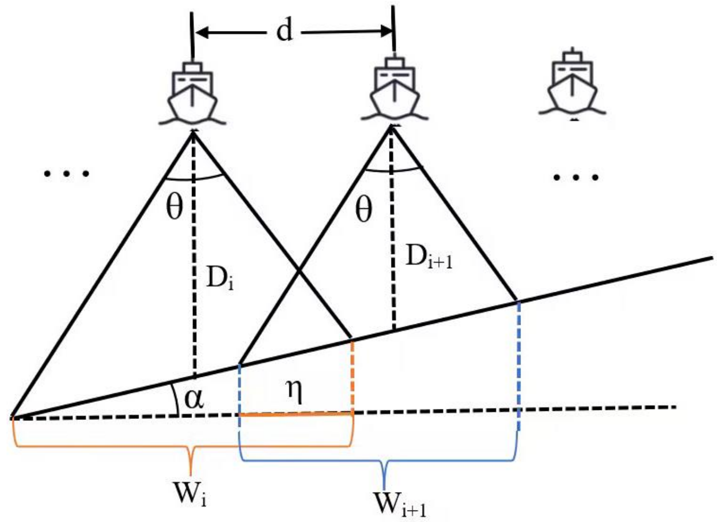

Figure 1. Schematic diagram of the calculation of the overlap rate between neighboring bands

(1) Relationship between angles and edges

If the angle of the multi-beam transducer is θ, the seabed slope is α, it can be obtained from the formula for the inner angle of a triangle

$$\begin{cases} \angle 1 = \frac{\pi-\theta}{2} - \alpha \\ \angle 2 = \frac{\pi-\theta}{2} + \alpha \end{cases} \tag{1}$$

By the sine theorem

$$\begin{cases} \frac{D_i}{\sin\angle 1} = \frac{W_{i,1}}{\sin\frac{\theta}{2}} \\ \frac{D_i}{\sin\angle 2} = \frac{W_{i,2}}{\sin\frac{\theta}{2}} \end{cases} \tag{2}$$

(2) Construction of the width model

According to the integration of formulas (1) and (2), the strip coverage width model is

$$W = W_1\cos\alpha + W_2\cos\alpha = D_i \cdot \cos\alpha \cdot \sin\frac{\theta}{2} \cdot \left(\frac{1}{\cos(\alpha-\frac{\theta}{2})} + \frac{1}{\cos(\alpha+\frac{\theta}{2})}\right) \tag{3}$$

2.2. Analysis of overlap rates

The calculation of the overlap rate needs to take the coverage width of the two sides of the survey line as a reference respectively, so the coverage width of the two strips needs to be calculated. According to the width model, the overlap rate model can be constructed as:[9]

$$\begin{cases} \eta_{i,1} = 1 - \frac{d}{W_i} \\ \eta_{i,2} = 1 - \frac{d}{W_{i+1}} \end{cases} \tag{4}$$

Where $\eta_{i,1}$ denotes the overlap rate when W takes W_i and $\eta_{i,2}$ denotes the overlap rate when W takes W_{i+1}.

3. Three-dimensional model

Since the coverage width of the survey vessel at any point along any direction in the sea area to be measured is different, here in this paper, only the survey line through the center point of the sea area is considered, and the schematic diagram is shown in Figure. 2.

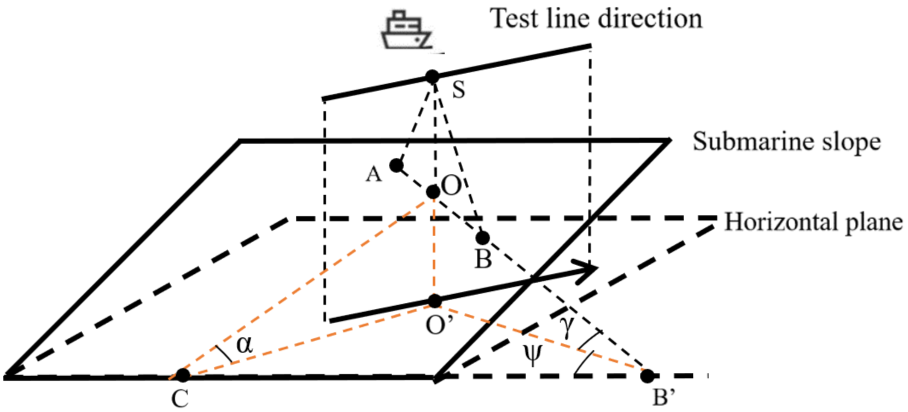

Figure 2. Three-dimensional model of the work of the survey ship

Assume that at a certain moment, the measuring ship reaches the position of point S. At this time, the intersection point of point S vertically downward with the seabed is set as O, and the intersection points of the outermost beam and the seabed are A and B, respectively, that is, the measuring line is perpendicular to the plane SAB. Extend SO, set the intersection of the extension line of SO and the dashed horizontal plane as O′, extend AB, set the intersection of the extension line of AB and the dashed horizontal plane as B′.

Take point C on the line of intersection of the submarine slope and the dashed horizontal so that the line B′C is perpendicular to the plane OO′C. It is easy to obtain that <OO′C = <OO′B′ = <O′CB′ $=\frac{\pi}{2}$

Let the dip of the seafloor slope, <OCO′ = α, <OB′O′ = β, <O′B′C = ψ, and <OB′O′ = γ. If the magnitude of γ can be found, the previous width model can be used.

(1) Analysis of the three-dimensional dimension

Analyzing RtΔOO′C, RtΔO′CB′ and RtΔOO′B′ we get:

$$\begin{cases} \tan\alpha = \frac{OO'}{O'C} \\ \sin\psi = \frac{O'C}{O'B'} \\ \tan\gamma = \frac{OO'}{O'B'} \end{cases} \tag{5}$$

Joining the three equations in formula (5), we get $\tan\gamma = \tan\alpha \cdot \sin\psi$.

(2) Analysis of the two-dimensional dimension

So to find out the magnitude of γ, we must first solve for the magnitude of ψ. The horizontal plane given in the question is analyzed, and the relationship between the vectors and the auxiliary lines is shown in Figure 3 (a). analysis, the relationship between the vectors and the auxiliary lines is shown in Figure 3 (a). Since the direction of vector $\overrightarrow{MN}$ is the same as the projection of the normal direction of the slope on the horizontal plane, MN ⊥ NC, and similarly, the direction of vector $\overrightarrow{MO'}$ is the same as the projection of the direction of the survey line on the horizontal plane, so MO′ is perpendicular to the plane SAB, and so MO′ ⊥ O′B′.

Combining these analyses, the angles in quadrilateral MNCO′ and triangle O′CB′ satisfy the following relations respectively:

$$\begin{cases} \beta + \angle MO'C = \pi \\ \psi + \angle B'O'C = \frac{\pi}{2} \end{cases} \tag{6}$$

Since MO′⊥ O′B′, it follows that:

$$\angle MO'C + \angle B'O'C = \frac{\pi}{2} \tag{7}$$

Substituting formula (7) into formula (6) yields:$\beta + \psi = \pi$.

Assuming that the depth of seawater at the center of the sea area is D_0, the depth of seawater here is set to be $D_{s,\beta}$ when the angle between the direction of the line of measurement and the normal direction of the seafloor slope is β in the horizontal plane and the distance of point S from the center of the sea area is s. The top view of the sea level is shown in Figure 3 (b).

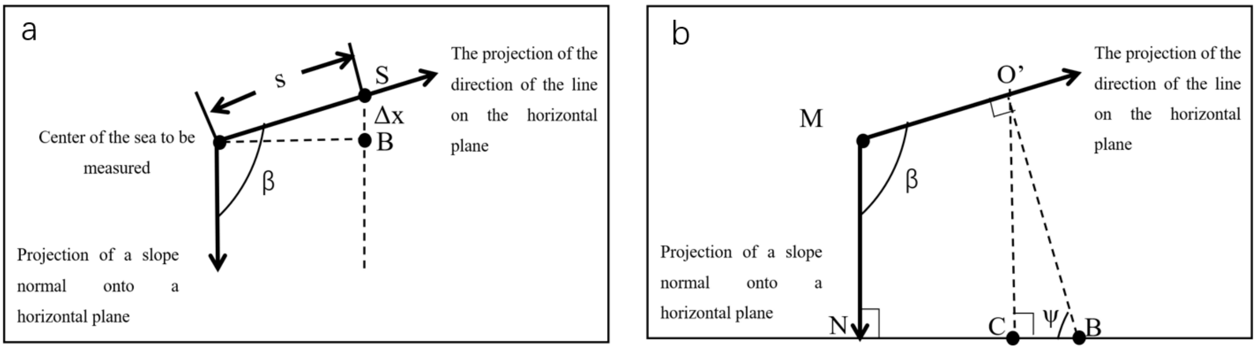

Figure 3. (a) Schematic of sea floor level; (b) Top view

The above figure shows that the displacement of S relative to the center point of the sea area in the direction of the projection of the normal direction of the slope along the bottom of the sea on the horizontal plane is:

$$\triangle x = s \cdot \cos\beta \tag{8}$$

Where, if $\cos\beta < 0$, denotes the direction of displacement in the opposite direction of the projection of the normal direction of the bottom slope along the horizontal plane.

Since the magnitude of the slope of the seabed slope is α, the depth of water at point S is:

$$D_{s,\beta} = D_0 + \triangle x \cdot \tan\alpha \tag{9}$$

Combined with the previously established width model, the three-dimensional model can be obtained as:

$$\begin{cases} W = D_{s,\beta} \cdot \cos\gamma \cdot \sin\frac{\theta}{2} \cdot \left(\frac{1}{\cos(\gamma-\frac{\theta}{2})} + \frac{1}{\cos(\gamma+\frac{\theta}{2})}\right) \\ \gamma = \arctan\left(\tan\alpha \cdot \sin(\pi-\beta)\right) \end{cases} \tag{10}$$

4. Line planning model

In the actual planning process of the survey line, it is necessary to always ensure that the overlap rate between neighboring strips is between 10% and 20%, so it is necessary to plan the navigation route of each survey line in turn according to the size of the overlap rate [10].

Assuming that the length of the ith measuring line is l_i, the total length of the measuring line is $\sum l_i$, and the spacing of the measuring line between the ith measuring line and the $i+1$th measuring line is d_i. Take the southwest corner of the rectangular area to be measured as the origin of coordinates, and take the direction of due east as the positive direction of the x-axis, and the direction of due north as the positive direction of the y-axis to set up the planar right-angle coordinate system. Since each measuring line is a straight line, let the function corresponding to the ith measuring line be

$$f_i(x) = k_i + b_i \tag{11}$$

Let the region that can be probed by the ith line be the set X_i and the region to be measured by the set U.

Consider a rectangular sea area with a north-south length of 2 nautical miles (3704 m) and an east-west width of 4 nautical miles (7408 m), with a seawater depth of 110 m at the center point of the sea area, deep in the west and shallow in the east, with a slope of 120°, a gradient of 1.5°, and a multi-beam transducer opening angle of 120°. In summary, the survey line planning model is established as follows:

The objective function is:

$$min\, Z = l_i \tag{12}$$

The constraints are:

① Overlap between neighboring bands is between 10 and 20 percent:

$$\begin{cases} \eta_{i,1} = \left(1 - \frac{d}{W_i}\right) \in [10\%, 20\%] \\ \eta_{i,2} = \left(1 - \frac{d}{W_{i+1}}\right) \in [10\%, 20\%] \end{cases} \tag{13}$$

② Survey ships can only work within the sea area to be surveyed:

$$\begin{cases} f_i(x) \in [0,3704] \\ x \in [0,7408] \end{cases} \tag{14}$$

③ Complete coverage of the entire sea area to be measured:

$$U \in (X_1 \cup X_2 \cup X_3 \cup \ldots) \tag{15}$$

In summary, the survey line planning model is:

$$min\ Z = l_i$$
$$s.t.\begin{cases}\eta_{i,1} = \left(1 - \frac{d}{W_i}\right) \in [10\%, 20\%] \\ \eta_{i,2} = \left(1 - \frac{d}{W_{i+1}}\right) \in [10\%, 20\%] \\ f_i(x) \in [0,3704] \\ x \in [0,7408] \\ U \in (X_1 \cup X_2 \cup X_3 \cup \dots)\end{cases} \quad (16)$$

5. Solving the line-planning model

The opening angle of the multibeam transducer is set to 120◦, the slope is set to be 15◦, and the depth of seawater at the center point of the sea area is set to 70 m. Using the geometrical model, the depth of seawater under different positions is obtained, and the coverage width of the survey line and the overlap rate with the previous survey line are shown in Table 1.

Table 1. Arithmetic example results for geometric model solving

Distance of the line from the center point /m	depth of sea /m	Coverage width /m	Overlap with previous line/%
-800	90.95	315.71	-
-600	85.71	297.53	32.78
-400	80.47	279.35	28.40
-200	75.24	261.17	23.42
0	70	242.99	17.69
200	64.76	224.81	11.04
400	59.53	206.63	3.21
600	54.29	188.45	-6.13
800	49.05	170.27	-17.46

The original setting is unchanged, now based on the original data, the three-dimensional model is utilized to solve the coverage width of the survey line under different survey line inclinations and different positions, and the calculation results are shown in Table 2.

Table 2. Arithmetic example results of the three-dimensional model solution

Coverage width /m		Measurement of the distance of the ship from the center point of the sea area mile							
		0	0.3	0.6	0.9	1.2	1.5	1.8	2.1
Measuring line direction angle /°	0	415.69	466.09	516.49	566.89	617.29	667.69	718.09	768.48
	45	416.12	451.79	487.47	523.14	558.82	594.49	630.16	665.84
	90	416.55	416.55	416.55	416.55	416.55	416.55	416.55	416.55
	135	416.22	380.45	344.77	309.10	273.42	237.75	202.08	166.40
	180	415.69	365.29	314.89	264.50	214.10	163.70	113.30	62.90
	225	416.12	380.45	344.77	309.10	273.42	237.75	202.08	166.40
	270	416.55	416.55	416.55	416.55	416.55	416.55	416.55	416.55
	315	416.12	451.79	487.47	523.14	558.82	594.49	630.16	665.84

In the rectangular sea area considered in 4, it is necessary to determine the direction of the survey line, which is broadly categorized into the following three cases:

① The measured line is in the north-south direction

When the direction of the survey line is north-south, the vertical distance from the same survey line to the seabed remains constant when the survey vessel works along each line, and then the first survey line needs to be planned from the easternmost or westernmost. Then plan the second line according to the probe range and overlap rate of the first line, and so on, as shown in Figure 4(a).

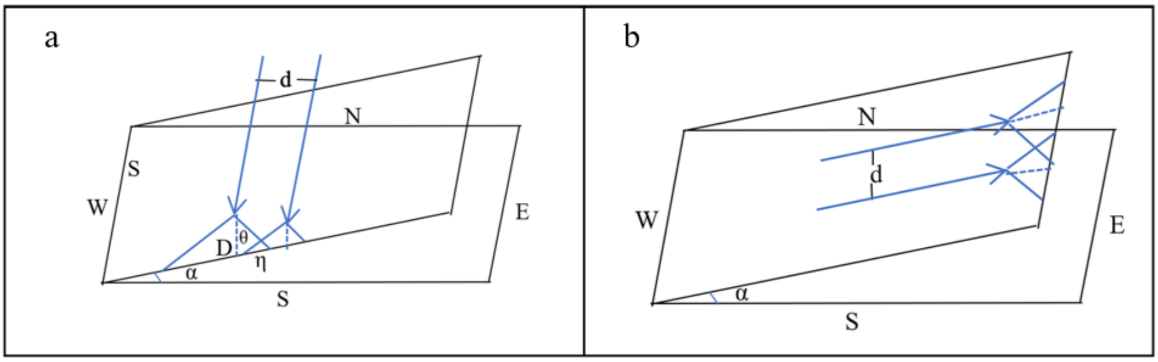

Figure 4. (a) Schematic of north-south alignment (b): Schematic of east-west alignment

To ensure that the final total length of the measurement line is the shortest, so set the overlap rate between the measurement lines is 10%, the results are shown in Table 3 and Table 4.

Table 3. Plot of lines where the first line is located in the westernmost part of the line

line number	The x-coordinate of the measurement line	x-coordinate corresponding to the western boundary of the coverage area	x-coordinate corresponding to the eastern boundary of the coverage area	Coverage width (m)
1	358.52	0.00	685.93	685.93
2	947.86	617.34	1249.70	632.36
…	…	…	…	…
33	7345.43	7318.85	7369.69	50.84
34	7389.11	7364.61	7411.48	46.87

Table 4. Line plan with the first line in the easternmost part of the line

line number	The x-coordinate of the measurement line	x-coordinate corresponding to the western boundary of the coverage area	x-coordinate corresponding to the eastern boundary of the coverage area	Coverage width（m）
1	7385.47	7360.80	7408.00	47.20
2	7341.48	7314.72	7365.92	51.20
…	…	…	…	…
33	898.81	565.96	1202.78	636.82
34	305.32	-55.73	635.04	690.77

The corresponding line planning diagrams are shown in Figure 5 and Figure 6 respectively.

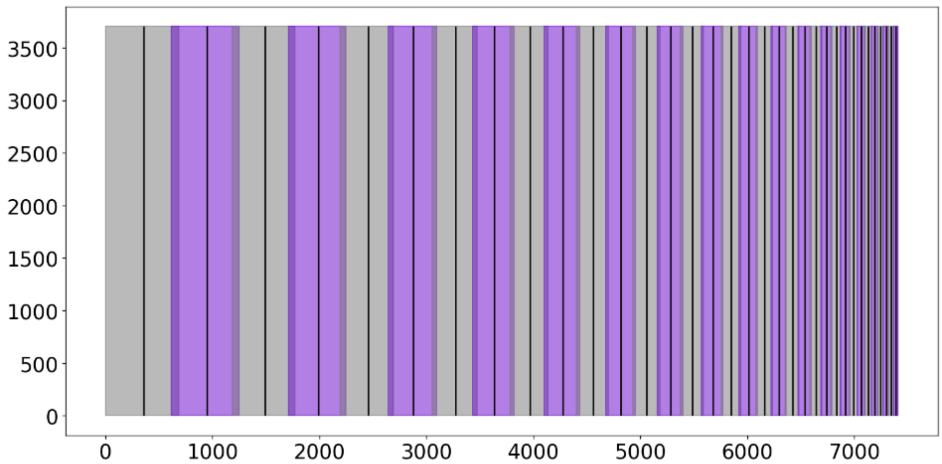

Figure 5. Plot of the first line in the westernmost part of the line

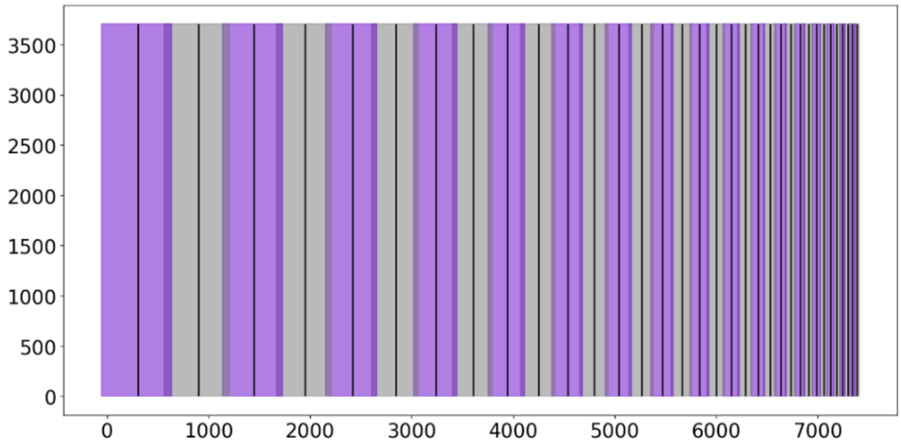

Figure 6. Line plan with the first line in the Far East

② The measured line is in an east-west direction

The survey line is planned as shown in Figure (4) b. The width of coverage of the strip is calculated to be 717.04 m in the westernmost part and 45.06 m in the easternmost part.

(1) If the overlap rate of neighboring lines in the easternmost part is 10%, the distance d between two neighboring lines is 40.55m. At this time, the overlap rate of neighboring lines in the westernmost part is 94.34%>20%, which does not meet the meaning of the question, so it is discarded.

(2) If the overlap rate of neighboring lines in the easternmost part is 20%, the distance d between two neighboring lines is 36.05m. At this time, the overlap rate of neighboring lines in the westernmost part is 94.97%>20%, which is not in line with the meaning of the question, so it is discarded.

(3) If the overlap rate of neighboring lines in the westernmost part is 10%, the spacing d between two neighboring lines is 645.34 m. Since the coverage width of the strip in the easternmost part is 45.06 m < the spacing between two neighboring lines, η < 0, the omission occurs, which does not conform to the meaning of the question, and it is discarded.

(4) If the overlap rate of adjacent lines in the westernmost part of the strip is 20%, then the spacing d between two adjacent lines is 573.63 m. Since the width of the strip in the easternmost part of the strip is 45.06 m < the spacing between two adjacent lines, η < 0, there is a leakage of measurements, which is not in line with the meaning of the question, so it is discarded.

Therefore, the requirement is not satisfied when the survey line is in the east-west direction.

The line of sighting is in the other direction

When the direction of the measuring line is neither parallel nor coincident with the x-axis nor with the y-axis, as shown in Figure 7.

Where the hexagon $A_1M_1A_2B_2N_1B_1$ is the coverage of the measurement line l_i (beyond the rectangular to-be-measured area is ignored). In order not to have a missed measurement, the coverage of the next measurement line l_{i+1} must include the point N_2, which is the boundary line C_1D_1 at the lower right of the coverage of l_{i+1}, and the point D_1 coincides with the point N_2. At this point, the overlap rate $\eta_{i,1}$=50%>20% between the strips corresponding to the survey line l_i and the survey line l_{i+1} is not in line with the meaning of the question rounded off.

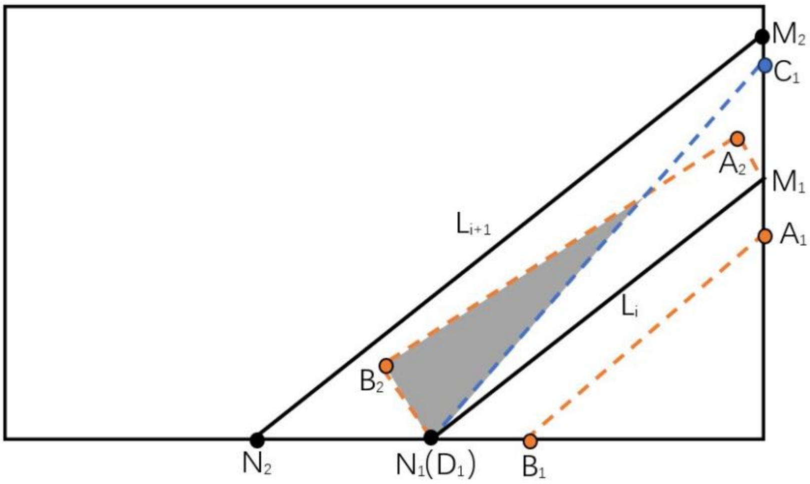

Figure 7. Schematic diagram of line measurement in other directions

In summary, the optimal line plan is: all lines are in a north-south direction, with a total of 34 lines and a total line length of 68 nautical miles (125,936 m).

6. Conclusion

In this paper, by analyzing the multibeam transducer opening angle, seabed slope line direction, the model of line coverage width and line planning model is established, according to the characteristics of the seabed in the sea area, analyzing the situation corresponding to different line directions, and designing the shortest line program, which provides a reference program for a certain range of multibeam surveying. In real life, this paper can not only bring help to the seabed topographic survey but also provide a certain degree of reference when using UAVs to photograph unknown areas, even mountainous areas.

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